18.701 Algebra I
Fall 2007

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Practice Quiz 3

(This is last year’s quiz.)

1. (15 points) Let $A, B$ be positive definite real symmetric matrices. Which of the following matrices are positive definite symmetric: $A^2, A^{-1}, AB, A + B$?

2. (20 points) Let $W$ be the subspace of $\mathbb{R}^3$ spanned by the vectors $(1, 1, 0)^t$ and $(0, 1, 1)^t$. Determine the orthogonal projection of the vector $e_1 = (1, 0, 0)^t$ to $W$.

3. (20 points) Let $A = R + Si$ be a hermitian matrix, with $R, S$ real.
   (i) Show that $R$ is symmetric and that $S$ is skew-symmetric.
   (ii) Show that if $A$ is a positive definite hermitian matrix, then $R$ is a real positive definite symmetrix matrix.

4. (15 points) What does the spectral theorem for normal operators say about the conjugacy classes in the unitary group $U_n$?

5. (15 points) Determine the type of the conic $x^2 - 4xy + 4y^2 + 3x - 2y - 2 = 0$.

6. (15 points) Let $G$ be the group of upper triangular real $n \times n$ matrices with diagonal entries 1. Determine the 1-parameter groups in $G$. Prove your assertions.