Advanced Topics in Abstract Algebra
Math 314.001 – Spring 2012
Homework Assignment for Chapter 15, Due (2/27)

- Work book exercises from Chapter 15: 2, 4*, 5+, 9#, 10*, 11.

(*) Indicates an extra credit problem.
(\textdagger) Use Mathematica to check your work.
(+ ) Hint: It would be helpful to have completed Exercise 4; alternatively, write everything in terms of elementary symmetric polynomials.
(#) You may assume the results of Exercise 8.

(S1) \textit{(Here we define Schur polynomials)} As introduced in class, let \( x \) denote the set \( \{ x_1, x_2, \ldots, x_n \} \). Given a sequence \( \alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n) \) of integers, define the polynomial \( D_\alpha(x) \) by

\[
D_\alpha(x) = \begin{vmatrix}
x_1^{\alpha_1} & x_1^{\alpha_2} & \cdots & x_1^{\alpha_n} \\
x_2^{\alpha_1} & x_2^{\alpha_2} & \cdots & x_2^{\alpha_n} \\
x_3^{\alpha_1} & x_3^{\alpha_2} & \cdots & x_3^{\alpha_n} \\
\vdots & \vdots & \ddots & \vdots \\
x_n^{\alpha_1} & x_n^{\alpha_2} & \cdots & x_n^{\alpha_n}
\end{vmatrix}.
\]

Let \( \rho = (n - 1, n - 2, \ldots, 1, 0) \). Given any sequence \( \lambda = (\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n) \) of nonnegative integers, define the rational function \( S_\lambda(x) \) by

\[
S_\lambda(x) = \frac{D_{\lambda + \rho}(x)}{D_\rho(x)},
\]

where “\( \rho + \lambda \)” indicates vector (component-wise) addition of the sequences.

Prove that \( a) \) \( S_\lambda(x) \) is a symmetric function, and \( b) \) it is actually a polynomial (not a more general rational function)\(^1\)

(S2) \textit{(Here we prove Newton’s Identities)} Use the formal power series \( S(t) \), \( H(t) \), and \( P(t) \) introduced in class to prove Theorem 15.6 in the text.\(^2\)

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\(^1\) The results of Exercises 10 and 11 are essential here.

\(^2\) This is the Newton. He found these identities in 1666, but they were known earlier to Girard (1629). I am not sure when the proof you are trying to recreate was first written down. Probably multiple centuries later.