1. (1 pt) Library/Rochester/setLinearAlgebra14TransfOfRn-ur_la_14_18.pg

Let $L$ be the line in $\mathbb{R}^3$ that consists of all scalar multiples of the vector $\begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$. Find the orthogonal projection of the vector $v = \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}$ onto $L$.

$$\text{proj}_Lv = \begin{bmatrix} \text{answer} \\ \text{answer} \\ \text{answer} \end{bmatrix}.$$ 

Answer(s) submitted:
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- (incorrect)

2. (1 pt) Library/Rochester/setLinearAlgebra21InnerProductSpaces-ur_la_21_9.pg

Let $M_1 = \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}$ and $M_2 = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}$.

Consider the inner product $\langle A, B \rangle = \text{trace}(A^T B)$ in the vector space $\mathbb{R}^{2 \times 2}$ of $2 \times 2$ matrices. Use the Gram-Schmidt process to determine an orthonormal basis for the subspace of $\mathbb{R}^{2 \times 2}$ spanned by the matrices $M_1$ and $M_2$.

$$\begin{bmatrix} \text{answer} & \text{answer} \\ \text{answer} & \text{answer} \end{bmatrix}.$$ 

Answer(s) submitted:
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- (incorrect)

3. (1 pt) Library/Rochester/setLinearAlgebra21InnerProductSpaces-ur_la_21_4.pg

If $f(x)$ and $g(x)$ are arbitrary polynomials of degree at most 2, then the mapping $\langle f, g \rangle = f(-2)g(-2) + f(0)g(0) + f(3)g(3)$ defines an inner product in $P_2$.

Use this inner product to find $\langle f, g \rangle$, $\|f\|$, $\|g\|$, and the angle $\alpha_{f,g}$ between $f(x)$ and $g(x)$ for $f(x) = 3x^2 + 6x + 9$ and $g(x) = 3x^2 - 6x - 6$.

$$\langle f, g \rangle = \text{answer}, \quad \|f\| = \text{answer}, \quad \|g\| = \text{answer}, \quad \alpha_{f,g} = \text{answer}.$$ 

Answer(s) submitted:
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- (incorrect)

4. (1 pt) Library/Rochester/setLinearAlgebra21InnerProductSpaces-ur_la_21_11.pg

Let $f(x) = -7$, $g(x) = -3x - 6$, and $h(x) = -7x^2$.

Consider the inner product $\langle p(x), q(x) \rangle = \int_0^4 p(x)q(x)dx$ in the vector space $C^0[0,1]$. Use the Gram-Schmidt process to determine an orthonormal basis for the subspace of $C^0[0,1]$ spanned by the functions $f(x)$, $g(x)$, and $h(x)$.

$$\langle f, g, h \rangle = \text{answer}, \quad \|f\| = \text{answer}, \quad \|g\| = \text{answer}, \quad \|h\| = \text{answer}, \quad \alpha_{f,g} = \text{answer}, \quad \alpha_{f,h} = \text{answer}, \quad \alpha_{g,h} = \text{answer}.$$ 

Answer(s) submitted:
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- (incorrect)

5. (1 pt) local/Library/Rochester/setLinearAlgebra17DotProductRn-ur_la_17_4.pg

Let $W$ be the subspace of $\mathbb{R}^3$ spanned by the vectors $\begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 6 \\ -15 \\ 4 \end{bmatrix}$. Find the matrix $A$ of the orthogonal projection onto $W$ with respect to the standard basis of $\mathbb{R}^3$.

$$A = \begin{bmatrix} \text{answer} & \text{answer} & \text{answer} \\ \text{answer} & \text{answer} & \text{answer} \\ \text{answer} & \text{answer} & \text{answer} \end{bmatrix}.$$ 

Answer(s) submitted:
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- (incorrect)
Suppose \( v_1, v_2, v_3 \) is an orthogonal set of vectors in \( \mathbb{R}^3 \). Let \( w \) be a vector in \( \text{Span}(v_1, v_2, v_3) \) such that
\[
\begin{align*}
v_1 \cdot v_1 &= 30, v_2 \cdot v_2 = 99, v_3 \cdot v_3 = 16, \\
w \cdot v_1 &= -30, w \cdot v_2 = -495, w \cdot v_3 = 32, \\
\end{align*}
\]
then \( w = \underline{v_1} + \underline{v_2} + \underline{v_3} \).

\[ \text{Answer(s) submitted:} \]
\[ \bullet \]
(incorrect)

7. (1 pt) Library/TCNJ/TCNJ_OogonalProjections/problem4.pg
All vectors and subspaces are in \( \mathbb{R}^n \).

Check the true statements below:

- A. The orthogonal projection \( \hat{y} \) of \( y \) onto a subspace \( W \) can sometimes depend on the orthogonal basis for \( W \) used to compute \( \hat{y} \).
- B. If \( y \) is in a subspace \( W \), then the orthogonal projection of \( y \) onto \( W \) is \( y \) itself.
- C. If the columns of an \( n \times p \) matrix \( U \) are orthonormal, then \( UU^T y \) is the orthogonal projection of \( y \) onto the column space of \( U \).
- D. If \( z \) is orthogonal to \( u_1 \) and \( u_2 \) and if \( W = \text{Span}\{u_1, u_2\} \), then \( z \) must be in \( W^\perp \).
- E. For each \( y \) and each subspace \( W \), the vector \( y - \text{proj}_W(y) \) is orthogonal to \( W \).

\[ \text{Answer(s) submitted:} \]
\[ \bullet \]
(incorrect)

8. (1 pt) Library/TCNJ/TCNJ_OogonalProjections/problem8.pg
Find the minimal distance from the point \( P = \begin{bmatrix} -10 \\ -4 \\ -8 \end{bmatrix} \) to the plane \( V \) of \( \mathbb{R}^3 \) spanned by \( \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} \) and \( \begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix} \).

The minimal distance is \underline{\underline{}}.

\[ \text{Answer(s) submitted:} \]
\[ \bullet \]
(incorrect)

9. (1 pt) Library/TCNJ/TCNJ_OogonalProjections/problem5.pg
All vectors and subspaces are in \( \mathbb{R}^n \).

Check the true statements below:

- A. If \( y = z_1 + z_2 \), where \( z_1 \) is in a subspace \( W \) and \( z_2 \) is in \( W^\perp \), then \( z_1 \) must be the orthogonal projection of \( y \) onto \( W \).
- B. The best approximation to \( y \) by elements of a subspace \( W \) is given by the vector \( y - \text{proj}_W(y) \).
- C. In the Orthogonal Decomposition Theorem, each term \( \hat{y} = \frac{y \cdot u_1}{u_1 \cdot u_1} u_1 + \ldots + \frac{y \cdot u_p}{u_p \cdot u_p} u_p \) is itself an orthogonal projection of \( y \) onto a subspace of \( W \).
- D. If an \( n \times p \) matrix \( U \) has orthonormal columns, then \( U^TUx = x \) for all \( x \) in \( \mathbb{R}^n \).
- E. If \( W \) is a subspace of \( \mathbb{R}^n \) and if \( y \) is in both \( W \) and \( W^\perp \), then \( y \) must be the zero vector.

\[ \text{Answer(s) submitted:} \]
\[ \bullet \]
(incorrect)

10. (1 pt) Library/TCNJ/TCNJ_LenghOrthogonality/problem1.pg
All vectors are in \( \mathbb{R}^n \).

Check the true statements below:

- A. If the distance from \( u \) to \( v \) is equal to the distance from \( u \) to \( -v \), then \( u \) and \( v \) are orthogonal.
- B. If vectors \( v_1, \ldots, v_p \) span a subspace \( W \) and if \( x \) is orthogonal to each \( v_j \) for \( j = 1, \ldots, p \), then \( x \) is in \( W^\perp \).
- C. For a square matrix \( A \), vectors in \( \text{Col}A \) are orthogonal to vectors in \( \text{Nul}A \).
- D. \( v \cdot v = \|v\|^2 \).
- E. For any scalar \( c \), \( u \cdot (cv) = c(u \cdot v) \).

\[ \text{Answer(s) submitted:} \]
\[ \bullet \]
(incorrect)

11. (1 pt) Library/TCNJ/TCNJ_LenghOrthogonality/problem6.pg
Find the angle \( \alpha \) between the vectors \( \begin{bmatrix} 2 \\ -2 \end{bmatrix} \) and \( \begin{bmatrix} 2 \\ -1 \end{bmatrix} \).

\[ \alpha = \underline{\underline{}}. \]

\[ \text{Answer(s) submitted:} \]
\[ \bullet \]
(incorrect)