

Totally monotonic sequences
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An elegant inequality of Mercer,

$$\frac{1}{2^n} \sum_{k=0}^n \binom{n}{k} x_k \leq \frac{1}{n+1} \sum_{k=0}^n x_k,$$

valid for all convex sequences (x_0, x_1, \dots, x_n) , leads very naturally to nine conjectures on totally monotonic sequences.

To explain how these conjectures arise, we first present a brief history of the subject: especially Hausdorff's groundbreaking work (Moment sequences, Summability Theory) and the surprising contributions of Hardy (Inequalities) and Rhoades (Operator Theory).