Parametric Summability and Its Applications

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Abstract. We study summability based on double sequences of complex constants as it is defined in "Linear Operators, General Theory" by Neilson Dunford, Jacob T. Schwartz. We define "power double sequence" or infinite "power matrix" as a certain generalization of double sequence and power series as follows.

Let $A = \{ a_{ij} \}$, $i = 0, 1, 2, \ldots , j = 0, 1, 2, \ldots$, be a double sequence of constants.

Let $f(z) = \sum_{j=0}^{\infty} b_j z^j$ and $g(z) = \sum_{i=0}^{\infty} c_i z^i$ be functions f and g with their Taylor series.

The *power double sequence* of first type induced by A and associated with $f(z)$ is defined as

$$P_{A;f}(z) = \{ a_{ij} b_{i+j} z^{i+j} \},\ i = 0, 1, 2 \ldots , j = 0, 1, 2, \ldots$$

The *power double sequence* of second type induced by A and associated with $f(z)$ and $g(z)$ is defined as

$$P_{A;f,g}(z) = \{ a_{ij} b_j c_i z^{i+j} \},\ i = 0, 1, 2 \ldots , j = 0, 1, 2, \ldots$$

We relate the summability of the power double sequences $P_{A;f}(z)$ and $P_{A;f,g}(z)$ to the summability and other properties of the double sequence A and functions f and g.

While others do investigate "power matrices" their definitions, as far as we were able to find, differ from our definition. Using this definition we extend some summability results for double sequences of constants to our power double sequences.

We investigate also other possible generalizations of double sequences and assess their usefulness in summability study.

Keywords. Double sequence, Summability, Power double sequence, Power matrix.

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