Asymptotically Equivalent Matrices in an Ideal Setting.

Jeff Connor* and Hafize Gök**

*Ohio University, Department of Mathematics, Athens, Ohio, USA
connorj@ohio.edu

**Afyonkarahisar Kocatepe University, Department of Mathematics,
Afyonkarahisar/TURKEY
hafize_1409@hotmail.com

Two sequences x and y are defined to be asymptotically equivalent if $\lim_k (x_k/y_k) = 1$ and a summability matrix is defined to be *asymptotically regular* if it maps nonnegative asymptotically equivalent sequences to asymptotically equivalent sequences. Pobyvanets(1980) introduced asymptotically equivalent matrices and provided a characterization of nonnegative asymptotically equivalent matrices. More recently Marouf(1993), Li(1997), Patterson(2003), and Patterson and Savas (2009) have explored variations of the definition of asymptotically equivalent sequences and studied nonnegative matrices which preserve these variations of asymptotic equivalence. Given an ideal I of subsets of $\mathbb{N}$ that contains the finite subsets of $\mathbb{N}$, a sequence is said to be I-convergent to l provided $\{k : |x_k - l| \geq \varepsilon\} \in I$ for all $\varepsilon > 0$, and two nonnegative sequences x and y are said to be $I-$asymptotically equivalent provided x_n/y_n is I-convergent to 1. In this note, given ideals I and J, we characterize nonnegative matrices which map I-asymptotically equivalent sequences to J-asymptotically equivalent sequences and establish similar results analogous to those of Marouf and Li.

To date, characterizations of matrices that preserve asymptotic equivalence (or its variants) are typically restricted to nonnegative matrices and nonnegative sequences, where in some cases the sequences are bounded below by a positive number. We also discuss the necessity of these hypotheses.