Some combinatorics of $\widehat{\mathfrak{sl}}_n$ crystals

(different models for $\widehat{\mathfrak{sl}}_n$ crystals and how they are related) [1]

Peter Tingley

Massachusetts Institute of Technology

Korea, September 2009

[1] Notes "Explicit crystal maps between cylindric plane partitions, multi-partitions and multi-segments" are available at www-math.mit.edu/∼ptingley/

Outline

1. Motivation
   - Crystals, Characters and Combinatorics
   - What does “understand" mean anyway?
   - Two examples
2. Some structures I understand
   - The multi-partition realization of $B(\Lambda)$
   - Understanding the infinity crystal
   - Relationship with the Kyoto path model
3. A structure I only partly understand
   - Fayers’ crystals
   - Relationship with monomial crystals (partly conjectural)

The adjoint representation of $\mathfrak{sl}_3$

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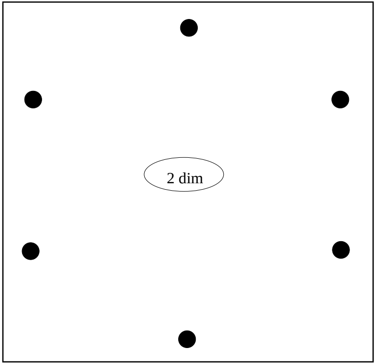

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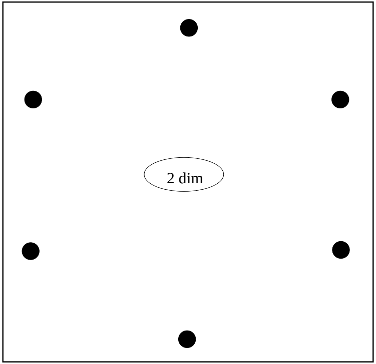

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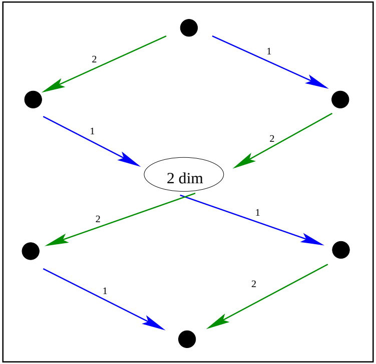

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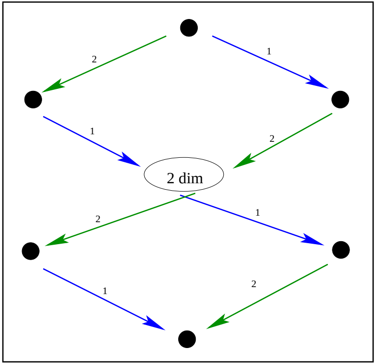

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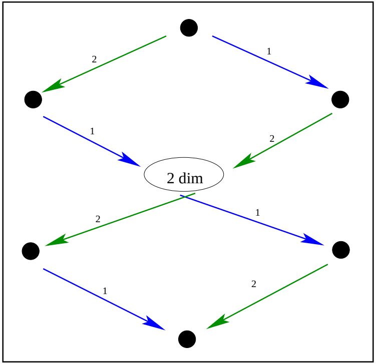

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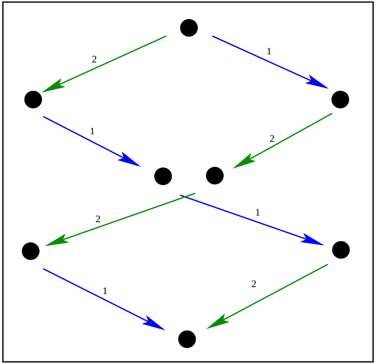

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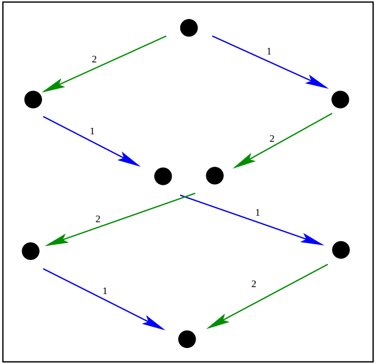

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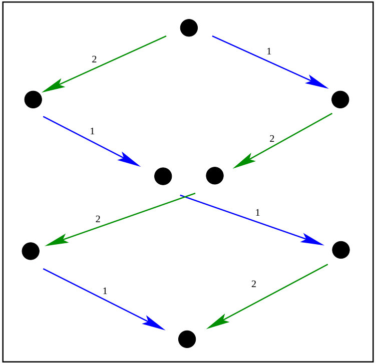

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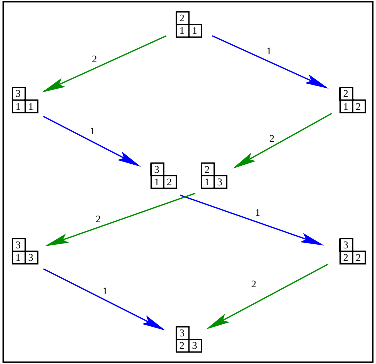

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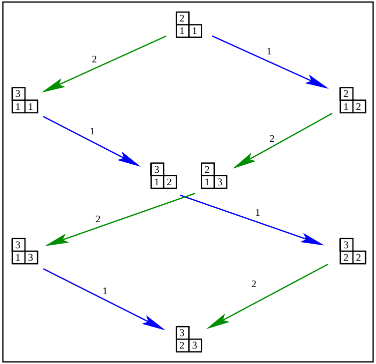

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- Then the combinatorics gives information about representation theory, and vise-versa.

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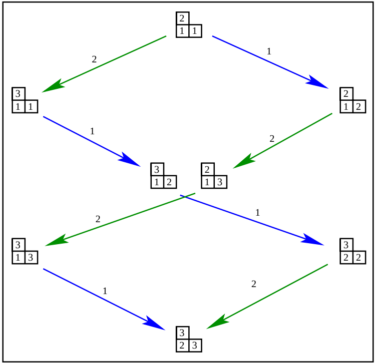

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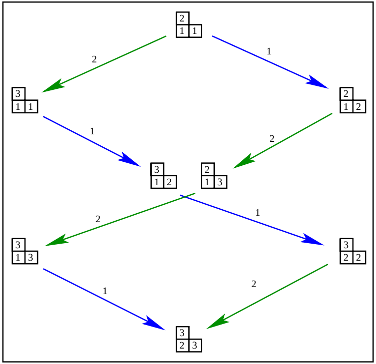

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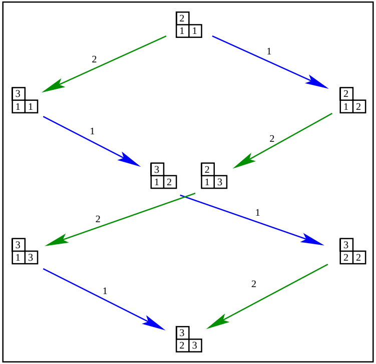

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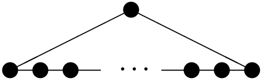

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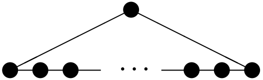

- $\widehat{\mathfrak{sl}}_n$ is (almost) generated by $\{E_i, F_i\}_{0 \leq i \leq n-1}$ subject to the relations that for each pair $0 \leq i < j \leq n-1$, $\{E_i, F_i, E_j, F_j\}$ generate a copy of

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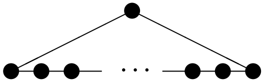

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- Fix $n \geq 3$. An (infinite) n-colored directed graph is an $\widehat{\mathfrak{sl}}_n$ crystal if, for each pair of colors c_i and c_j, the graph consisting of all edges of those 2 colors is

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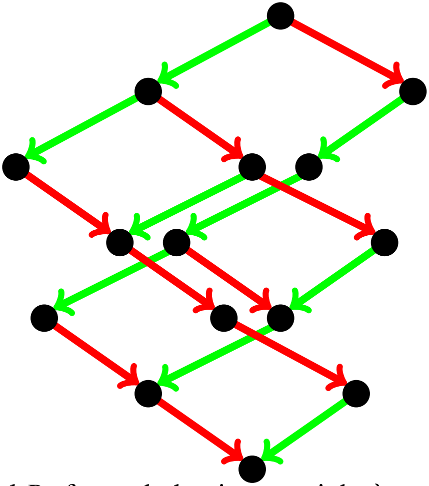

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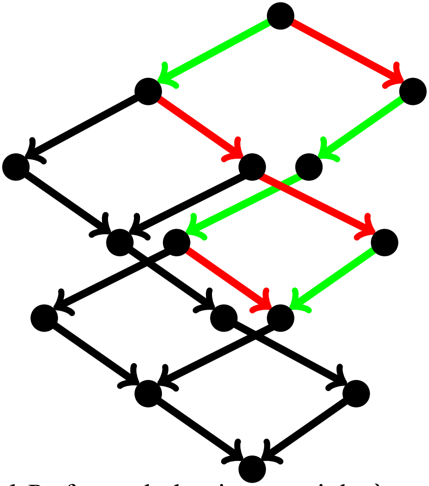

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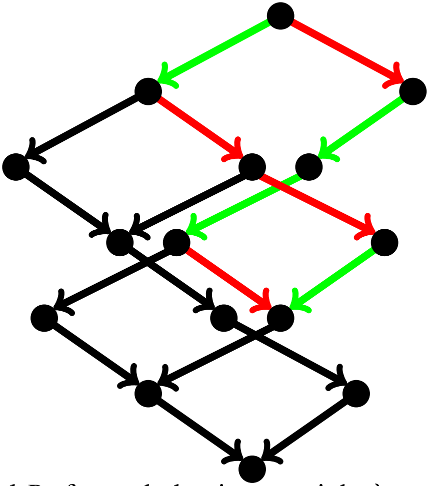

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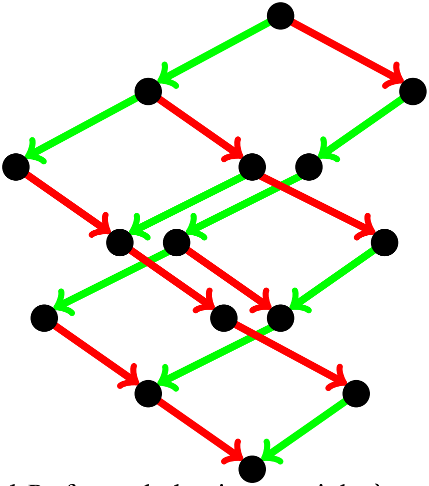

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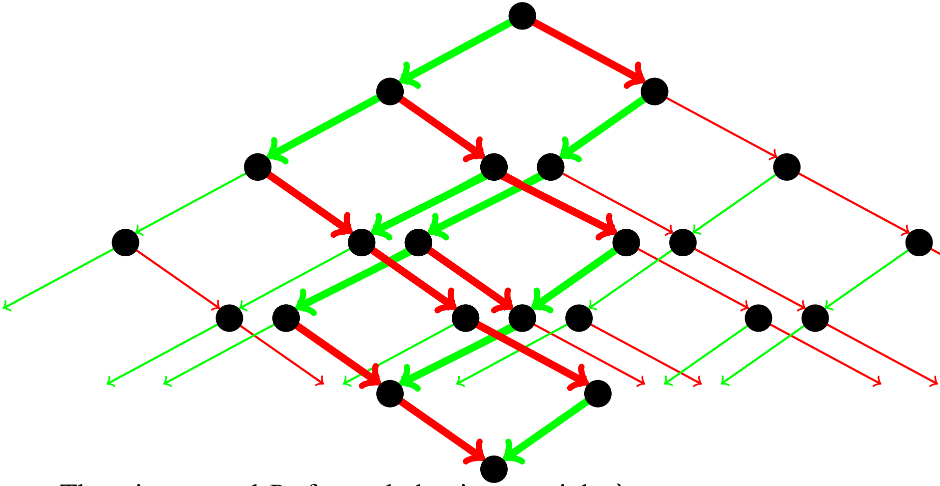

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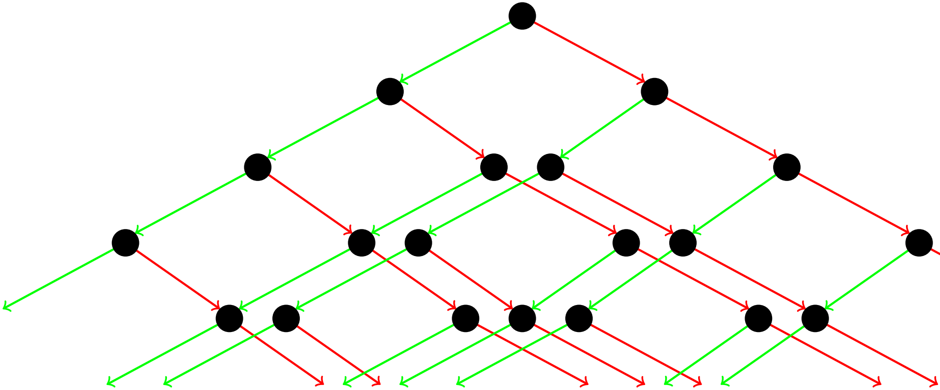

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The Misra-Miwa-Hayashi realizarion of B_{Λ_0} for $\widehat{\mathfrak{sl}}_3$

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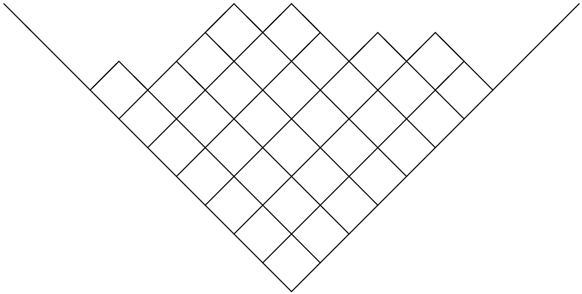

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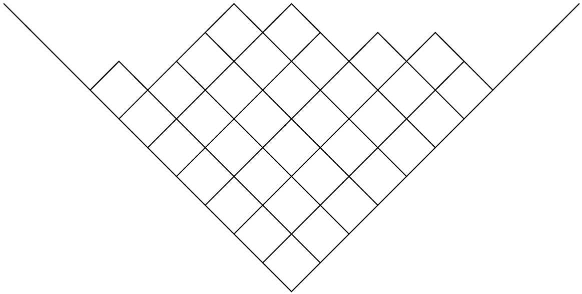

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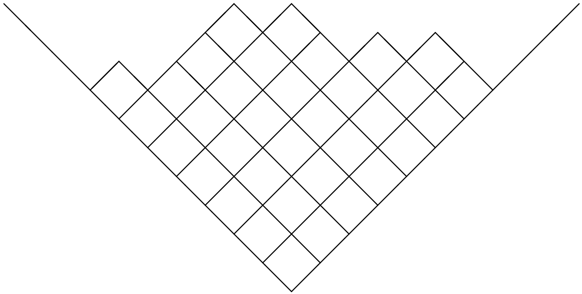

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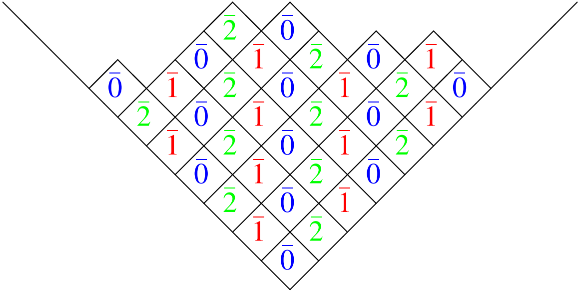

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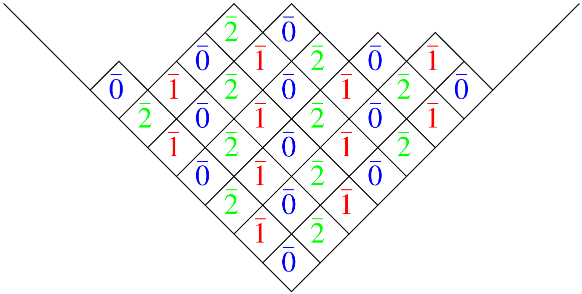

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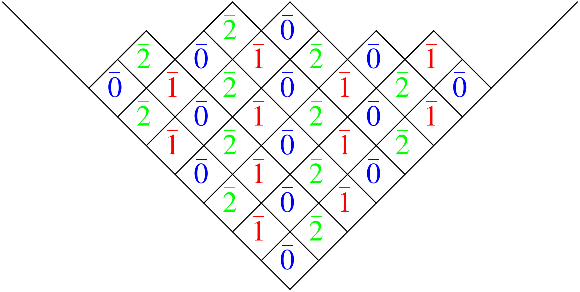

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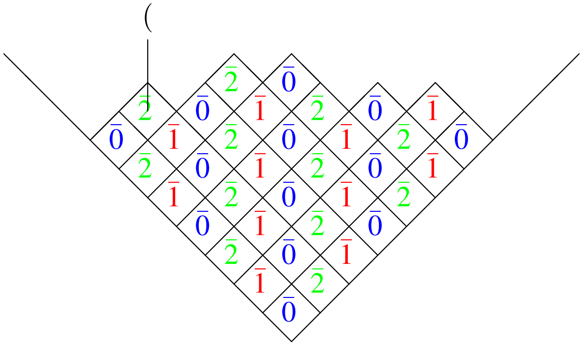

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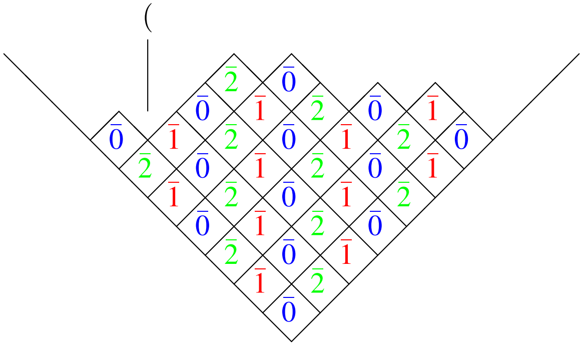

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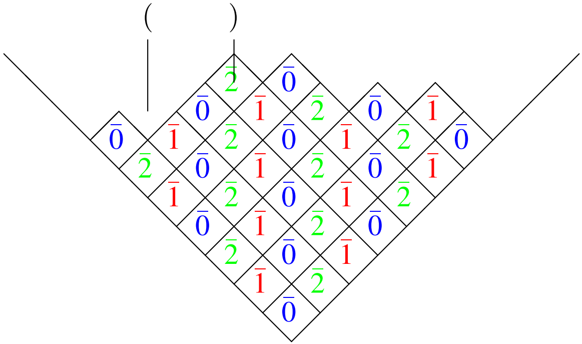

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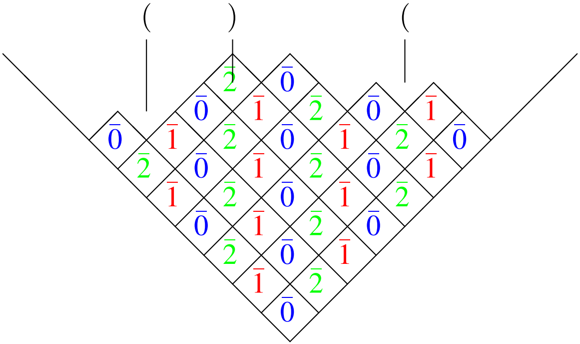

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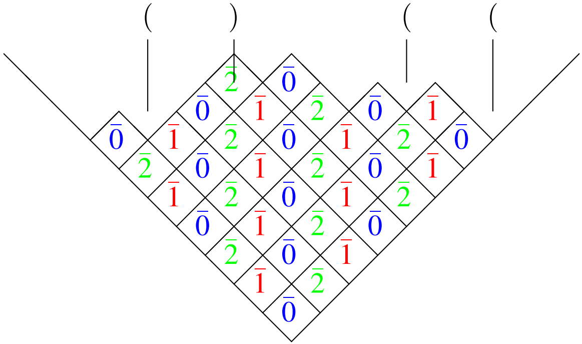

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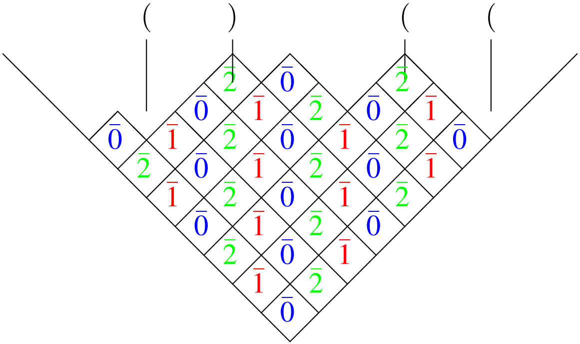

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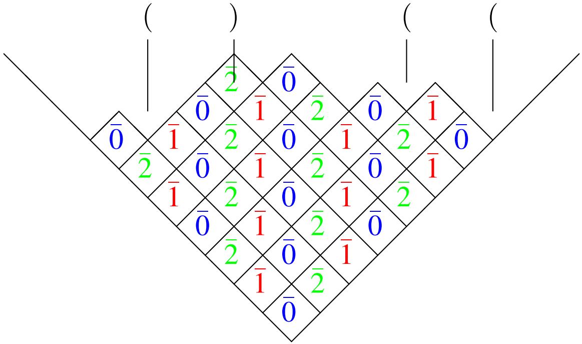

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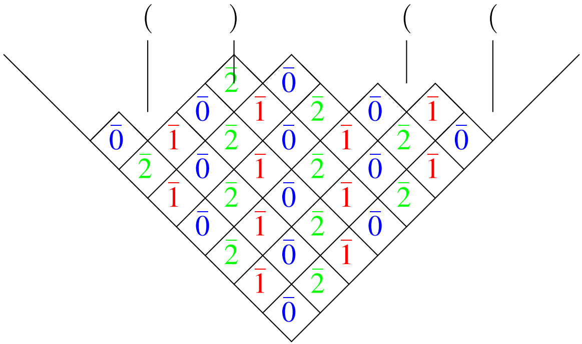

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- $E_{\bar{2}}$ would send this partition to 0.

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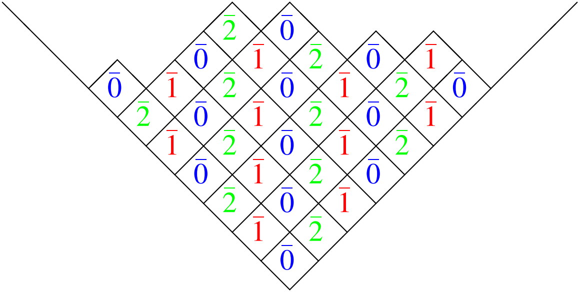

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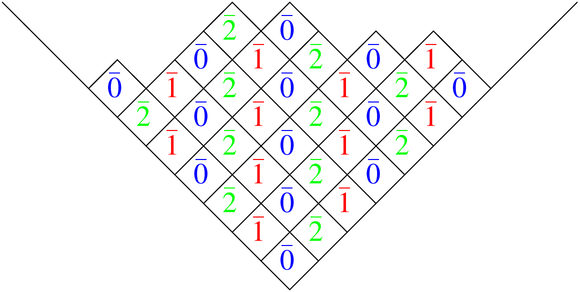

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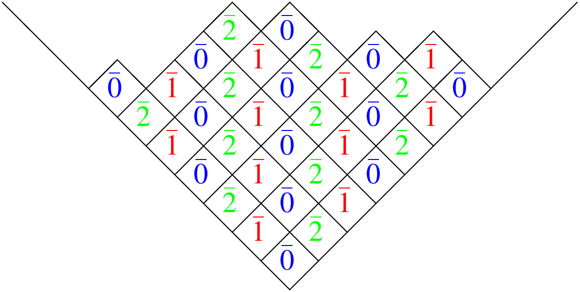

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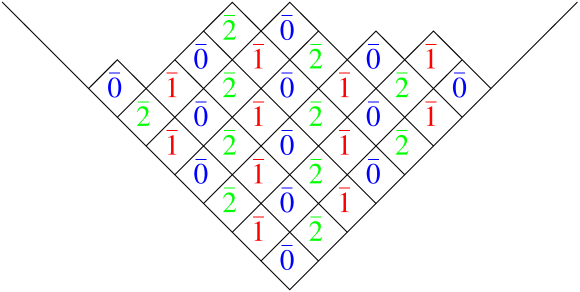

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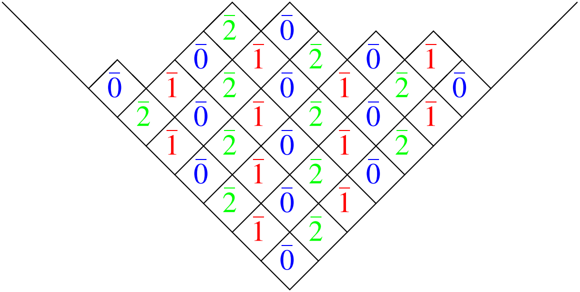

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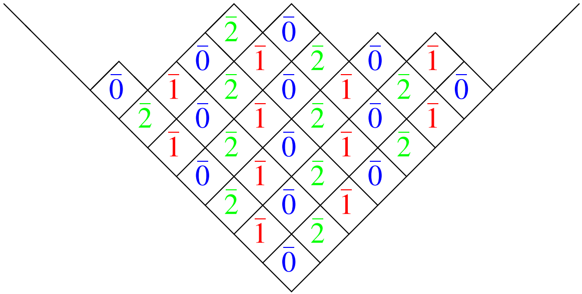

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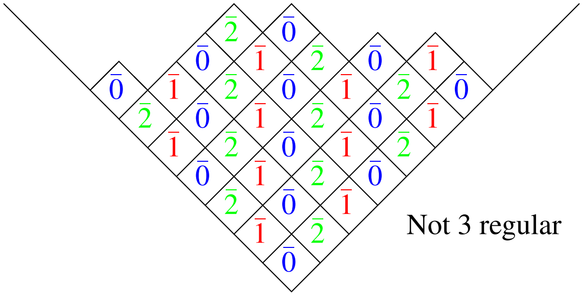
- The subcrystal generated by the empty partition consists exactly of the 3-regular partitions are a single copy of B_{Λ_0} (no 3 rows of same length).

The Misra-Miwa-Hayashi realizarion of B_{Λ_0} for $\widehat{\mathfrak{sl}}_3$



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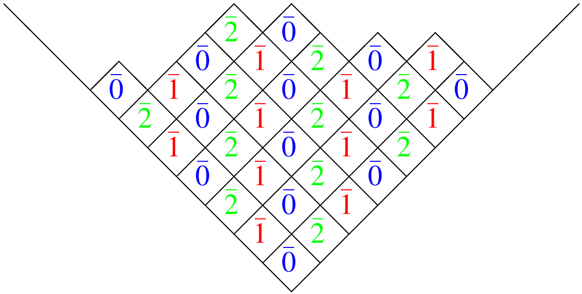

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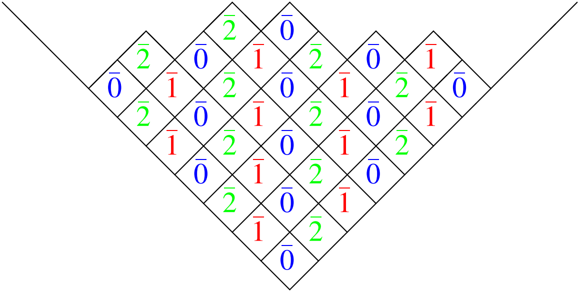

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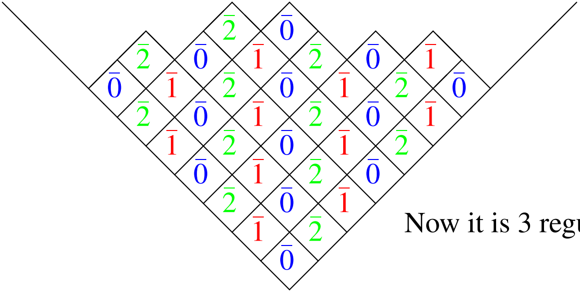

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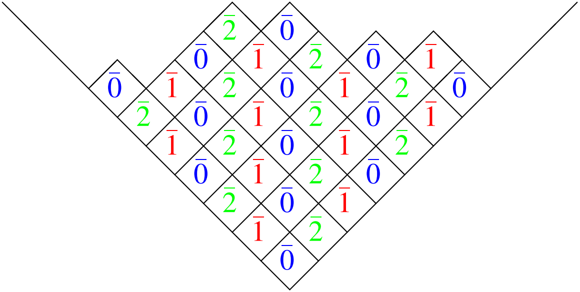

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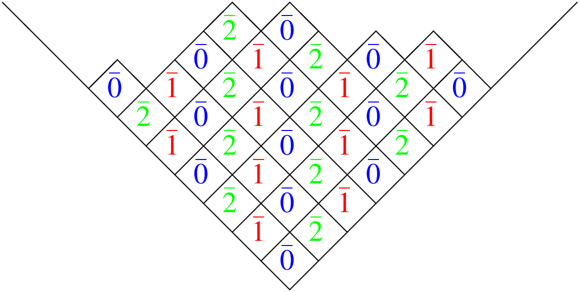

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- The subcrystal generated by the empty partition consists exactly of the 3-regular partitions are a single copy of B_{Λ_0} (no 3 rows of same length).
- Now we can say we understand the model.

Nakajima's monomial crystal

Nakajima's monomial crystal

- Consider monomials on variables $Y_{\bar{i},k}^{\pm 1}$, $\bar{i} \in \mathbb{Z}/n\mathbb{Z}$, $k \in \mathbb{Z}$

Nakajima's monomial crystal

$$Y_{\bar{1},15}Y_{\bar{2},14}Y_{\bar{1},13}^{-2}Y_{\bar{0},10}Y_{\bar{1},9}\ Y_{\bar{3},9}\ Y_{\bar{1},7}\ Y_{\bar{3},7}^{-1}\ Y_{\bar{1},5}^{-1}\ Y_{\bar{0},4}^{-1}\ Y_{\bar{1},1}$$

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- Consider monomials on variables $Y_{\bar{i},k}^{\pm 1}$, $\bar{i} \in \mathbb{Z}/n\mathbb{Z}$, $k \in \mathbb{Z}$ (here $n = 4$).

Nakajima's monomial crystal

$$Y_{\bar{1},15}Y_{\bar{2},14}Y^{-2}_{\bar{1},13}Y_{\bar{0},10}Y_{\bar{1},9}\ Y_{\bar{3},9}\ Y_{\bar{1},7}\ Y^{-1}_{\bar{3},7}\ Y^{-1}_{\bar{1},5}\ Y^{-1}_{\bar{0},4}\ Y_{\bar{1},1}$$

- Consider monomials on variables $Y^{\pm 1}_{\bar{i},k}$, $\bar{i} \in \mathbb{Z}/n\mathbb{Z}$, $k \in \mathbb{Z}$ (here $n = 4$).
- Define operators $E_{\bar{i}}$ and $F_{\bar{i}}$ on this set. We show $E_{\bar{1}}, F_{\bar{1}}$.

Nakajima's monomial crystal

$$Y_{\bar{1},15}Y_{\bar{2},14}Y^{-2}_{\bar{1},13}Y_{\bar{0},10}Y_{\bar{1},9}\ Y_{\bar{3},9}\ Y_{\bar{1},7}\ Y^{-1}_{\bar{3},7}\ Y^{-1}_{\bar{1},5}\ Y^{-1}_{\bar{0},4}\ Y_{\bar{1},1}$$

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Nakajima's monomial crystal

(

$$Y_{\bar{1},15}Y_{\bar{2},14}Y_{\bar{1},13}^{-2}Y_{\bar{0},10}Y_{\bar{1},9}\ Y_{\bar{3},9}\ Y_{\bar{1},7}\ Y_{\bar{3},7}^{-1}\ Y_{\bar{1},5}^{-1}\ Y_{\bar{0},4}^{-1}\ Y_{\bar{1},1}$$

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Nakajima's monomial crystal

( ))

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Nakajima's monomial crystal

$$\begin{array}{ccccccccccc} (& & &)) & & (& & & & & \\ Y_{\bar{1},15} & Y_{\bar{2},14} & Y^{-2}_{\bar{1},13} & Y_{\bar{0},10} & Y_{\bar{1},9} & Y_{\bar{3},9} & Y_{\bar{1},7} & Y^{-1}_{\bar{3},7} & Y^{-1}_{\bar{1},5} & Y^{-1}_{\bar{0},4} & Y_{\bar{1},1} \end{array}$$

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Nakajima's monomial crystal

$$\begin{array}{ccccccccccc} (& & &)) & & (& & (& &) & \\ Y_{\bar{1},15} & Y_{\bar{2},14} & Y_{\bar{1},13}^{-2} & Y_{\bar{0},10} & Y_{\bar{1},9} & Y_{\bar{3},9} & Y_{\bar{1},7} & Y_{\bar{3},7}^{-1} & Y_{\bar{1},5}^{-1} & Y_{\bar{0},4}^{-1} & Y_{\bar{1},1} \end{array}$$

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Nakajima's monomial crystal

$$\begin{matrix} (& &)) & & (& & (& &) & & (\\ Y_{\bar{1},15} & Y_{\bar{2},14} & Y_{\bar{1},13}^{-2} & Y_{\bar{0},10} & Y_{\bar{1},9} & Y_{\bar{3},9} & Y_{\bar{1},7} & Y_{\bar{3},7}^{-1} & Y_{\bar{1},5}^{-1} & Y_{\bar{0},4}^{-1} & Y_{\bar{1},1} \end{matrix}$$

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$$\begin{array}{ccccccccccc} (& &)) & & (& & (& &) & & (\\ Y_{\bar{1},15} Y_{\bar{2},14} & & Y^{-2}_{\bar{1},13} & Y_{\bar{0},10} & Y_{\bar{1},9} & Y_{\bar{3},9} & Y_{\bar{1},7} & Y^{-1}_{\bar{3},7} & Y^{-1}_{\bar{1},5} & Y^{-1}_{\bar{0},4} & Y_{\bar{1},1} \end{array}$$

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- Put a "(" for every $Y_{\bar{1},k}$ and a ")" for every $Y^{-1}_{\bar{1},k}$, ordered left to right by decreasing k.
- $F_{\bar{1}}$ multiplies m by $A^{-1}_{\bar{1},k+1} := Y^{-1}_{\bar{1},k} Y^{-1}_{\bar{1},k+2} Y_{\bar{0},k+1} Y_{\bar{2},k+1}$, where the first uncanceled "(" corresponds to a $Y_{\bar{1},k}$.

Nakajima's monomial crystal

$$\begin{matrix} (& &)) & & (& & (& &) & & (\\ Y_{\bar{1},15} Y_{\bar{2},14} Y^{-2}_{\bar{1},13} Y_{\bar{0},10} Y_{\bar{1},9} & Y_{\bar{3},9} & Y_{\bar{1},7} & Y^{-1}_{\bar{3},7} & Y^{-1}_{\bar{1},5} & Y^{-1}_{\bar{0},4} & Y_{\bar{1},1} \end{matrix}$$

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Nakajima's monomial crystal

$$\begin{matrix} (& &)) & (& (&) & (\end{matrix}$$

$$Y_{\bar{1},15}Y_{\bar{2},14}Y_{\bar{1},13}^{-2}Y_{\bar{0},10}Y_{\bar{1},9}\ Y_{\bar{3},9}\ Y_{\bar{1},7}\ Y_{\bar{3},7}^{-1}\ Y_{\bar{1},5}^{-1}\ Y_{\bar{0},4}^{-1}\ Y_{\bar{1},1}$$

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- $F_{\bar{1}}$ multiplies m by $A_{\bar{1},k-1} := Y_{\bar{1},k-2}Y_{\bar{1},k}Y_{\bar{0},k-1}^{-1}Y_{\bar{2},k-1}^{-1}$, where the first uncanceled ")" corresponds to a $Y_{\bar{1},k}^{-1}$.

Nakajima's monomial crystal

$$\begin{matrix} (& &)) & & (& & (& &) & & (\\ Y_{\bar{1},15} & Y_{\bar{2},14} & Y_{\bar{1},13}^{-2} & Y_{\bar{0},10} & Y_{\bar{1},9} & Y_{\bar{3},9} & Y_{\bar{1},7} & Y_{\bar{3},7}^{-1} & Y_{\bar{1},5}^{-1} & Y_{\bar{0},4}^{-1} & Y_{\bar{1},1} \end{matrix}$$

Nakajima's monomial crystal

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$$\Big\downarrow F_{\bar{1}}$$

Nakajima's monomial crystal

$$\begin{array}{ccccccccccc} (& &)) & & \overset{*}{(} & & (& &) & & (\\ Y_{\bar{1},15} & Y_{\bar{2},14} & Y_{\bar{1},13}^{-2} & Y_{\bar{0},10} & Y_{\bar{1},9} & Y_{\bar{3},9} & Y_{\bar{1},7} & Y_{\bar{3},7}^{-1} & Y_{\bar{1},5}^{-1} & Y_{\bar{0},4}^{-1} & Y_{\bar{1},1} \end{array}$$

$$\Big\downarrow F_{\bar{1}}$$

Nakajima's monomial crystal

$$\begin{array}{ccccccccccc} (& &)) & \overset{*}{(} & & (& &) & & (& \end{array}$$

$$Y_{\bar{1},15} Y_{\bar{2},14} Y_{\bar{1},13}^{-2} Y_{\bar{0},10} Y_{\bar{1},9} \; Y_{\bar{3},9} \; Y_{\bar{1},7} \; Y_{\bar{3},7}^{-1} \; Y_{\bar{1},5}^{-1} \; Y_{\bar{0},4}^{-1} \; Y_{\bar{1},1}$$

$$\downarrow F_{\bar{1}}$$

$$A_{\bar{1},10}^{-1} Y_{\bar{1},15} Y_{\bar{2},14} Y_{\bar{1},13}^{-2} Y_{\bar{0},10} Y_{\bar{1},9} \; Y_{\bar{3},9} \; Y_{\bar{1},7} \; Y_{\bar{3},7}^{-1} \; Y_{\bar{1},5}^{-1} \; Y_{\bar{0},4}^{-1} \; Y_{\bar{1},1}$$

Nakajima's monomial crystal

$$\begin{array}{c} (\quad\quad)) \quad\quad \overset{*}{(} \quad\quad (\quad\quad) \quad\quad (\\ Y_{\bar{1},15}Y_{\bar{2},14}Y_{\bar{1},13}^{-2}Y_{\bar{0},10}Y_{\bar{1},9}\ Y_{\bar{3},9}\ Y_{\bar{1},7}\ Y_{\bar{3},7}^{-1}\ Y_{\bar{1},5}^{-1}\ Y_{\bar{0},4}^{-1}\ Y_{\bar{1},1} \end{array}$$

$$\Big\downarrow F_{\bar{1}}$$

$$A_{\bar{1},10}^{-1}Y_{\bar{1},15}Y_{\bar{2},14}Y_{\bar{1},13}^{-2}Y_{\bar{0},10}Y_{\bar{1},9}\ Y_{\bar{3},9}\ Y_{\bar{1},7}\ Y_{\bar{3},7}^{-1}\ Y_{\bar{1},5}^{-1}\ Y_{\bar{0},4}^{-1}\ Y_{\bar{1},1}$$

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Nakajima's monomial crystal

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$$Y_{\bar{1},15}Y_{\bar{2},14}Y^{-2}_{\bar{1},13}Y_{\bar{0},10}Y_{\bar{1},9}\ Y_{\bar{3},9}\ Y_{\bar{1},7}\ Y^{-1}_{\bar{3},7}\ Y^{-1}_{\bar{1},5}\ Y^{-1}_{\bar{0},4}\ Y_{\bar{1},1}$$

$$\Big\downarrow F_{\bar{1}}$$

$$A^{-1}_{\bar{1},10}Y_{\bar{1},15}Y_{\bar{2},14}Y^{-2}_{\bar{1},13}Y_{\bar{0},10}Y_{\bar{1},9}\ Y_{\bar{3},9}\ Y_{\bar{1},7}\ Y^{-1}_{\bar{3},7}\ Y^{-1}_{\bar{1},5}\ Y^{-1}_{\bar{0},4}\ Y_{\bar{1},1}$$

$$||$$

$$Y^{-1}_{\bar{1},9}\ Y^{-1}_{\bar{1},11}Y_{\bar{0},10}Y_{\bar{2},10}Y_{\bar{1},15}Y_{\bar{2},14}Y^{-2}_{\bar{1},13}Y_{\bar{0},10}Y_{\bar{1},9}\ Y_{\bar{3},9}\ Y_{\bar{1},7}\ Y^{-1}_{\bar{3},7}\ Y^{-1}_{\bar{1},5}\ Y^{-1}_{\bar{0},4}\ Y_{\bar{1},1}$$

Nakajima's monomial crystal

$$\begin{matrix} (& &)) & \overset{*}{(} & (&) & (\end{matrix}$$

$$Y_{\bar{1},15}Y_{\bar{2},14}Y^{-2}_{\bar{1},13}Y_{\bar{0},10}Y_{\bar{1},9}\ Y_{\bar{3},9}\ Y_{\bar{1},7}\ Y^{-1}_{\bar{3},7}\ Y^{-1}_{\bar{1},5}\ Y^{-1}_{\bar{0},4}\ Y_{\bar{1},1}$$

$$\Big\downarrow F_{\bar{1}}$$

$$A^{-1}_{\bar{1},10}Y_{\bar{1},15}Y_{\bar{2},14}Y^{-2}_{\bar{1},13}Y_{\bar{0},10}Y_{\bar{1},9}\ Y_{\bar{3},9}\ Y_{\bar{1},7}\ Y^{-1}_{\bar{3},7}\ Y^{-1}_{\bar{1},5}\ Y^{-1}_{\bar{0},4}\ Y_{\bar{1},1}$$

$$||$$

$$Y^{-1}_{\bar{1},9}\ Y^{-1}_{\bar{1},11}Y_{\bar{0},10}Y_{\bar{2},10}Y_{\bar{1},15}Y_{\bar{2},14}Y^{-2}_{\bar{1},13}Y_{\bar{0},10}Y_{\bar{1},9}\ Y_{\bar{3},9}\ Y_{\bar{1},7}\ Y^{-1}_{\bar{3},7}\ Y^{-1}_{\bar{1},5}\ Y^{-1}_{\bar{0},4}\ Y_{\bar{1},1}$$

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Nakajima's monomial crystal

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$$Y_{\bar{1},15}Y_{\bar{2},14}Y^{-2}_{\bar{1},13}Y_{\bar{0},10}Y_{\bar{1},9}\;Y_{\bar{3},9}\;Y_{\bar{1},7}\;Y^{-1}_{\bar{3},7}\;Y^{-1}_{\bar{1},5}\;Y^{-1}_{\bar{0},4}\;Y_{\bar{1},1}$$

$$\Big\downarrow F_{\bar{1}}$$

$$A^{-1}_{\bar{1},10}Y_{\bar{1},15}Y_{\bar{2},14}Y^{-2}_{\bar{1},13}Y_{\bar{0},10}Y_{\bar{1},9}\;Y_{\bar{3},9}\;Y_{\bar{1},7}\;Y^{-1}_{\bar{3},7}\;Y^{-1}_{\bar{1},5}\;Y^{-1}_{\bar{0},4}\;Y_{\bar{1},1}$$

$$\|$$

$$Y^{-1}_{\bar{1},9}\;Y^{-1}_{\bar{1},11}Y_{\bar{0},10}Y_{\bar{2},10}Y_{\bar{1},15}Y_{\bar{2},14}Y^{-2}_{\bar{1},13}Y_{\bar{0},10}Y_{\bar{1},9}\;Y_{\bar{3},9}\;Y_{\bar{1},7}\;Y^{-1}_{\bar{3},7}\;Y^{-1}_{\bar{1},5}\;Y^{-1}_{\bar{0},4}\;Y_{\bar{1},1}$$

$$\|$$

$$Y_{\bar{1},15}Y_{\bar{2},14}Y^{-2}_{\bar{1},13}Y^{-1}_{\bar{1},11}Y^{2}_{\bar{0},10}Y_{\bar{2},10}Y_{\bar{3},9}\;Y_{\bar{1},7}\;Y^{-1}_{\bar{3},7}\;Y^{-1}_{\bar{1},5}\;Y^{-1}_{\bar{0},4}\;Y_{\bar{1},1}$$

Nakajima's monomial crystal

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$$Y_{\bar{1},15} Y_{\bar{2},14} Y^{-2}_{\bar{1},13} Y_{\bar{0},10} Y_{\bar{1},9} \; Y_{\bar{3},9} \; Y_{\bar{1},7} \; Y^{-1}_{\bar{3},7} \; Y^{-1}_{\bar{1},5} \; Y^{-1}_{\bar{0},4} \; Y_{\bar{1},1}$$

- The component generated by a dominant monomial is a highest weight crystal

Nakajima's monomial crystal

$$\begin{array}{ccccccccccc} (& &)) & & \overset{*}{(} & & (& &) & & (\\ Y_{\bar{1},15} & Y_{\bar{2},14} & Y_{\bar{1},13}^{-2} & Y_{\bar{0},10} & Y_{\bar{1},9} & Y_{\bar{3},9} & Y_{\bar{1},7} & Y_{\bar{3},7}^{-1} & Y_{\bar{1},5}^{-1} & Y_{\bar{0},4}^{-1} & Y_{\bar{1},1} \end{array}$$

- The component generated by a dominant monomial is a highest weight crystal (provided n is even, and some parity conditions hold).

Nakajima’s monomial crystal

$$\Big(\quad \Big)\Big) \quad \overset{*}{\Big(} \quad \Big(\quad \Big) \quad \Big($$

$$Y_{\bar{1},15} Y_{\bar{2},14} Y^{-2}_{\bar{1},13} Y_{\bar{0},10} Y_{\bar{1},9}\; Y_{\bar{3},9}\; Y_{\bar{1},7}\; Y^{-1}_{\bar{3},7}\; Y^{-1}_{\bar{1},5}\; Y^{-1}_{\bar{0},4}\; Y_{\bar{1},1}$$

- The component generated by a dominant monomial is a highest weight crystal (provided n is even, and some parity conditions hold).
  CAUTION: other components are not all crystals

Nakajima's monomial crystal

$$\begin{array}{ccccccccccc} (& &)) & & \overset{*}{(} & & (& &) & & (\\ Y_{\bar{1},15} & Y_{\bar{2},14} & Y^{-2}_{\bar{1},13} & Y_{\bar{0},10} & Y_{\bar{1},9} & Y_{\bar{3},9} & Y_{\bar{1},7} & Y^{-1}_{\bar{3},7} & Y^{-1}_{\bar{1},5} & Y^{-1}_{\bar{0},4} & Y_{\bar{1},1} \end{array}$$

- The component generated by a dominant monomial is a highest weight crystal (provided n is even, and some parity conditions hold). CAUTION: other components are not all crystals Hernandez and Nakajima have described some other components that are crystals.

Nakajima's monomial crystal

$$\begin{array}{ccccccccccc} (& &)) & & \overset{*}{(} & & (& &) & & (\\ Y_{\bar{1},15} & Y_{\bar{2},14} & Y^{-2}_{\bar{1},13} & Y_{\bar{0},10} & Y_{\bar{1},9} & Y_{\bar{3},9} & Y_{\bar{1},7} & Y^{-1}_{\bar{3},7} & Y^{-1}_{\bar{1},5} & Y^{-1}_{\bar{0},4} & Y_{\bar{1},1} \end{array}$$

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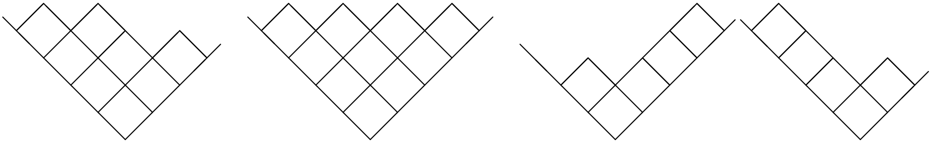
- I do know an explicit isomorphism with modification of the Misra-Miwa model due to Fayers. I'll mention this at the end.

The multi-partition realization of $B(\Lambda)$ (JMMO, FLOTW)



$\lambda^{(0)}$ $\lambda^{(1)}$ $\lambda^{(2)}$ $\lambda^{(3)}$

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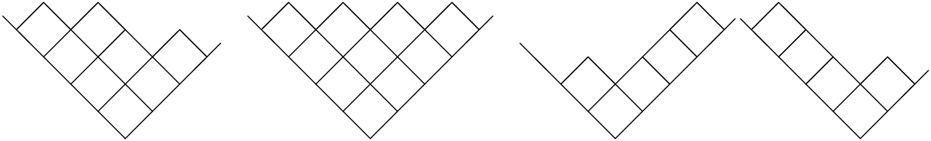

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- For $\ell = 4$, use 4-tuples of charged partitions.

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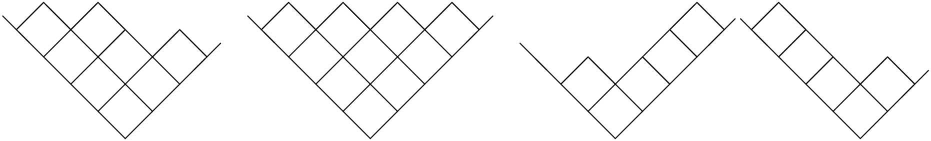

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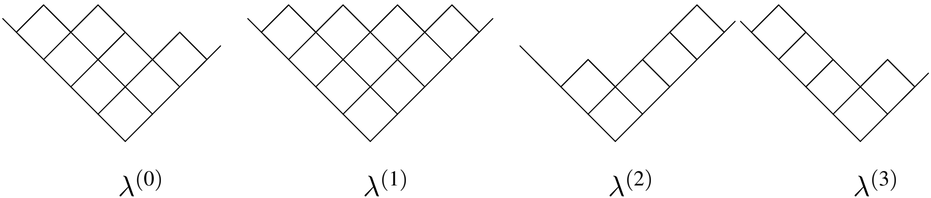

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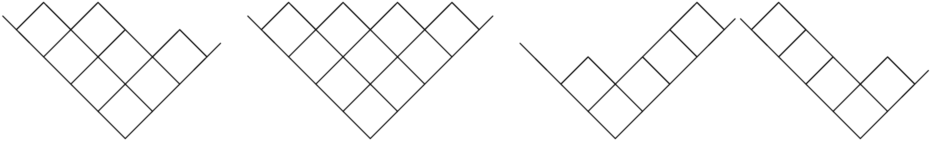

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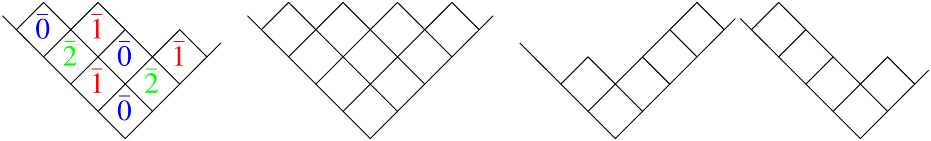

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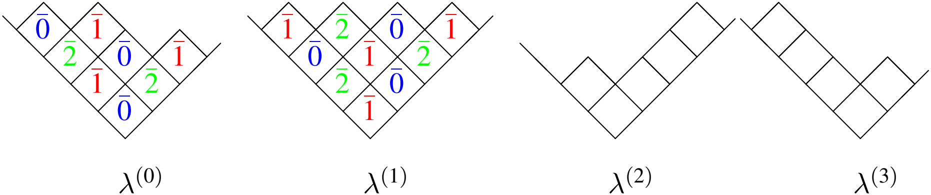

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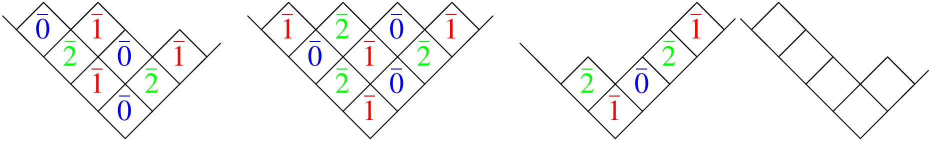

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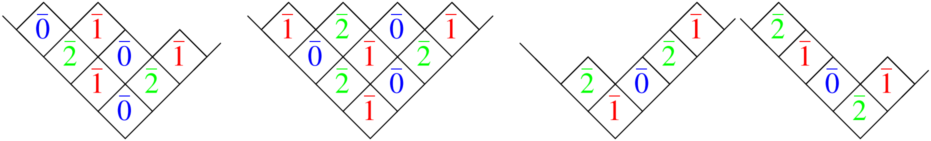

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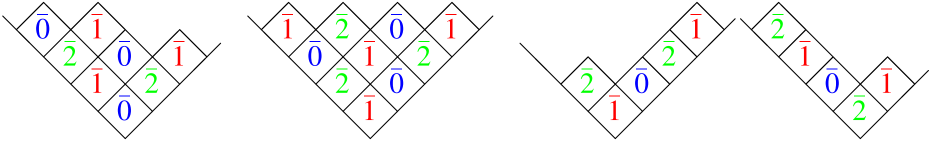

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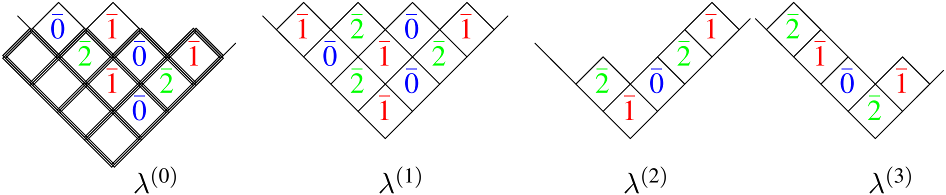

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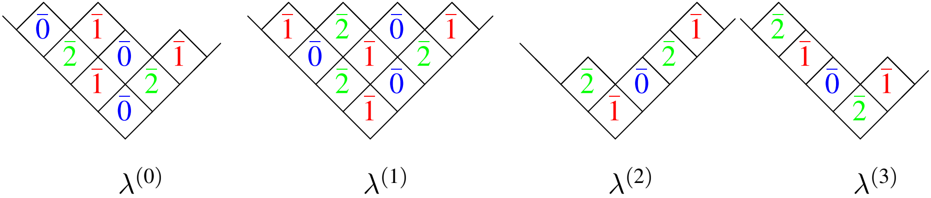

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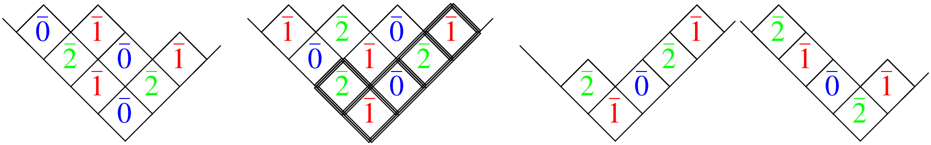

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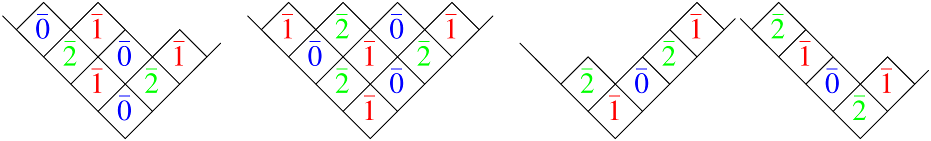

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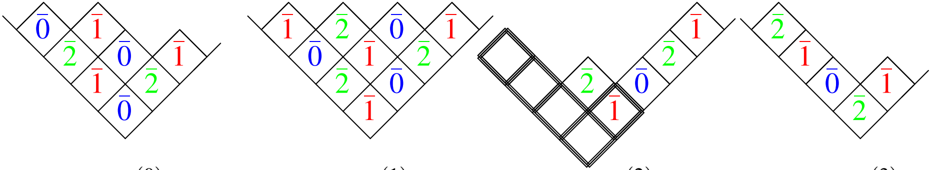

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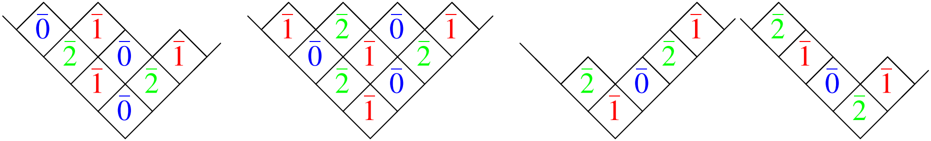

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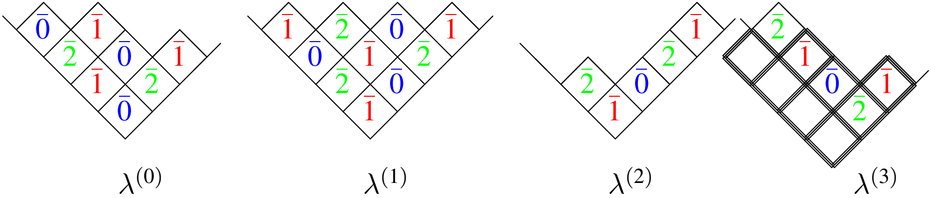

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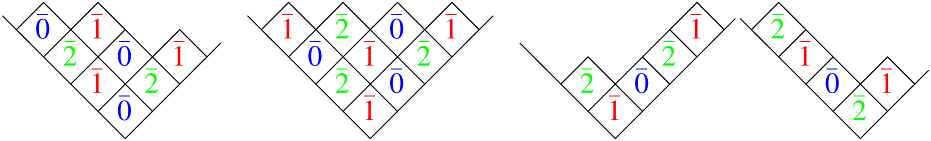

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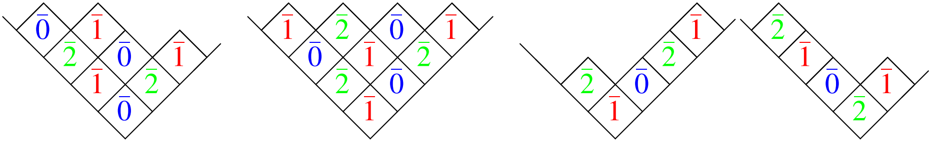

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- Again F_0 will add a box colored $\bar{0}$.

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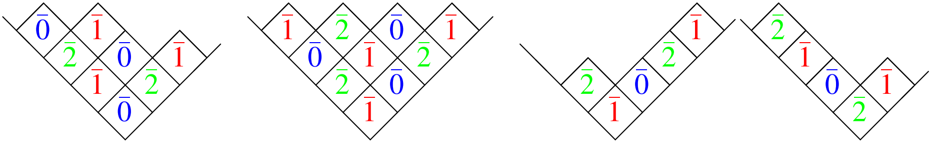

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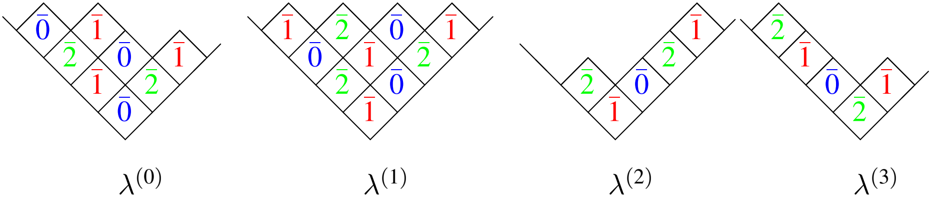

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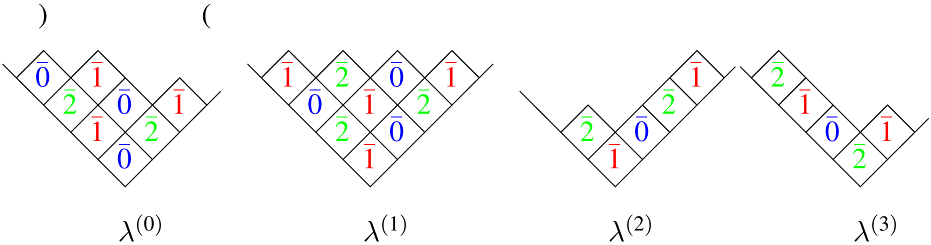

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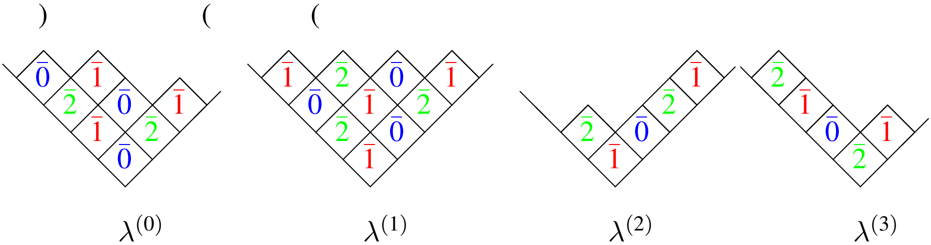

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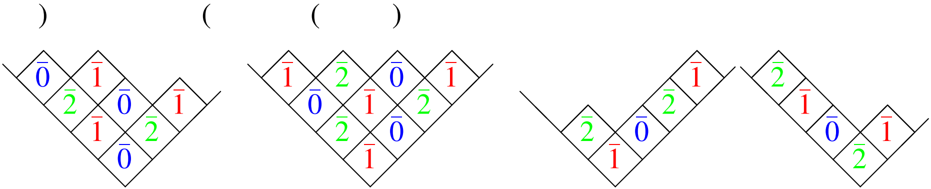

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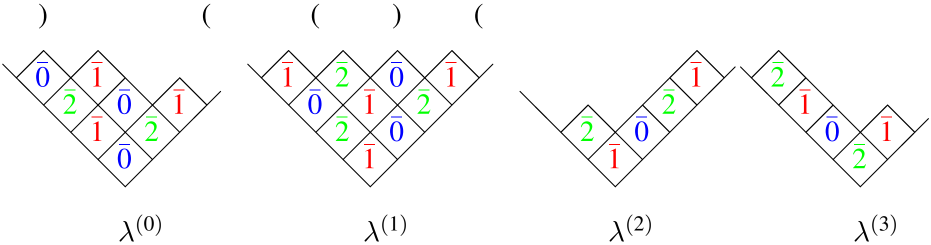

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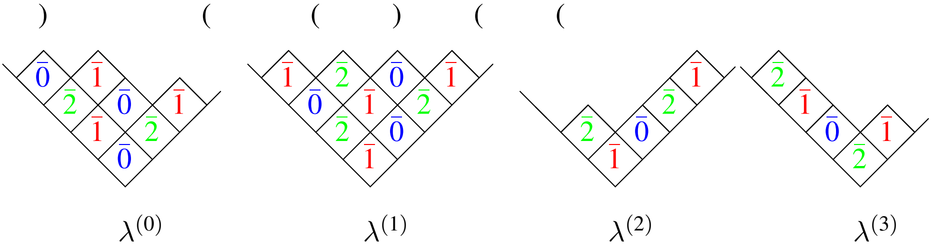

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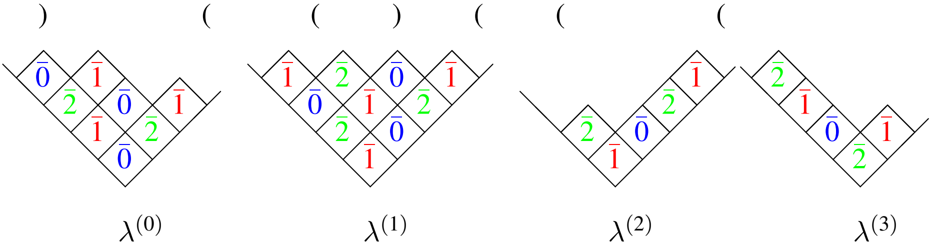

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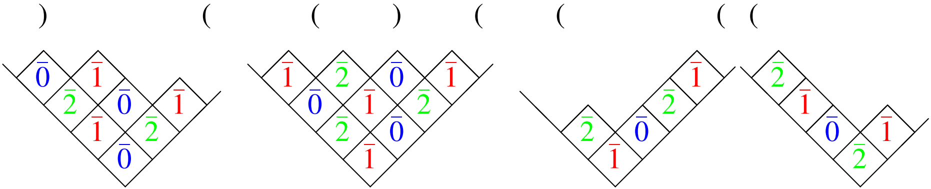

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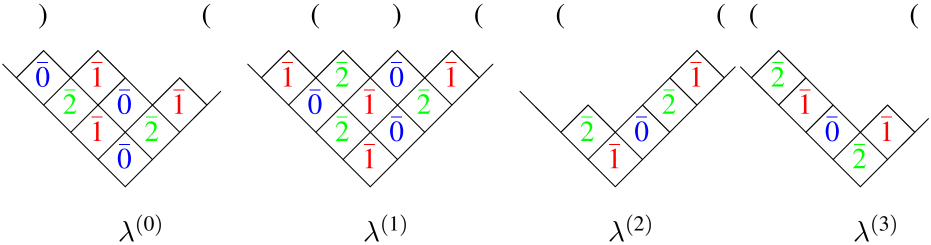

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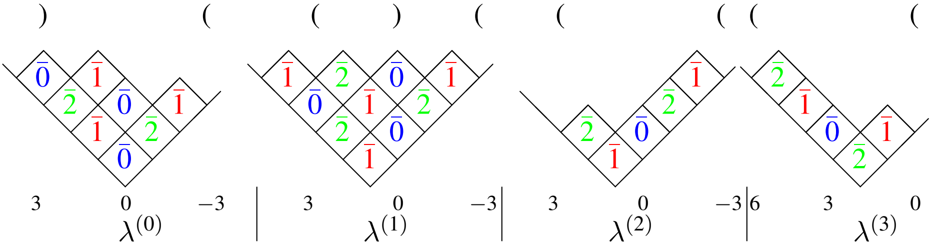

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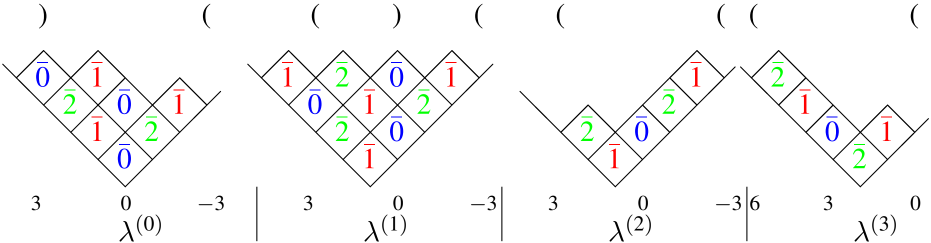

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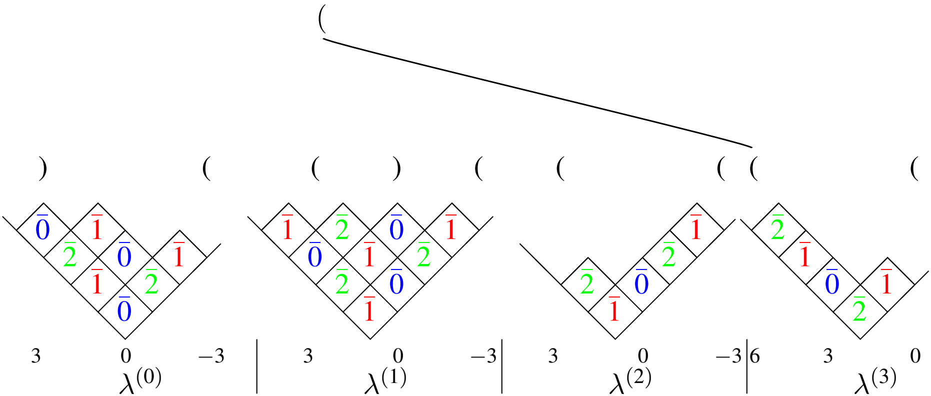

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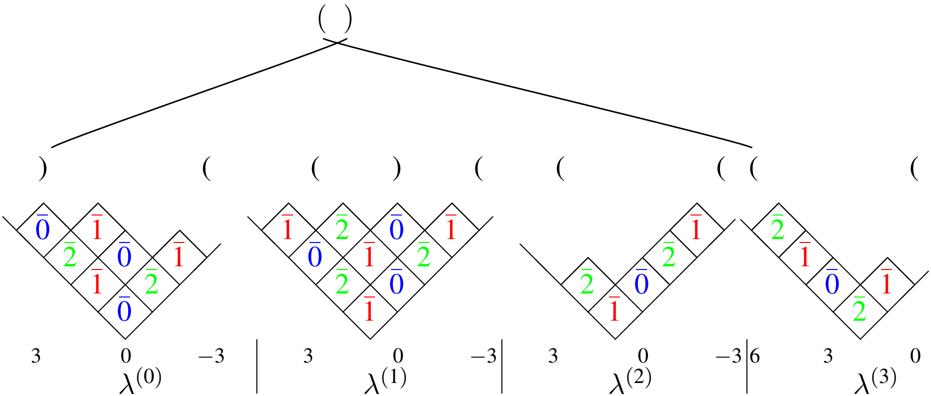

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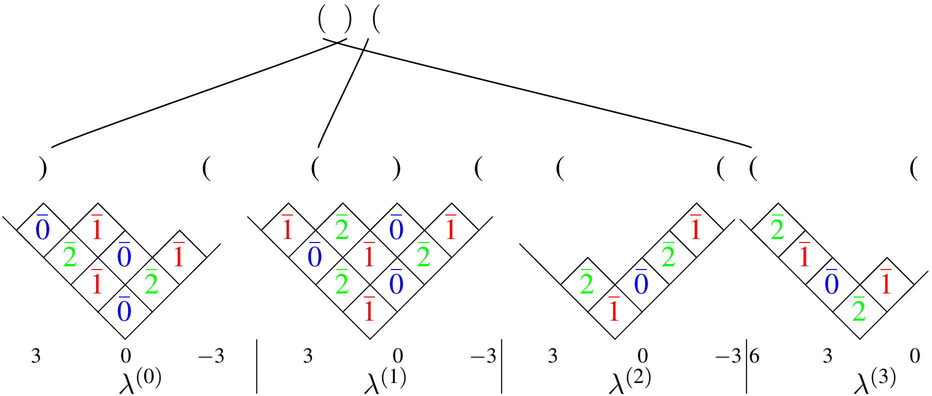

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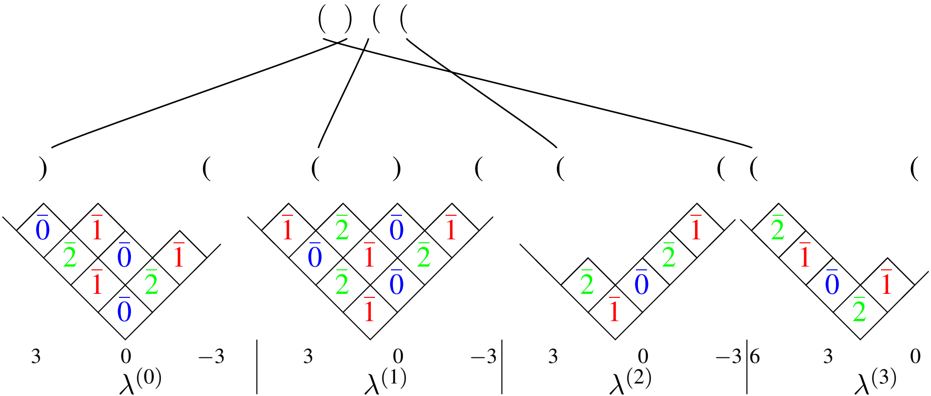
The multi-partition realization of $B(\Lambda)$ (JMMO, FLOTW)



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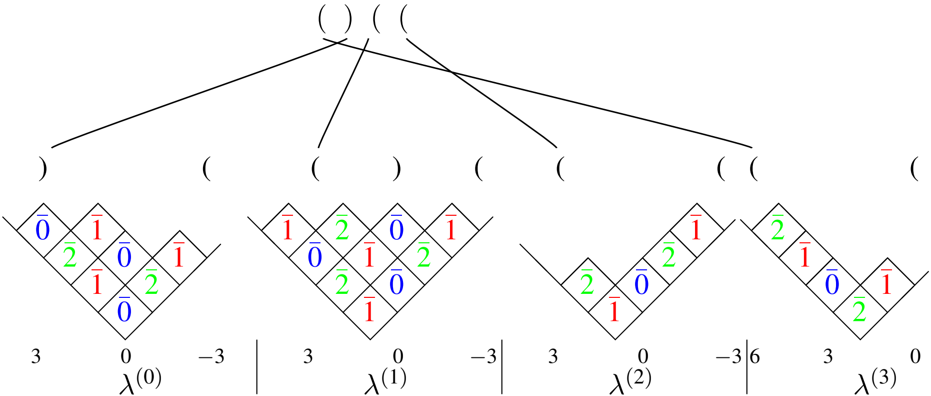

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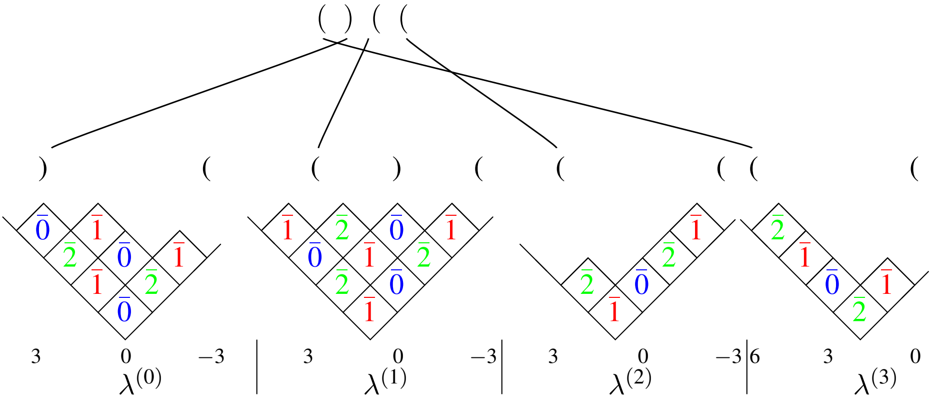

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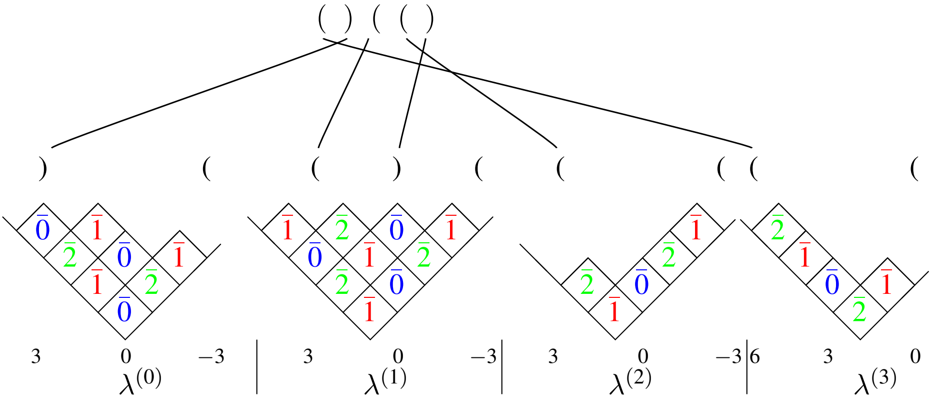

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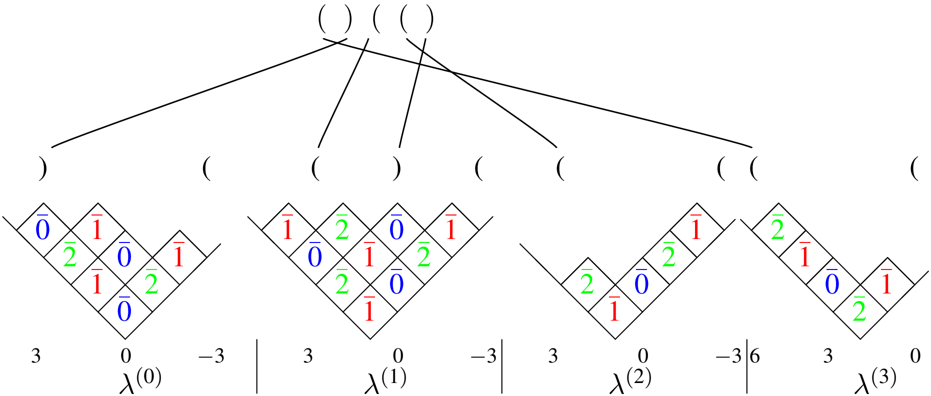

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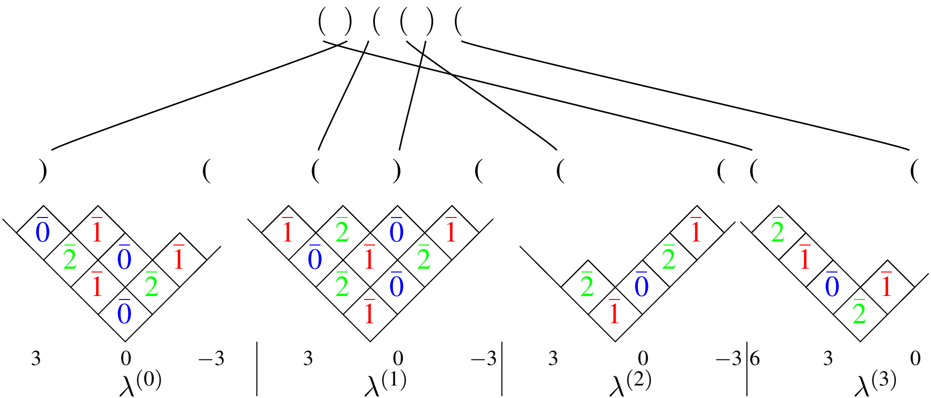

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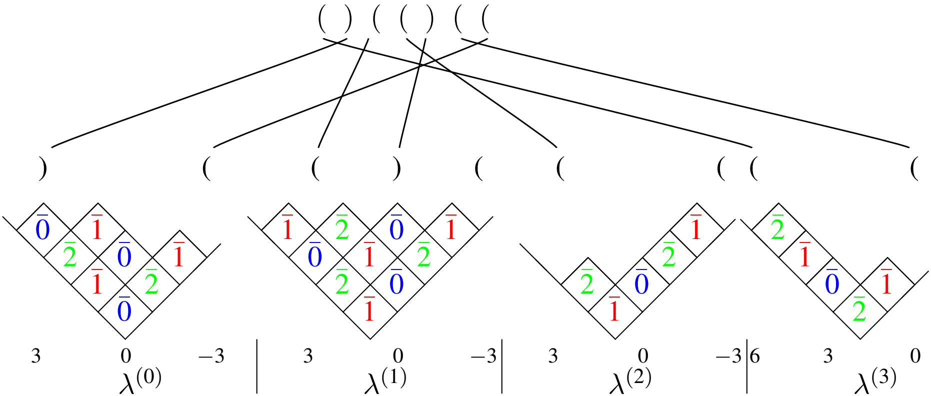

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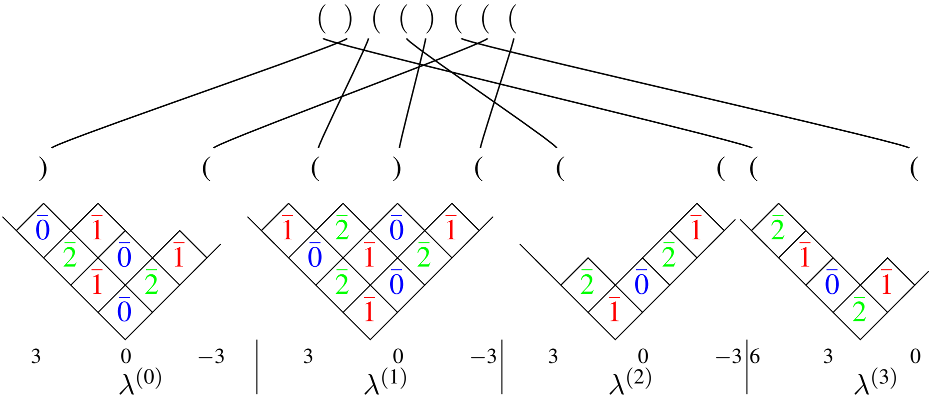

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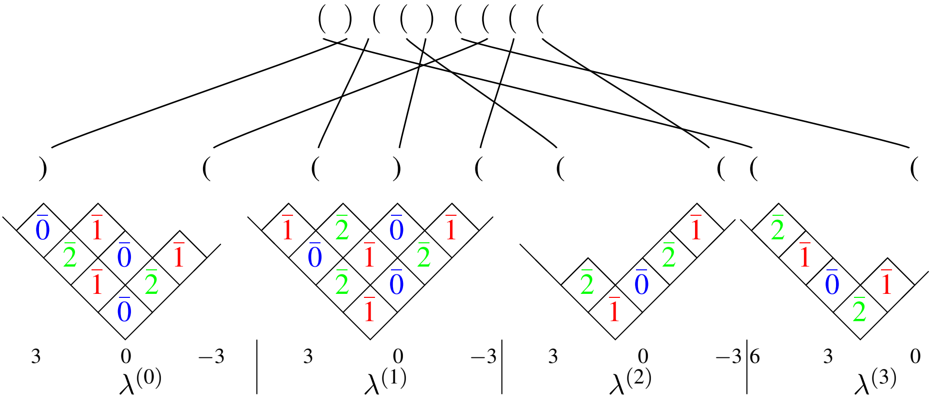

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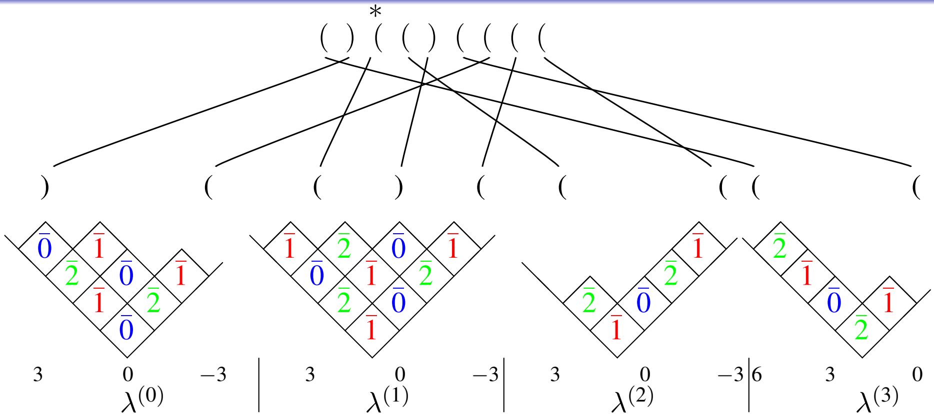

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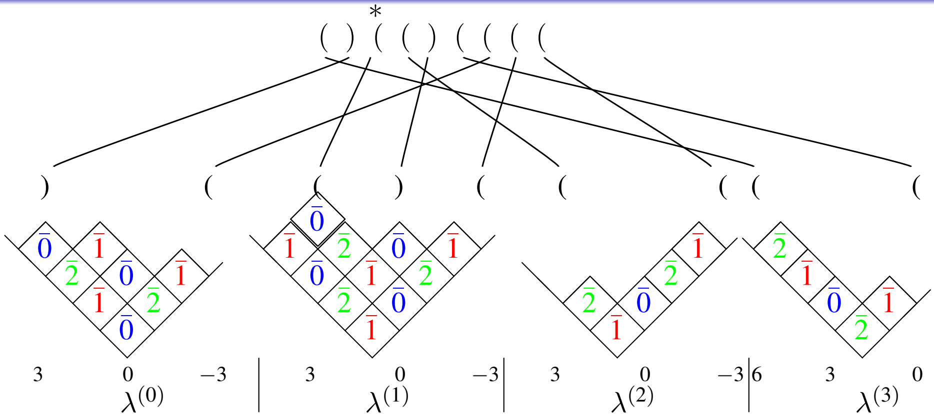

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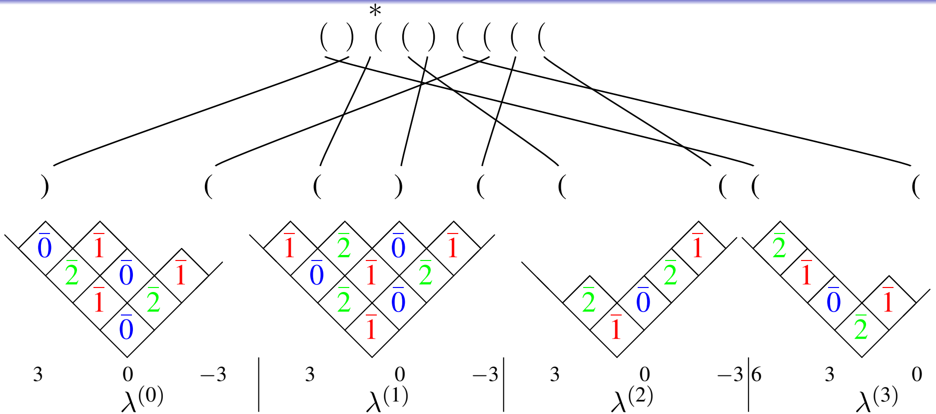

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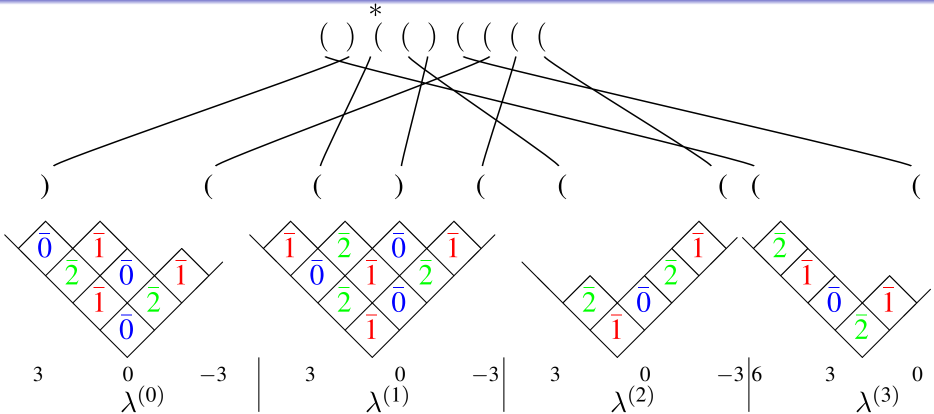

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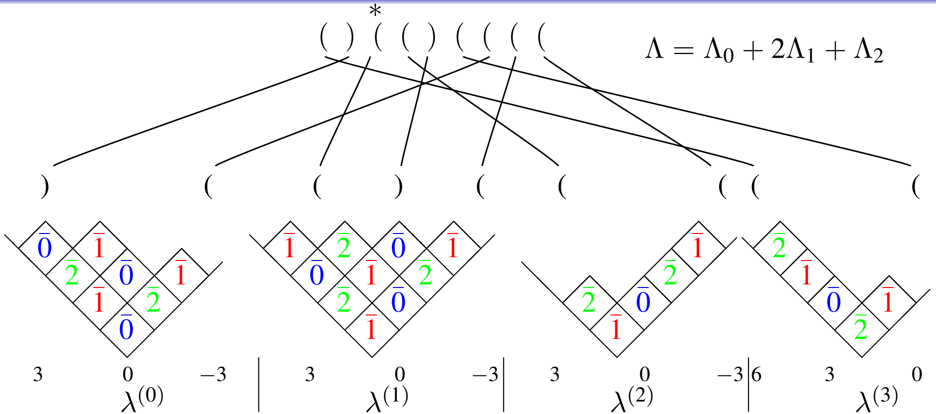

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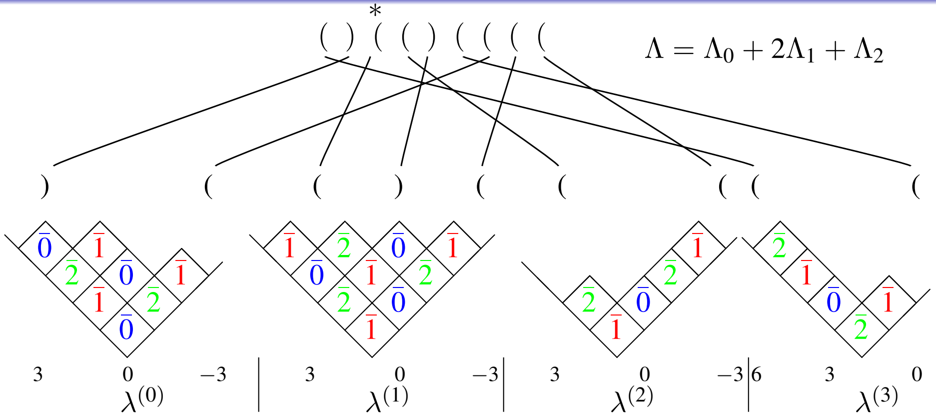

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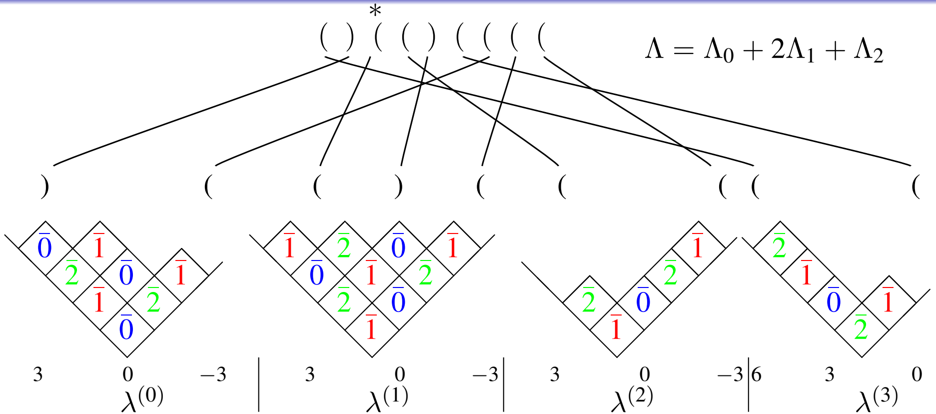

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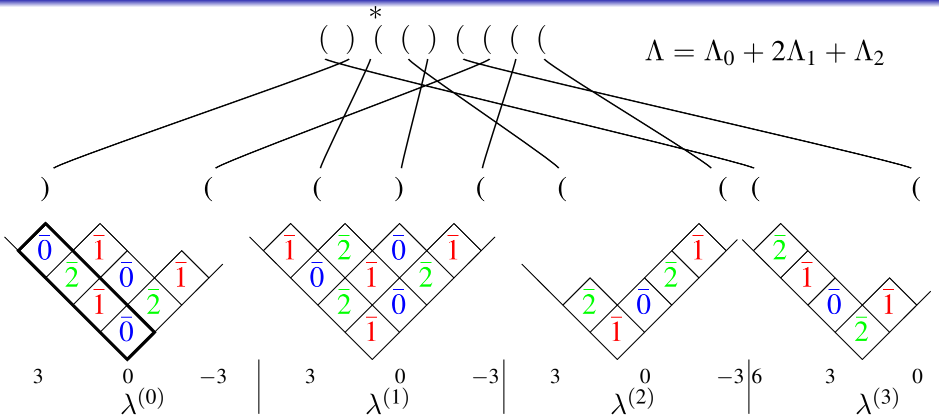

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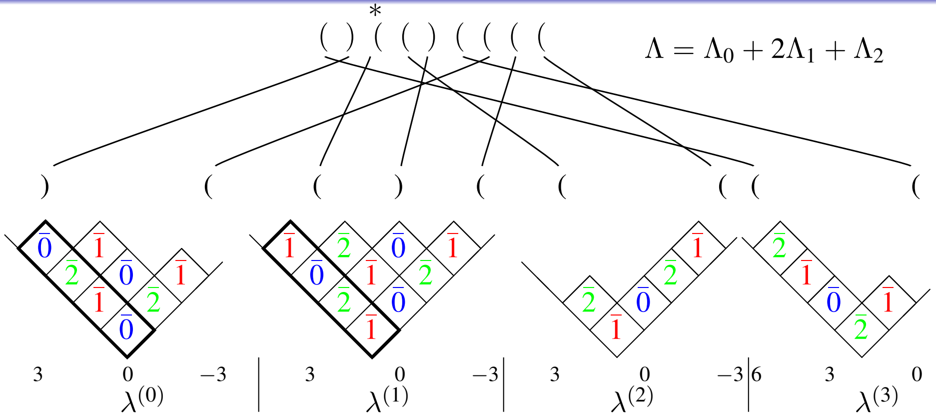

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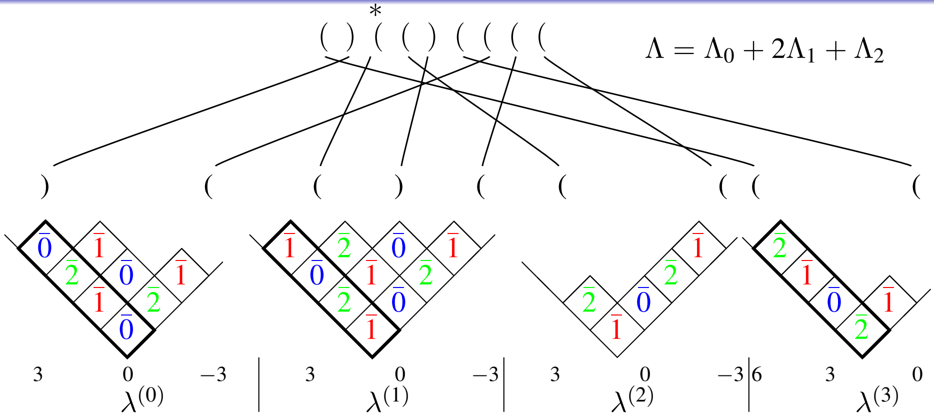

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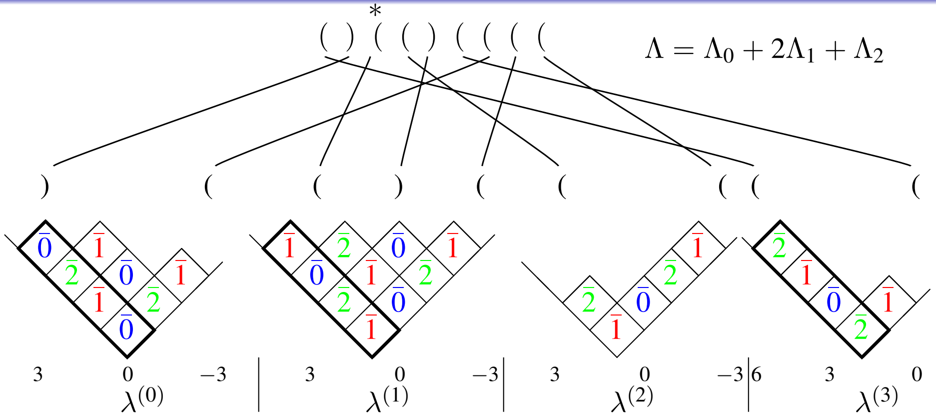

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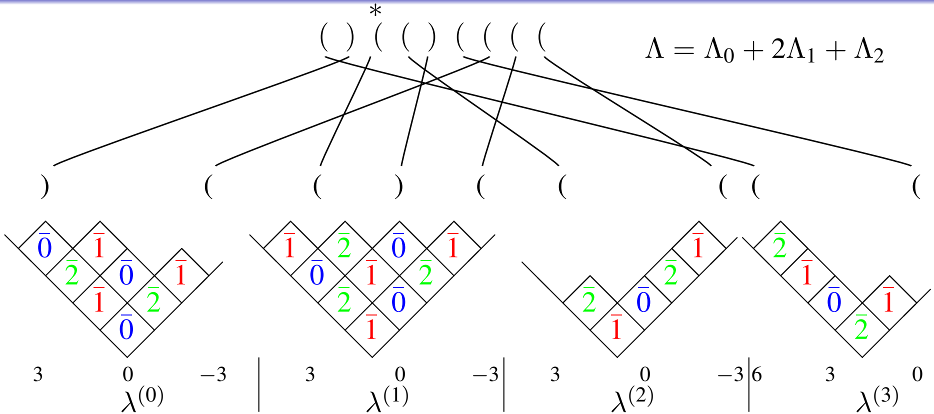

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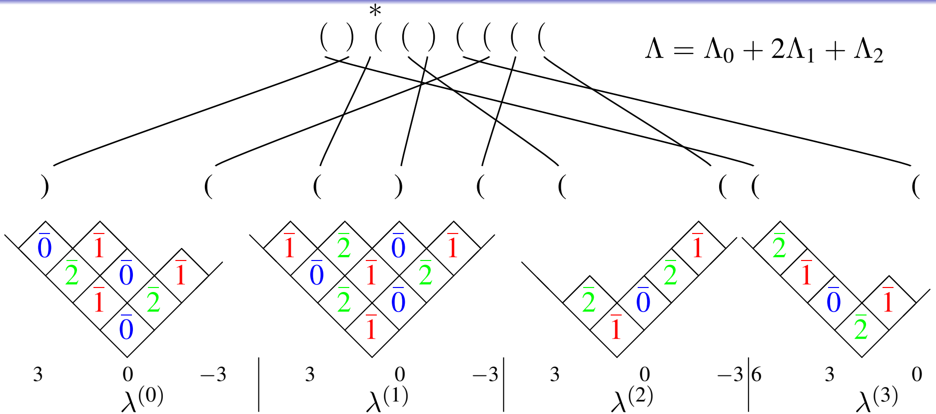

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- I would say we do understand this model for $B(\Lambda)$.

Understanding embeddings and $B(\infty)$

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- An ℓ-tuple of partitions satisfying the shifted containment conditions fits together into a three dimensional picture called a “cylindric partition".

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- Consider $n = 3$, $\ell = 2$, and multi-charge $(\overline{0}, \overline{1})$.

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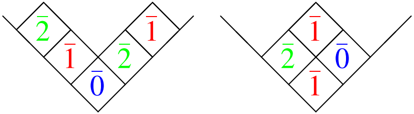

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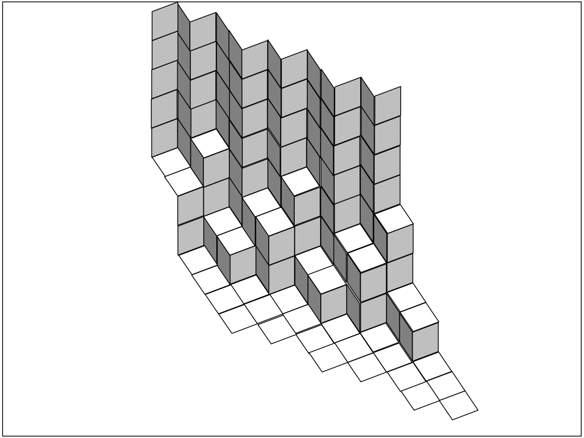

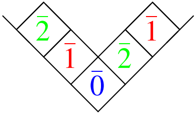

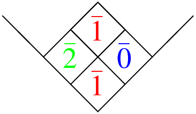

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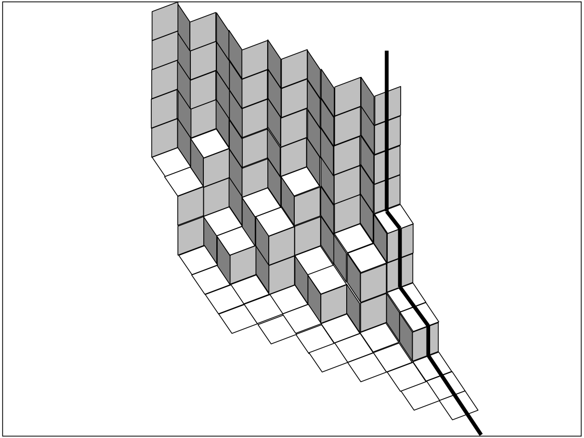

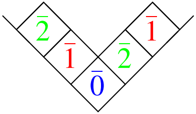

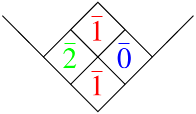

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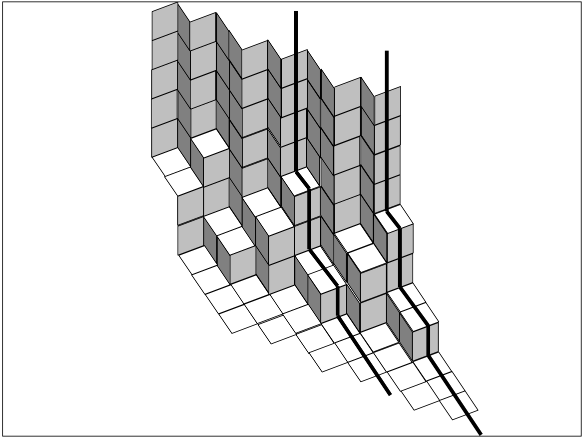

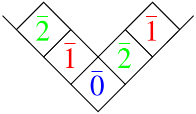

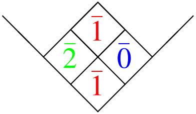

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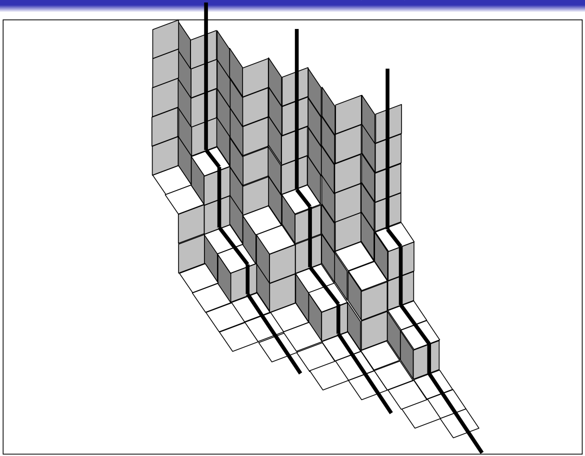

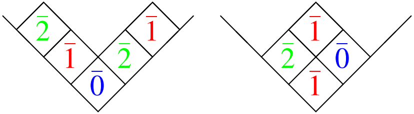

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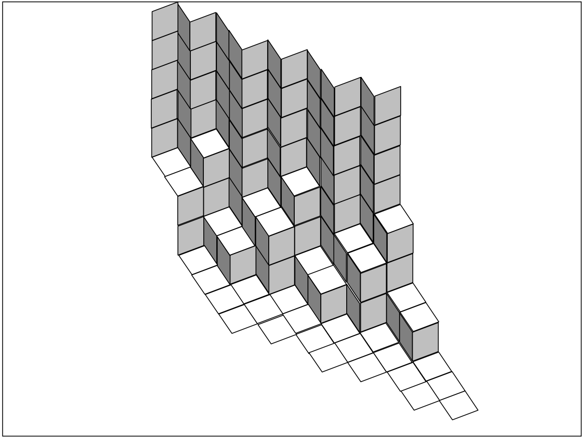

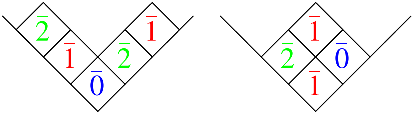

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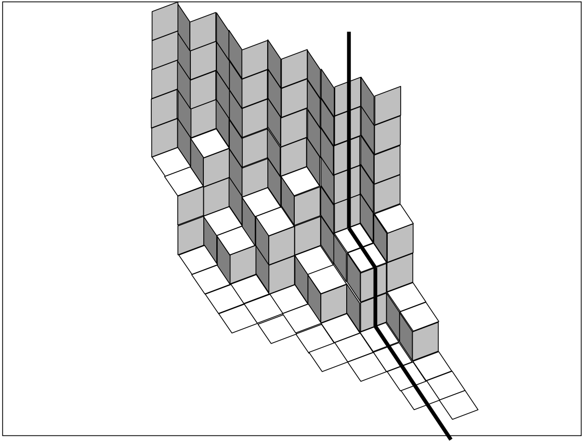

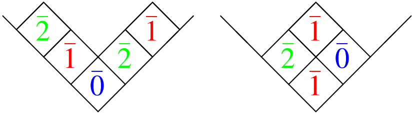

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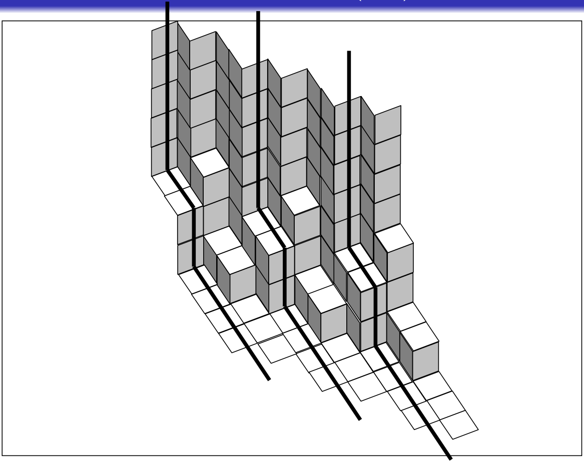

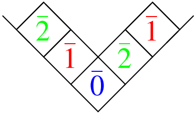

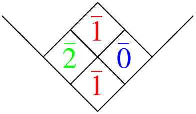

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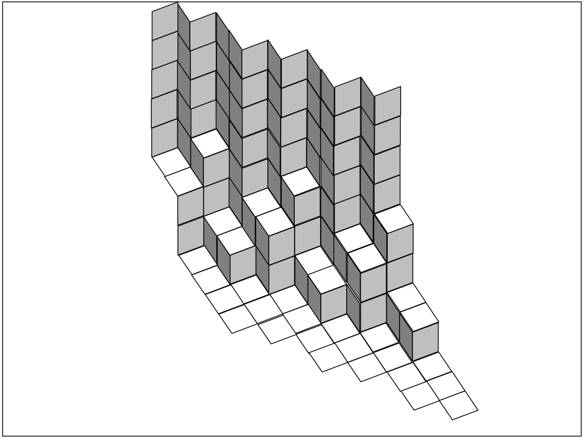

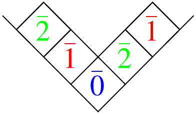

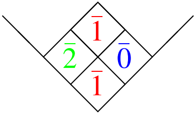

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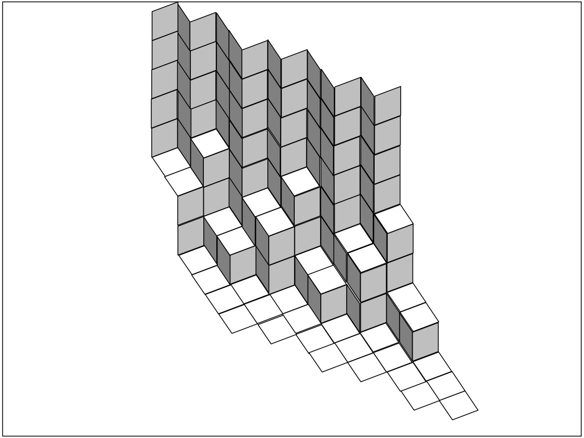

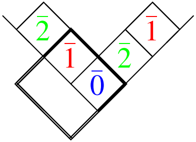

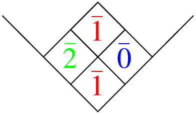

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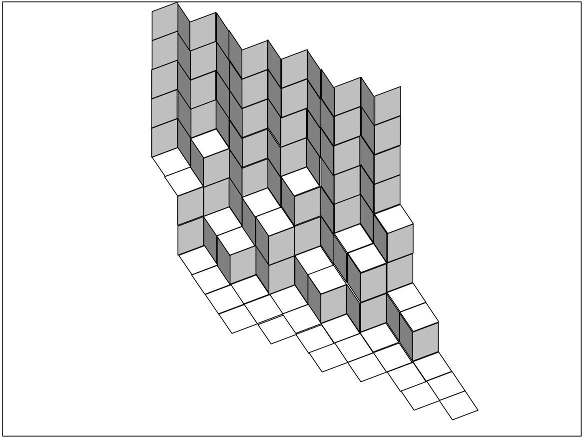

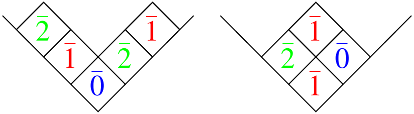

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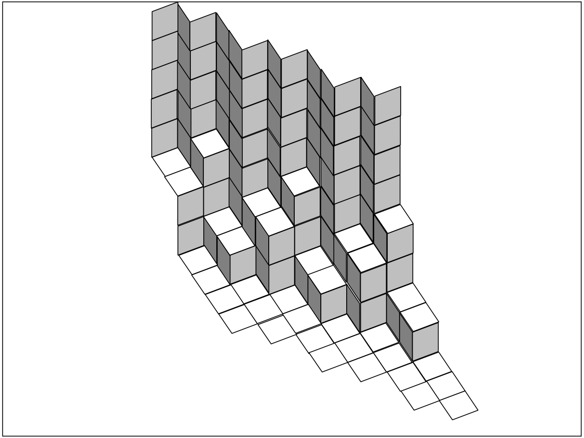

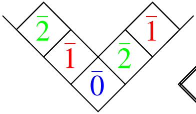

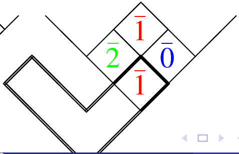

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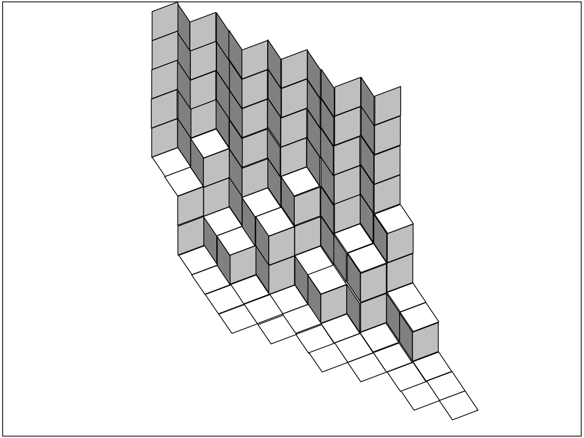

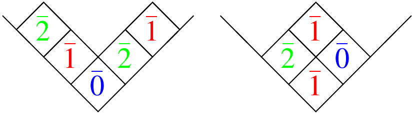

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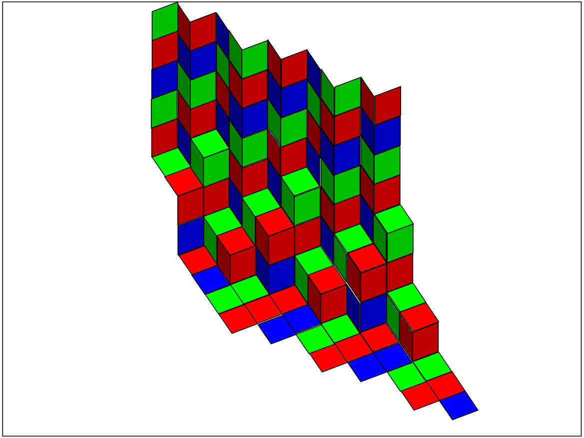
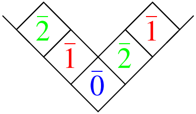
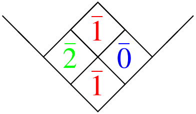
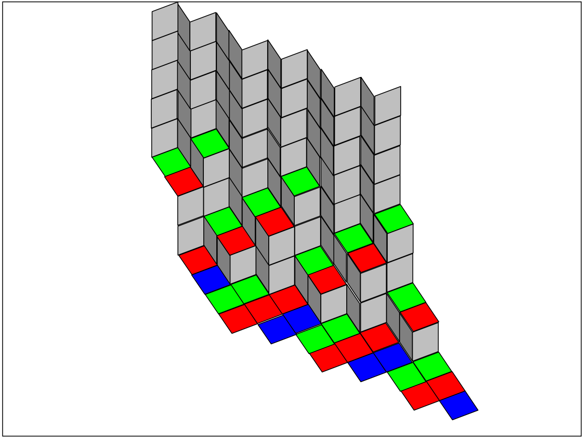
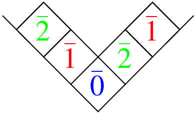


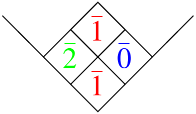




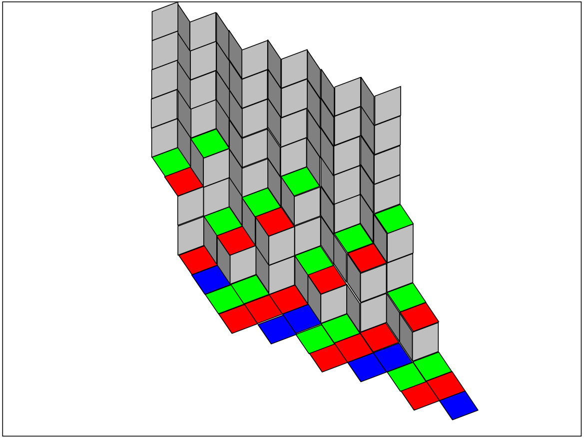
Understanding embeddings and $B(\infty)$





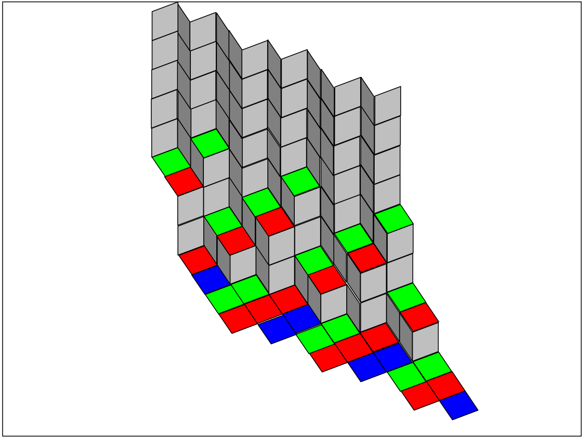


Understanding embeddings and $B(\infty)$

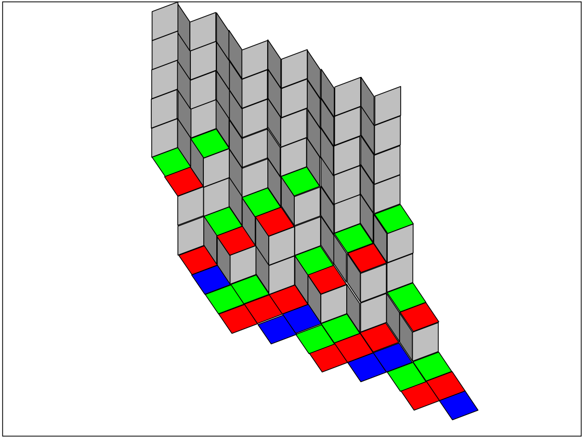


- A cylindric partition is in $B(\Lambda)$ if and only if it does not have three differently colored piles of the same height.

Understanding embeddings and $B(\infty)$

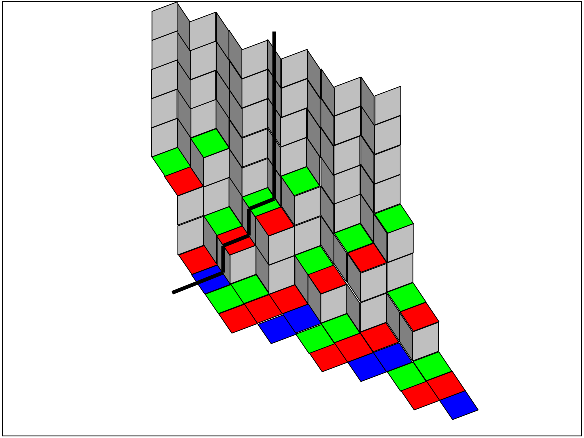


Understanding embeddings and $B(\infty)$



- To understand the embedding $B(\Lambda_0 + \Lambda_1) \hookrightarrow B(2\Lambda_1 + \Lambda_1)$, consider the "dual" n tuple of partitions.

Understanding embeddings and $B(\infty)$



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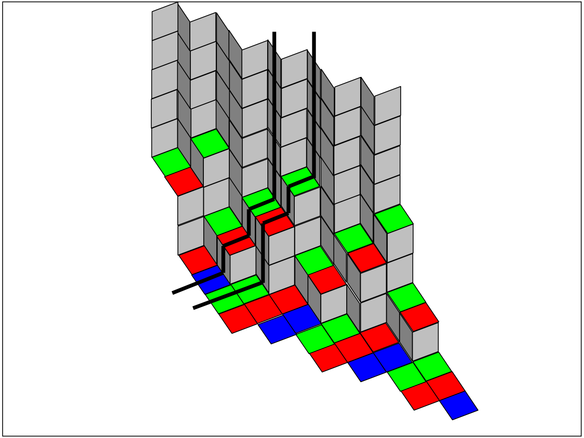
Understanding embeddings and $B(\infty)$



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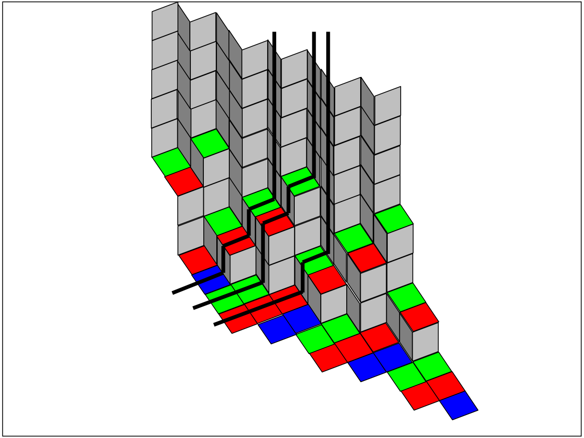
Understanding embeddings and $B(\infty)$



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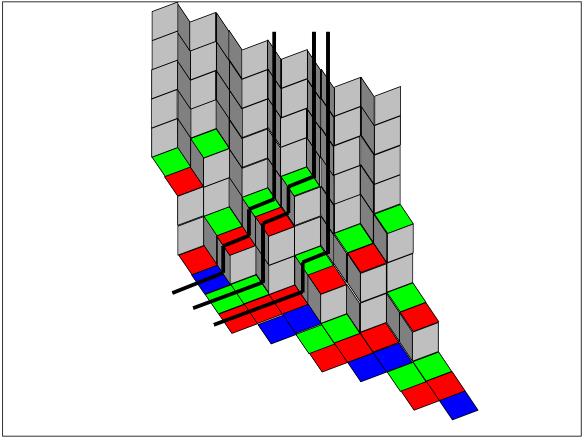
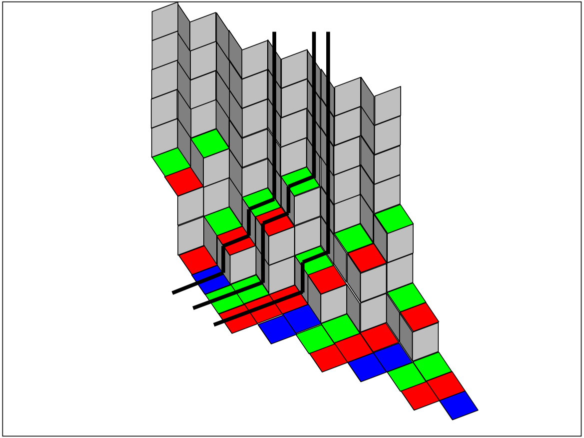
Understanding embeddings and $B(\infty)$

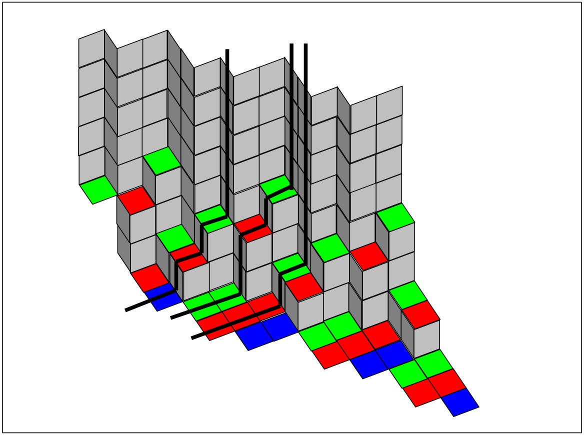


Understanding embeddings and $B(\infty)$



- Just shift the cylindric partition so that this dual n-tuple does not change.

Understanding embeddings and $B(\infty)$



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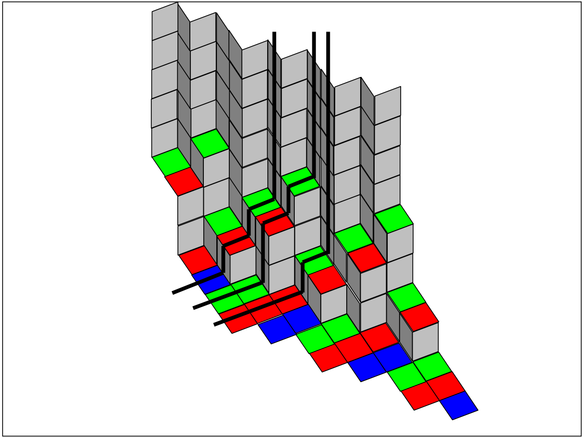
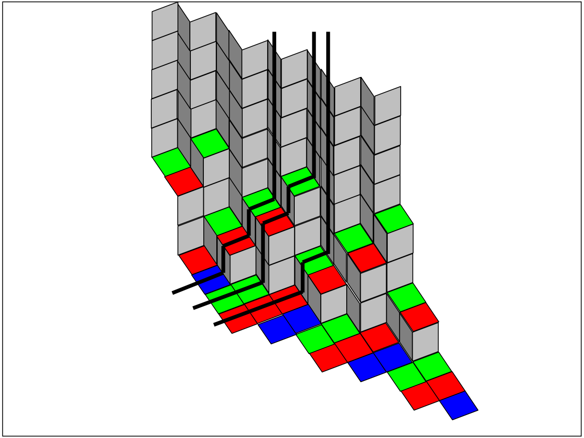
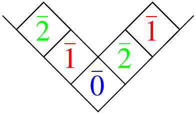
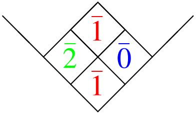
Understanding embeddings and $B(\infty)$



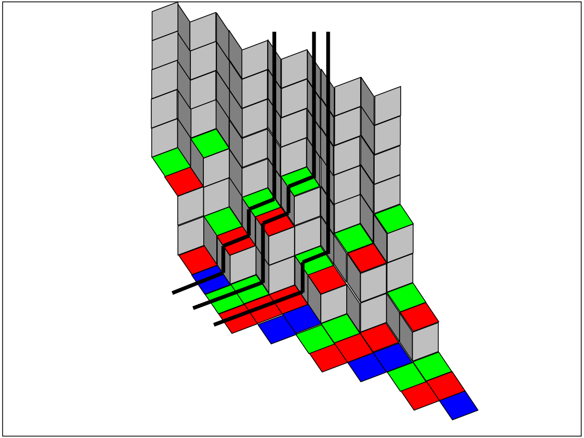
Understanding embeddings and $B(\infty)$



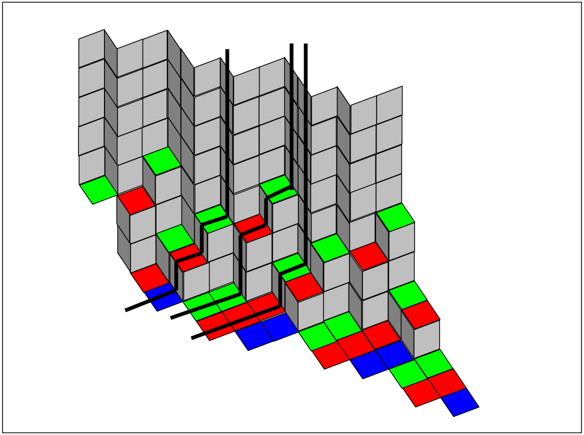




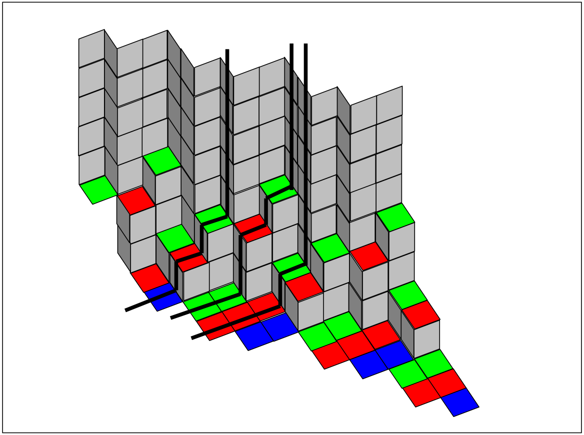
Understanding embeddings and $B(\infty)$

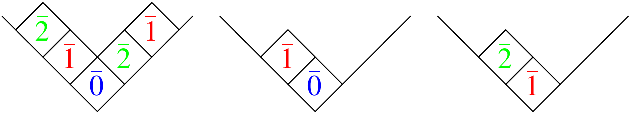


Understanding embeddings and $B(\infty)$

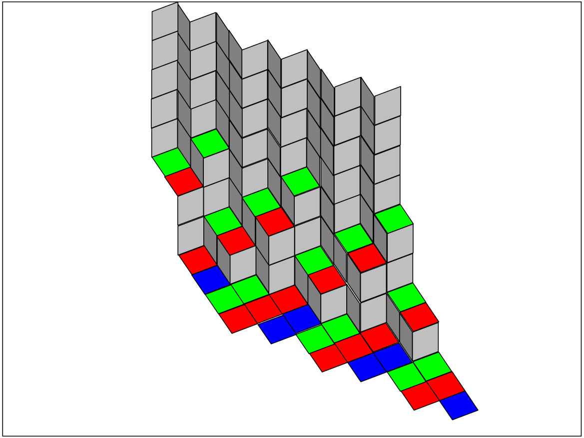


Understanding embeddings and $B(\infty)$

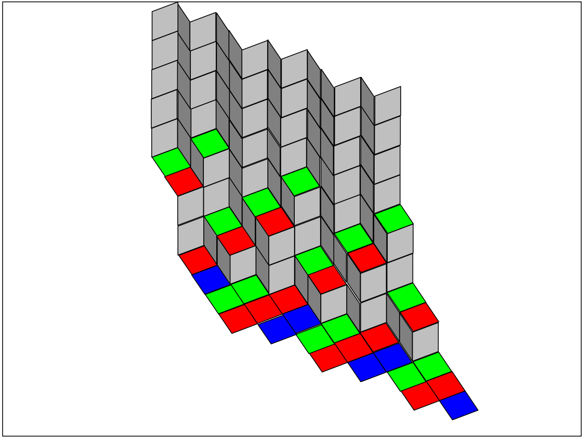




Understanding embeddings and $B(\infty)$

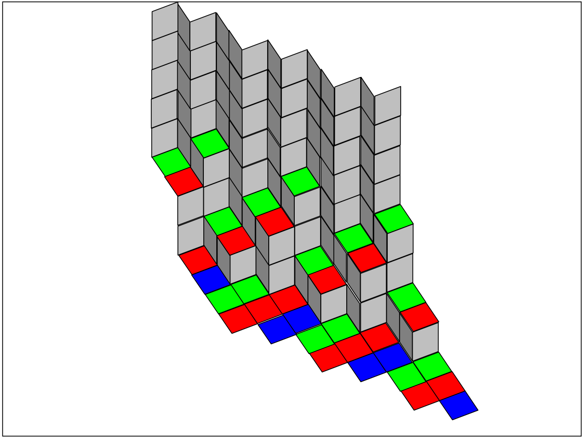


Understanding embeddings and $B(\infty)$

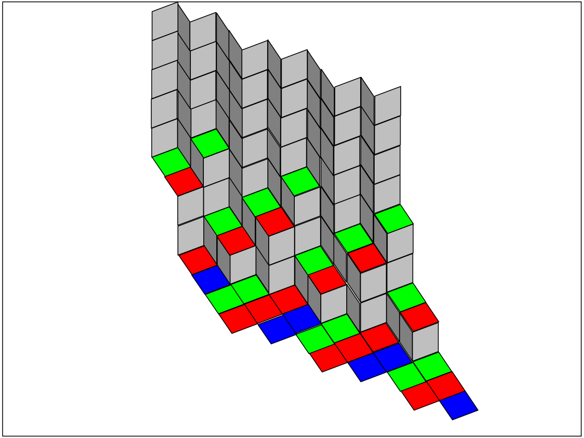


- For $B(\infty)$, just record the vertical piles, not the arrangement into an ℓ-tuple of partitions.

Understanding embeddings and $B(\infty)$

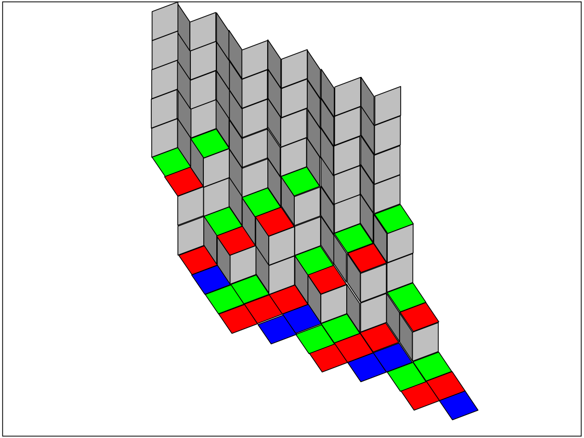


Understanding embeddings and $B(\infty)$



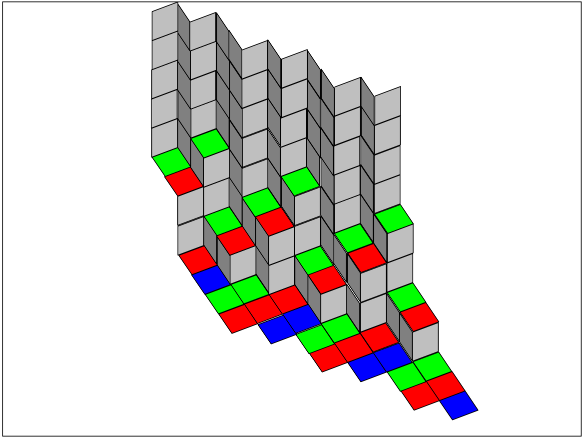
$\bar{2}$
$\bar{1}$
$\bar{0}$

Understanding embeddings and $B(\infty)$



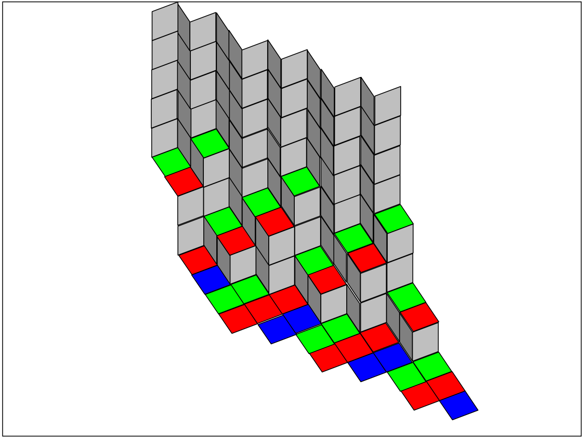
$\bar{2}$	
$\bar{1}$	$\bar{1}$
$\bar{0}$	$\bar{0}$

Understanding embeddings and $B(\infty)$



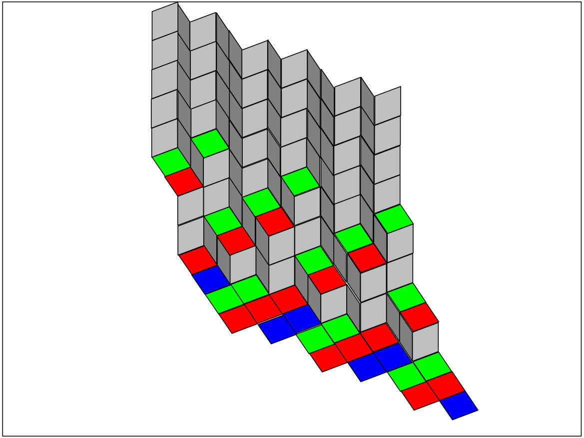
$\bar{2}$		
$\bar{1}$	$\bar{1}$	$\bar{2}$
$\bar{0}$	$\bar{0}$	$\bar{1}$

Understanding embeddings and $B(\infty)$



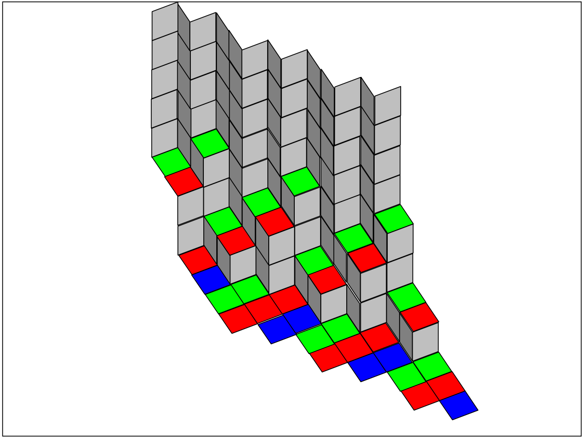
$\bar{2}$			
$\bar{1}$	$\bar{1}$	$\bar{2}$	
$\bar{0}$	$\bar{0}$	$\bar{1}$	$\bar{1}$

Understanding embeddings and $B(\infty)$



$\bar{2}$				
$\bar{1}$	$\bar{1}$	$\bar{2}$		
$\bar{0}$	$\bar{0}$	$\bar{1}$	$\bar{1}$	$\bar{2}$

Understanding embeddings and $B(\infty)$

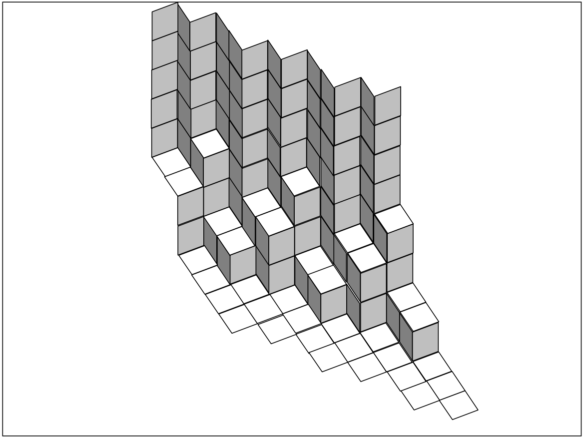


$\bar{2}$				
$\bar{1}$	$\bar{1}$	$\bar{2}$		
$\bar{0}$	$\bar{0}$	$\bar{1}$	$\bar{1}$	$\bar{2}$

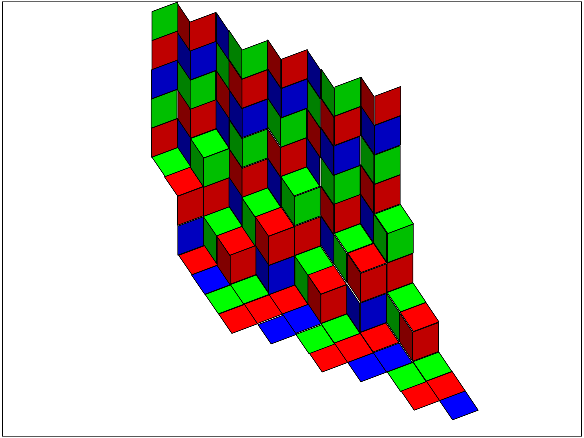
A “multi-segment"

Relation to the Kyoto path model

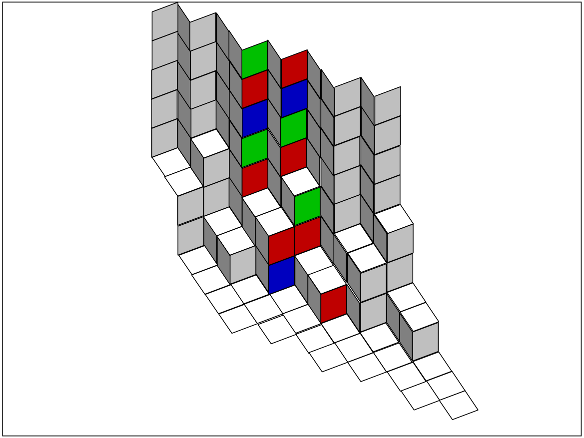
Relation to the Kyoto path model



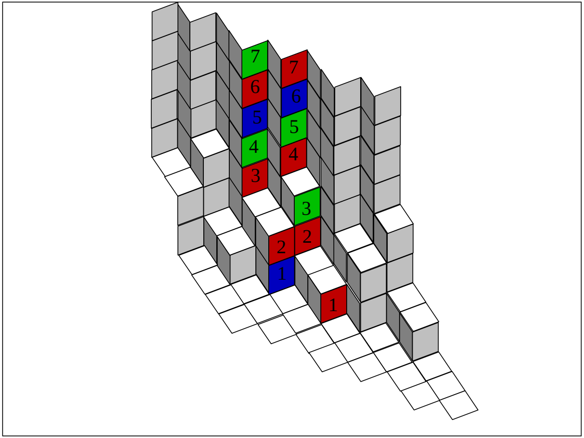
Relation to the Kyoto path model



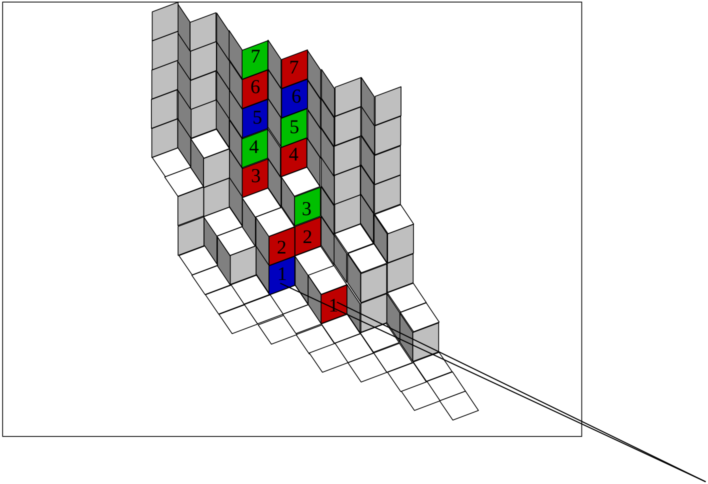
Relation to the Kyoto path model



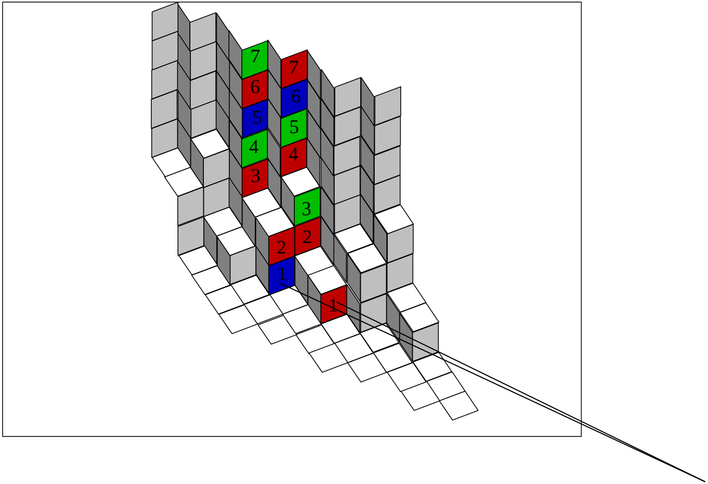
Relation to the Kyoto path model

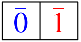


Relation to the Kyoto path model



Relation to the Kyoto path model





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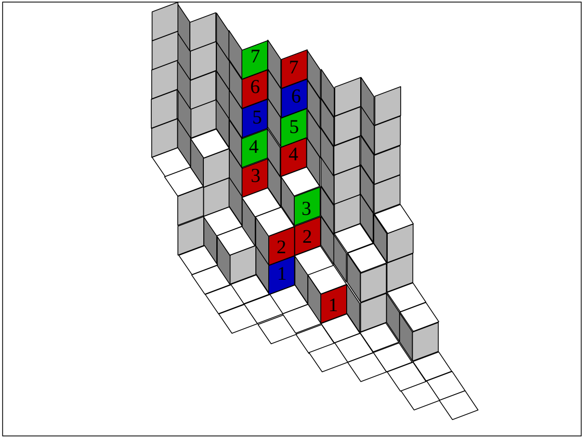
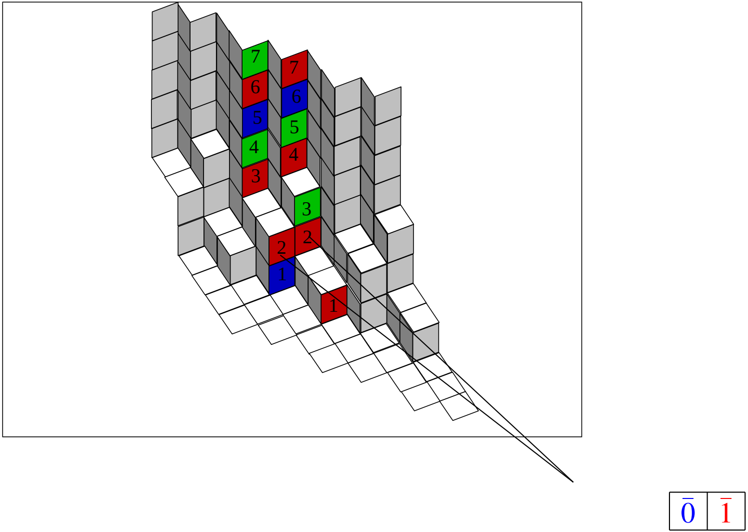
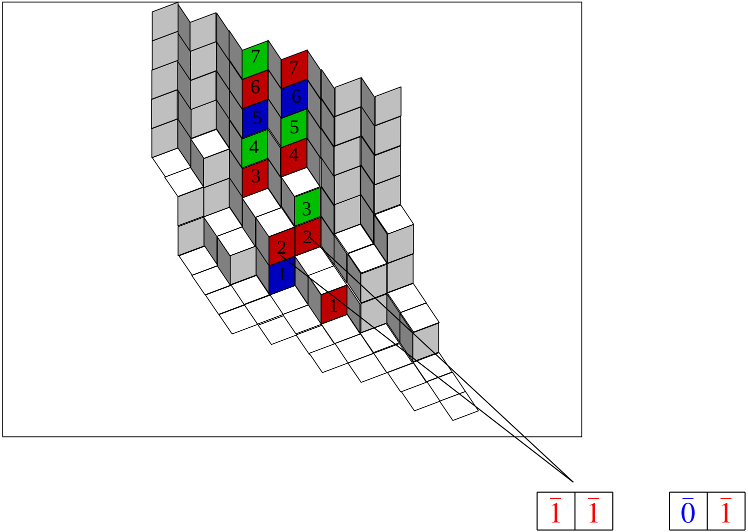


$\bar{0}$	$\bar{1}$

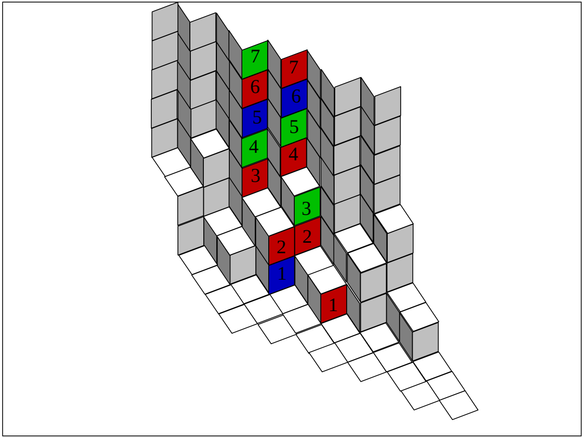
Relation to the Kyoto path model



Relation to the Kyoto path model



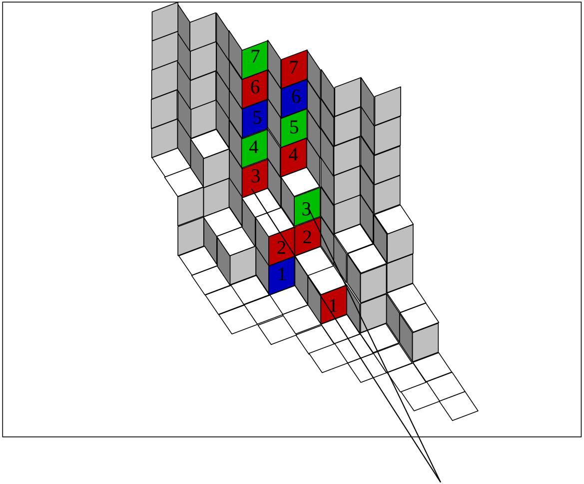
Relation to the Kyoto path model



$\bar{1}$	$\bar{1}$

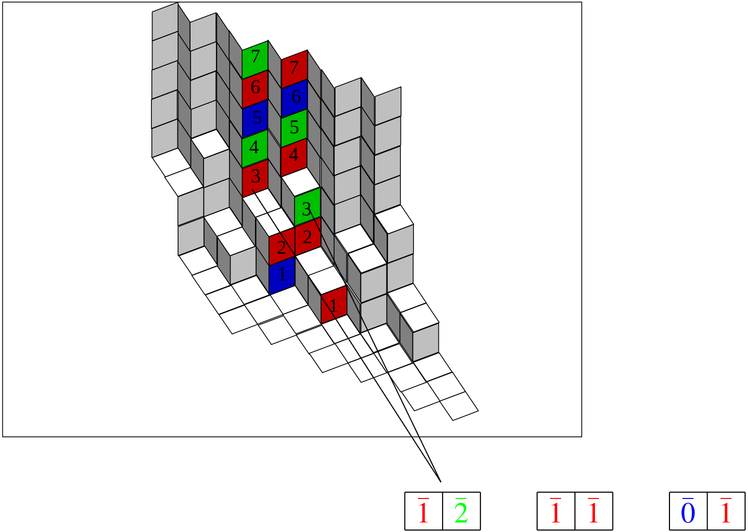
$\bar{0}$	$\bar{1}$

Relation to the Kyoto path model



$\bar{1}$ | $\bar{1}$ $\qquad$ $\bar{0}$ | $\bar{1}$

Relation to the Kyoto path model



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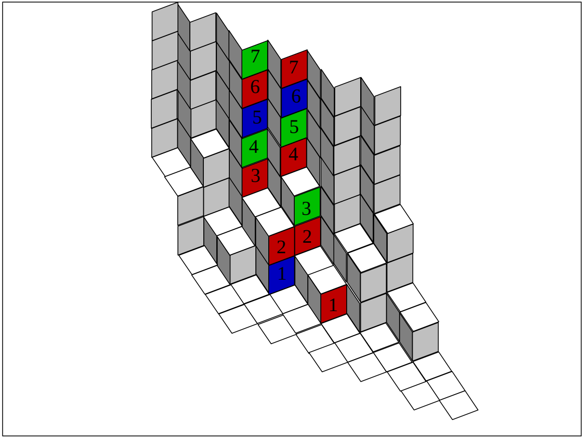
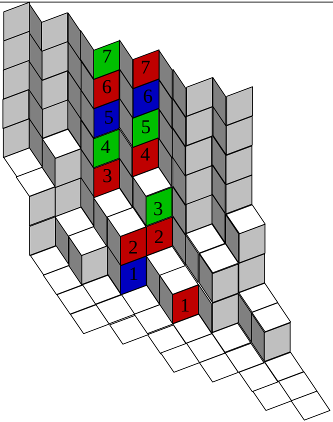


$\bar{1}$	$\bar{2}$

$\bar{1}$	$\bar{1}$

$\bar{0}$	$\bar{1}$

Relation to the Kyoto path model



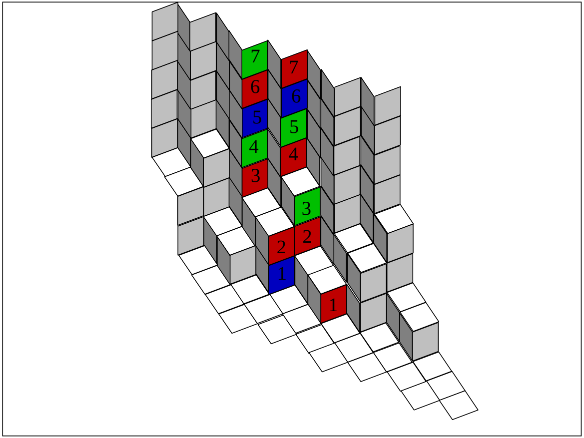
$\bar{1}$	$\bar{2}$

$\bar{1}$	$\bar{2}$

$\bar{1}$	$\bar{1}$

$\bar{0}$	$\bar{1}$

Relation to the Kyoto path model



$\bar{0}$	$\bar{2}$

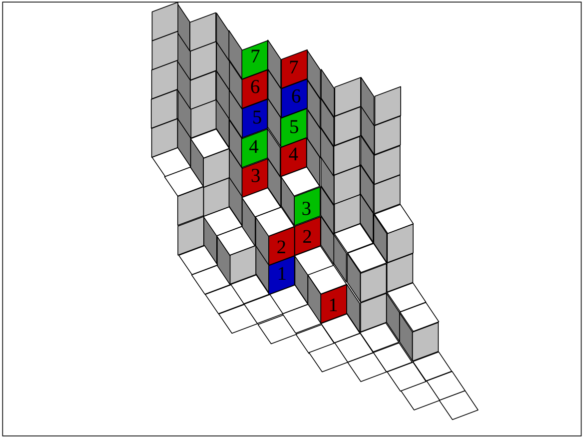
$\bar{1}$	$\bar{2}$

$\bar{1}$	$\bar{2}$

$\bar{1}$	$\bar{1}$

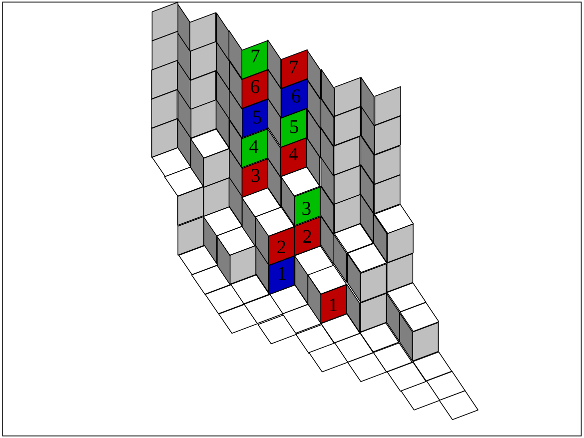
$\bar{0}$	$\bar{1}$

Relation to the Kyoto path model



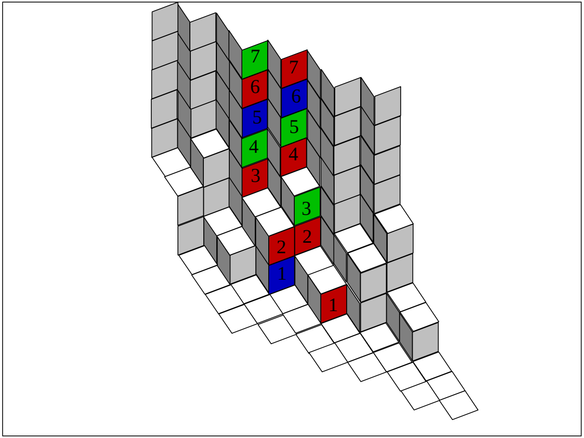
$\bar{0}$	$\bar{1}$	$\bar{0}$	$\bar{2}$	$\bar{1}$	$\bar{2}$	$\bar{1}$	$\bar{2}$	$\bar{1}$	$\bar{1}$	$\bar{0}$	$\bar{1}$

Relation to the Kyoto path model



$\bar{1}$	$\bar{2}$	$\bar{0}$	$\bar{1}$	$\bar{0}$	$\bar{2}$	$\bar{1}$	$\bar{2}$	$\bar{1}$	$\bar{2}$	$\bar{1}$	$\bar{1}$	$\bar{0}$	$\bar{1}$

Relation to the Kyoto path model



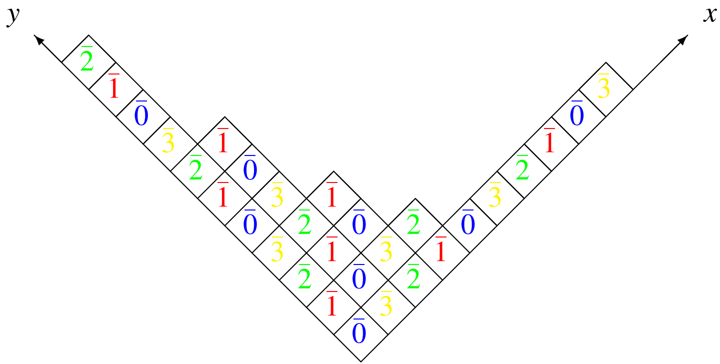
$$\cdots \quad \boxed{\bar{1} \mid \bar{2}} \otimes \boxed{\bar{0} \mid \bar{1}} \otimes \boxed{\bar{0} \mid \bar{2}} \otimes \boxed{\bar{1} \mid \bar{2}} \otimes \boxed{\bar{1} \mid \bar{2}} \otimes \boxed{\bar{1} \mid \bar{1}} \otimes \boxed{\bar{0} \mid \bar{1}}$$

The “horizontal" crystal

The "horizontal" crystal

- Define new operators $E_{\bar{i}}$ and $F_{\bar{i}}$ on the set of partitions.

The "horizontal" crystal



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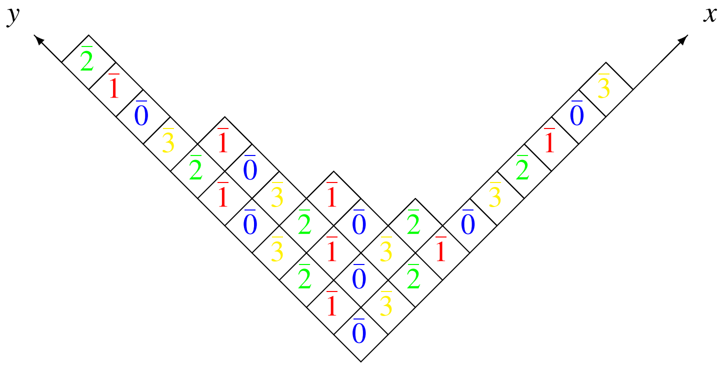
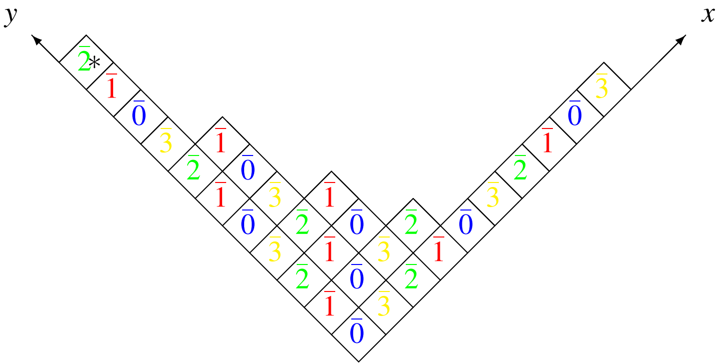
The "horizontal" crystal



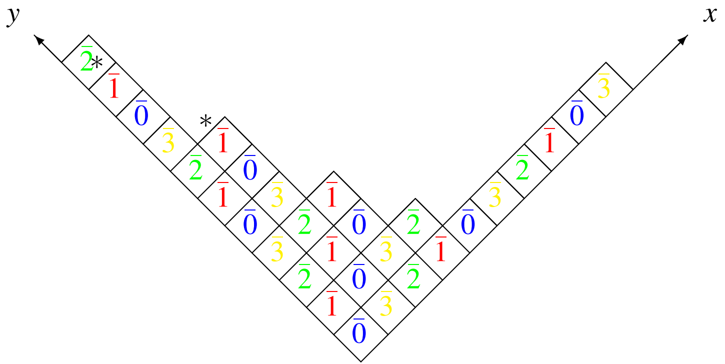
- Define new operators $E_{\bar{i}}$ and $F_{\bar{i}}$ on the set of partitions.
- for $\bar{i} = \bar{2}$, constuct a string of brackets as before, but ordered lexicographically by height, then right to left.

The "horizontal" crystal



- Define new operators $E_{\bar{i}}$ and $F_{\bar{i}}$ on the set of partitions.
- for $\bar{i} = \bar{2}$, constuct a string of brackets as before, but ordered lexicographically by height, then right to left.

The "horizontal" crystal



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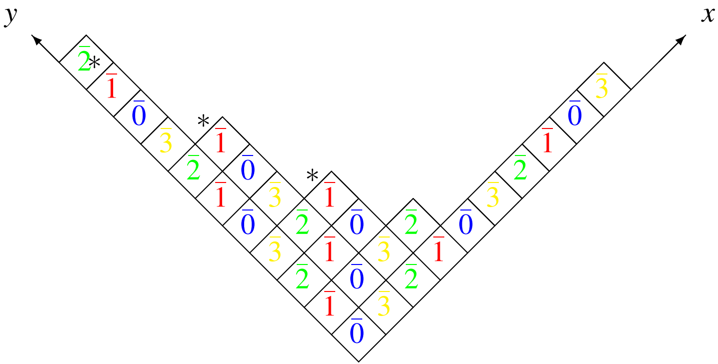
The "horizontal" crystal



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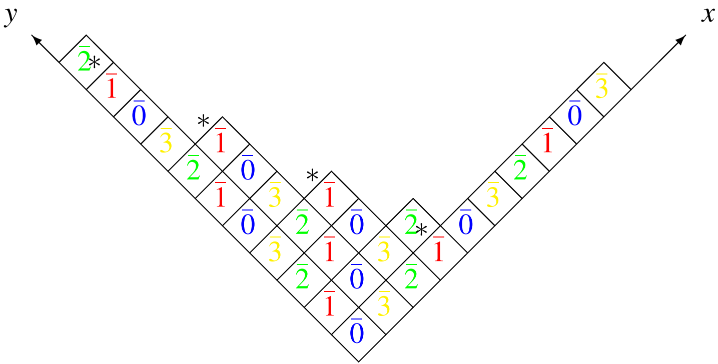
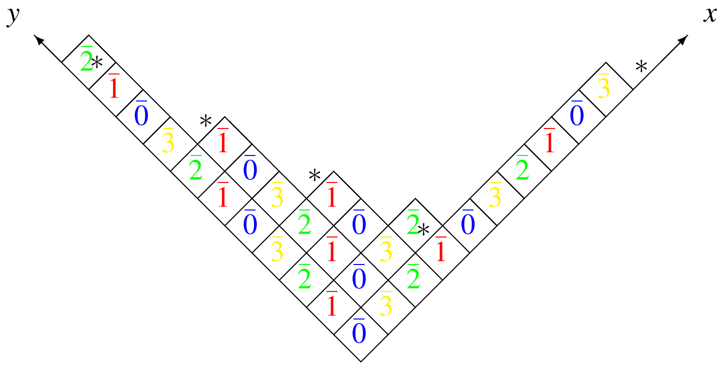
The “horizontal" crystal



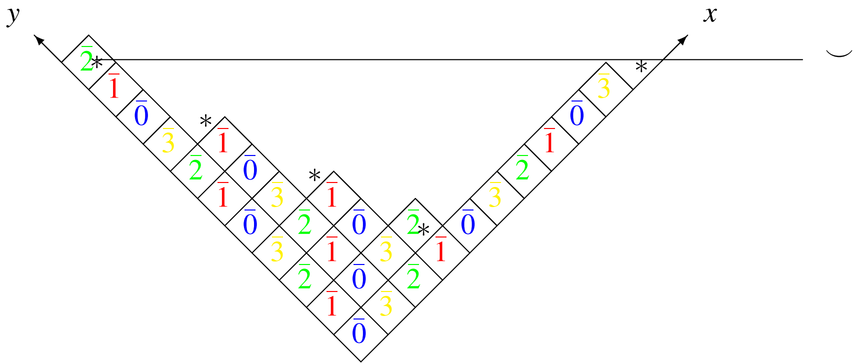
- Define new operators $E_{\bar{i}}$ and $F_{\bar{i}}$ on the set of partitions.
- for $\bar{i} = \bar{2}$, constuct a string of brackets as before, but ordered lexicographically by height, then right to left.

The "horizontal" crystal



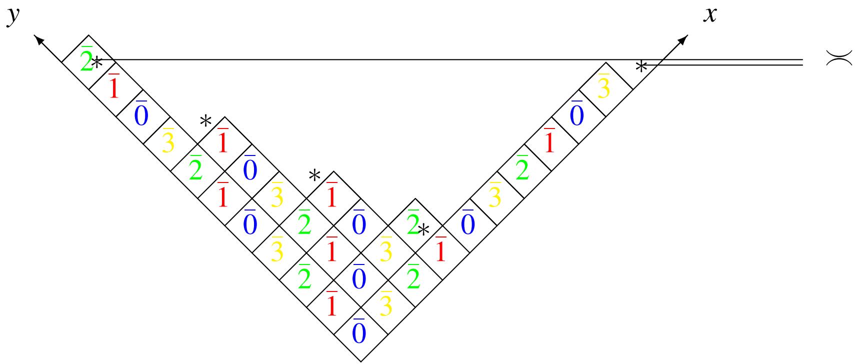
- Define new operators $E_{\bar{i}}$ and $F_{\bar{i}}$ on the set of partitions.
- for $\bar{i} = \bar{2}$, constuct a string of brackets as before, but ordered lexicographically by height, then right to left.

The "horizontal" crystal



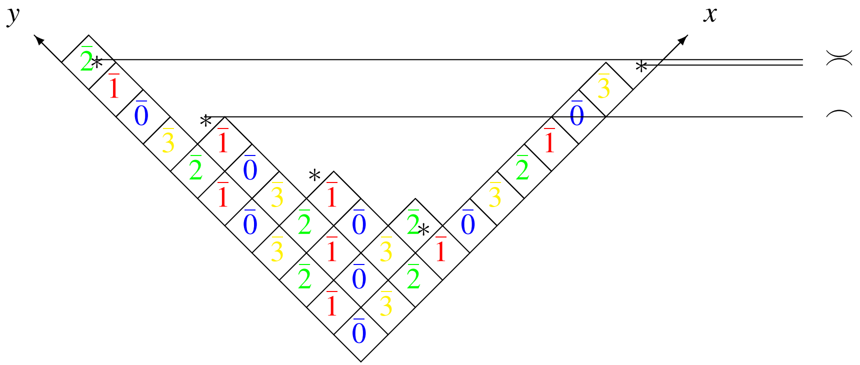
- Define new operators $E_{\bar{i}}$ and $F_{\bar{i}}$ on the set of partitions.
- for $\bar{i} = \bar{2}$, constuct a string of brackets as before, but ordered lexicographically by height, then right to left.

The "horizontal" crystal



- Define new operators $E_{\bar{i}}$ and $F_{\bar{i}}$ on the set of partitions.
- for $\bar{i} = \bar{2}$, constuct a string of brackets as before, but ordered lexicographically by height, then right to left.

The "horizontal" crystal



- Define new operators $E_{\bar{i}}$ and $F_{\bar{i}}$ on the set of partitions.
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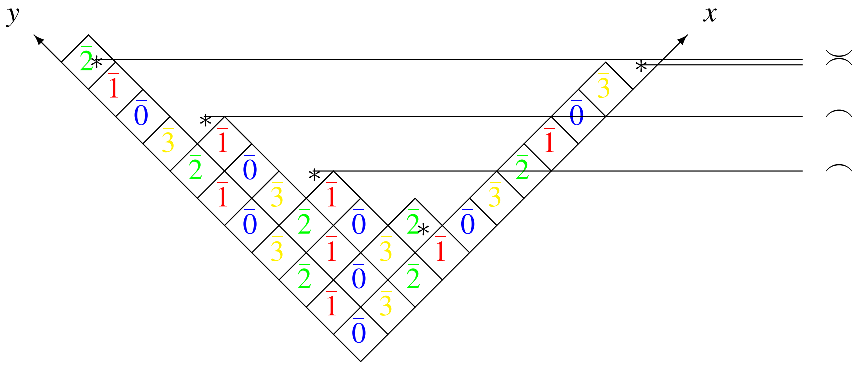

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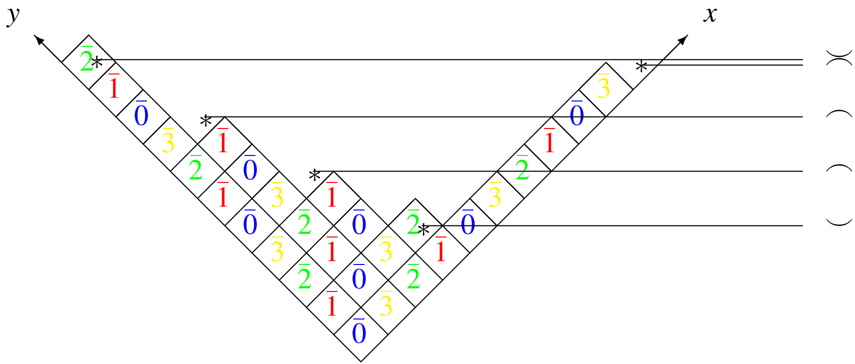

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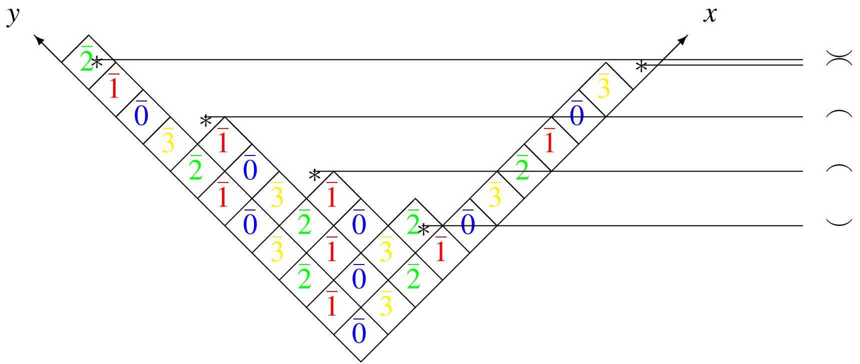

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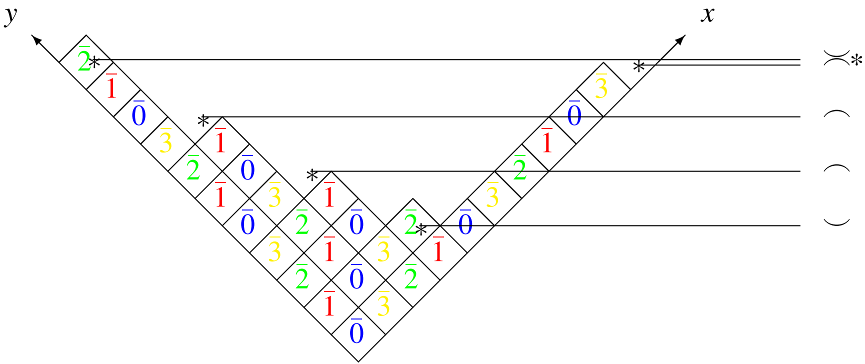

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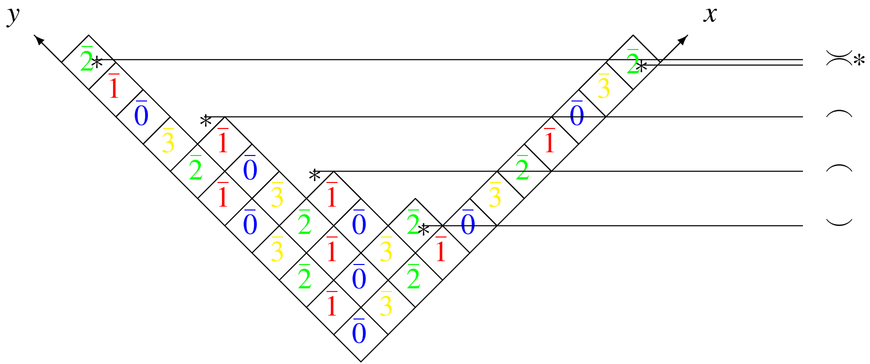

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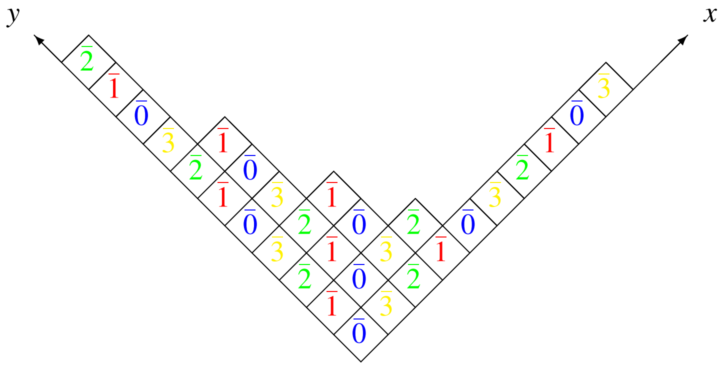

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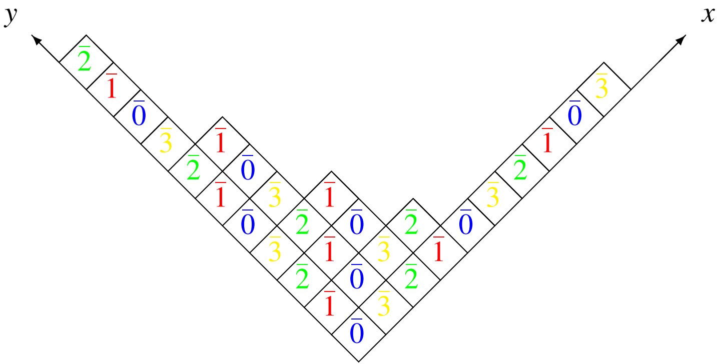

- The component generated by the empty partition is a copy of $B(\Lambda_0)$.

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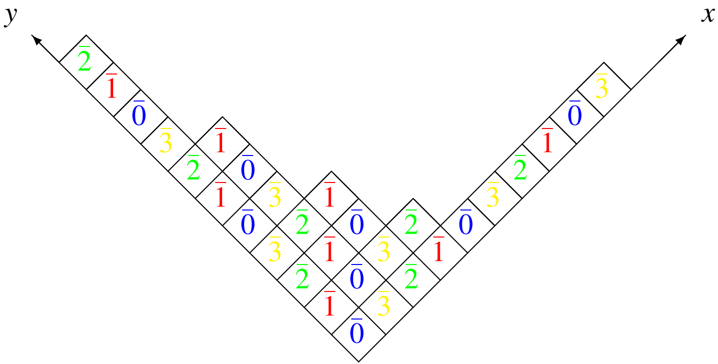

- The component generated by the empty partition is a copy of $B(\Lambda_0)$.
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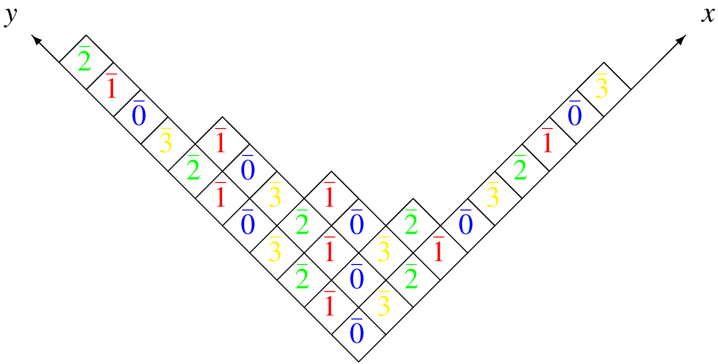

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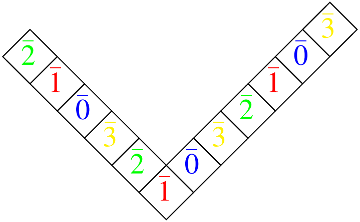

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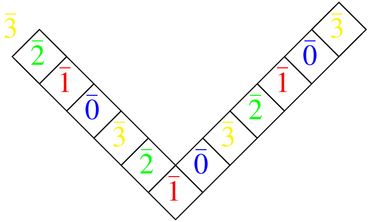

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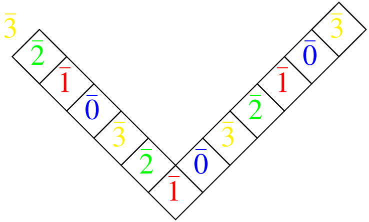

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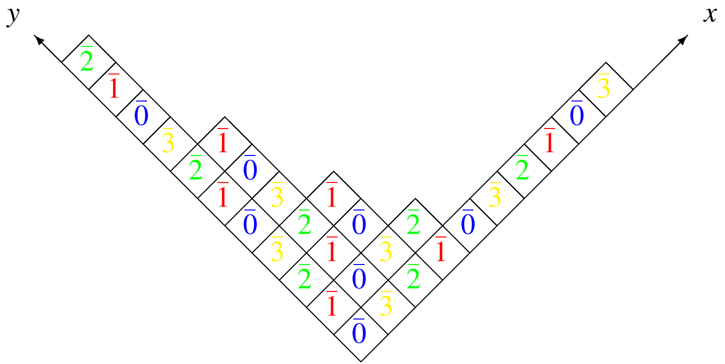

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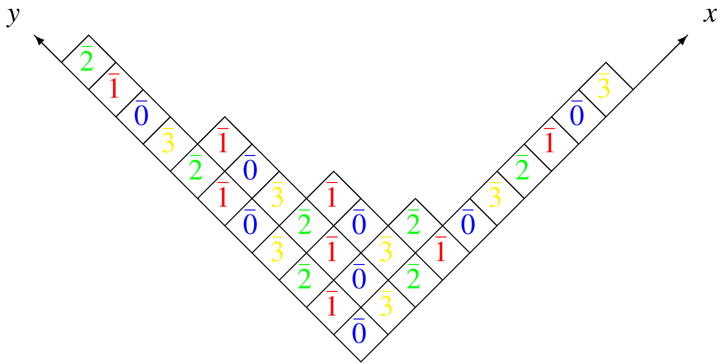
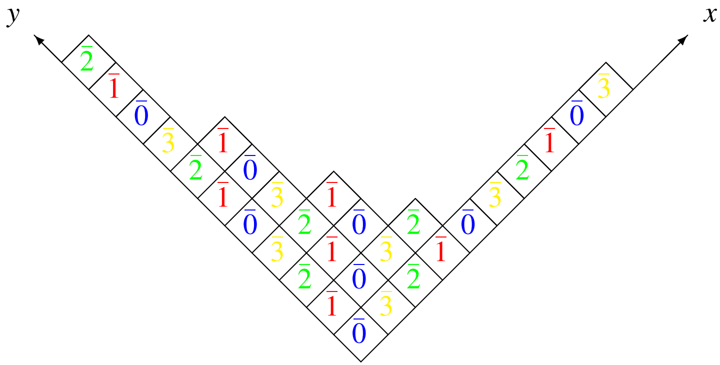


- One can actually read the boxes according to ANY slope

The "horizontal" crystal



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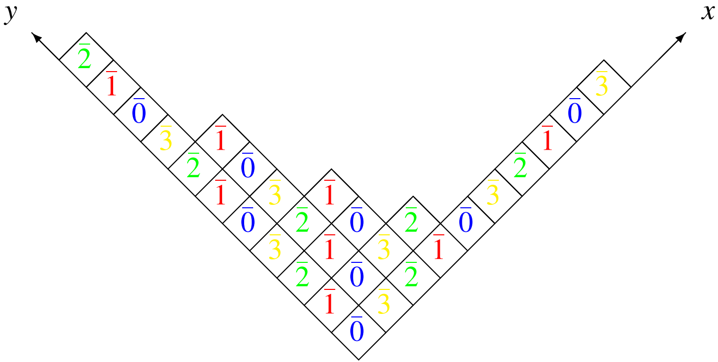

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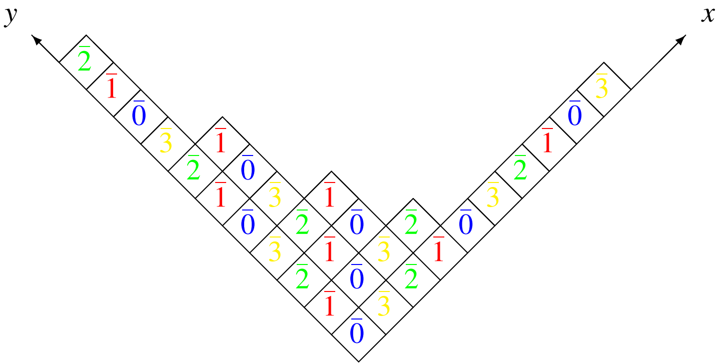

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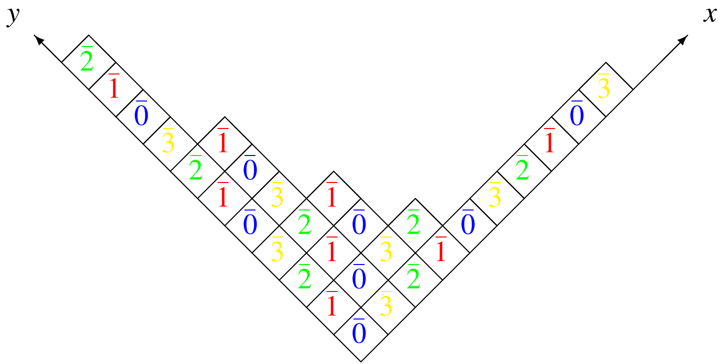


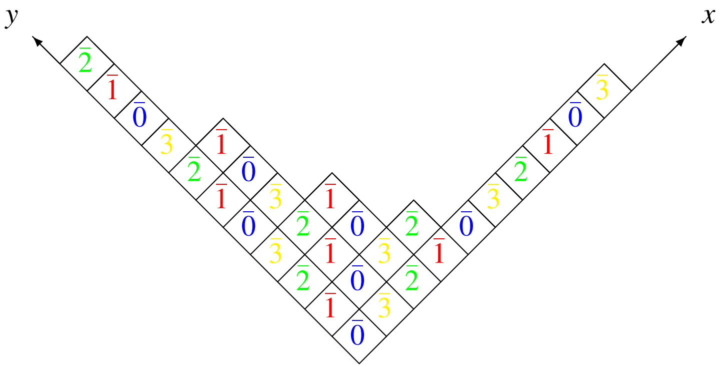
- One can actually read the boxes according to ANY slope
- The same result is true, although definition of "illegal hook" is a bit more complicated.

Horizontal to monomial

Horizontal to monomial

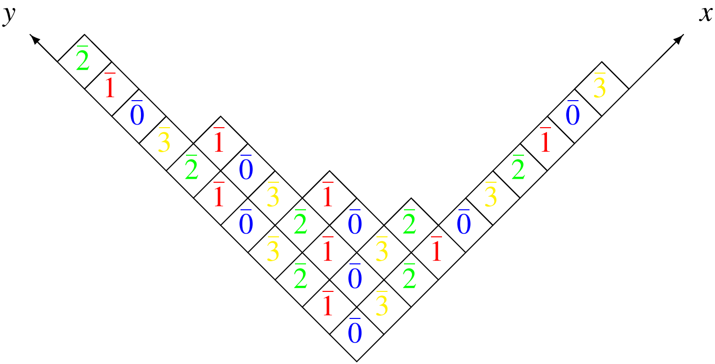
- There is a natural isomorphism between $B(\Lambda_0)$ realized using the horizontal crystal and $B(\Lambda_0)$ realized using the monimial crystal.

Horizontal to monomial



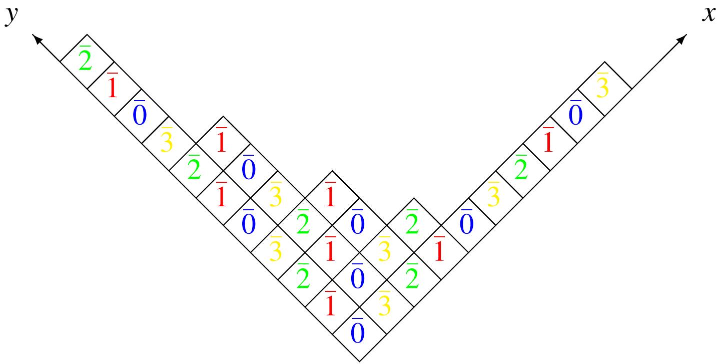
- There is a natural isomorphism between $B(\Lambda_0)$ realized using the horizontal crystal and $B(\Lambda_0)$ realized using the monimial crystal.

Horizontal to monomial

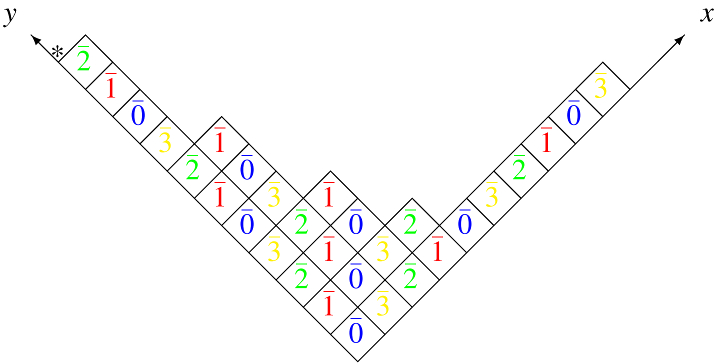


- There is a natural isomorphism between $B(\Lambda_0)$ realized using the horizontal crystal and $B(\Lambda_0)$ realized using the monimial crystal.
- Each “inner" corner corresponds to a Y and each “outer" corner to a Y^{-1}

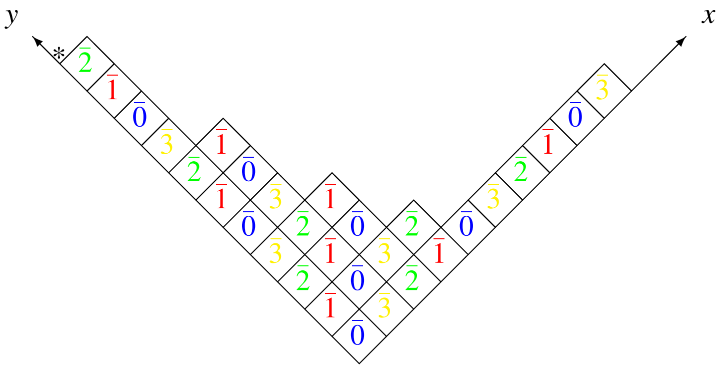
Horizontal to monomial



Horizontal to monomial

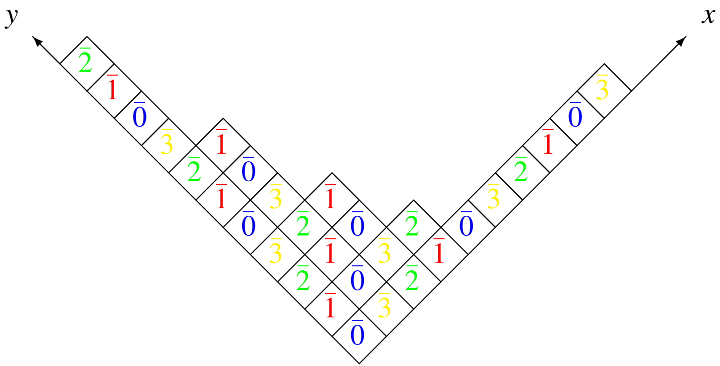


Horizontal to monomial



$Y_{\bar{3},11}$

Horizontal to monomial



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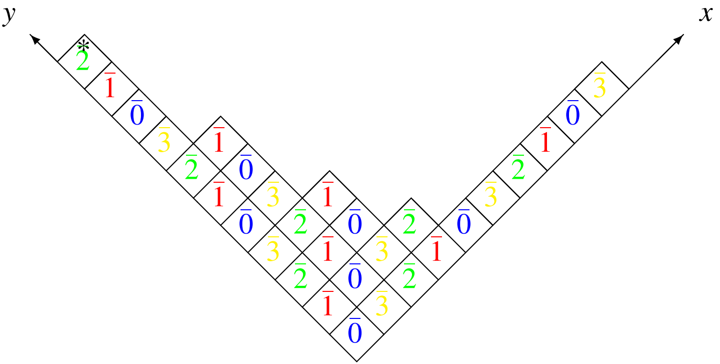

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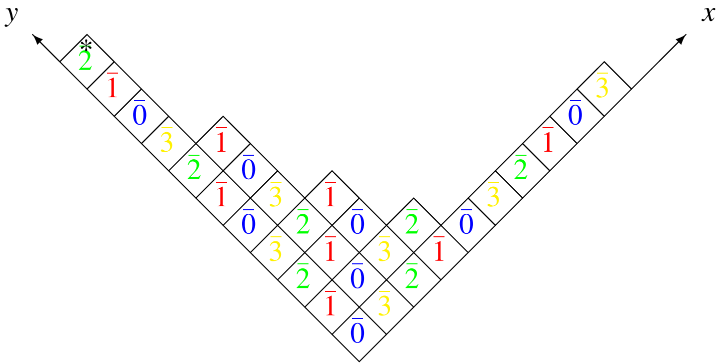
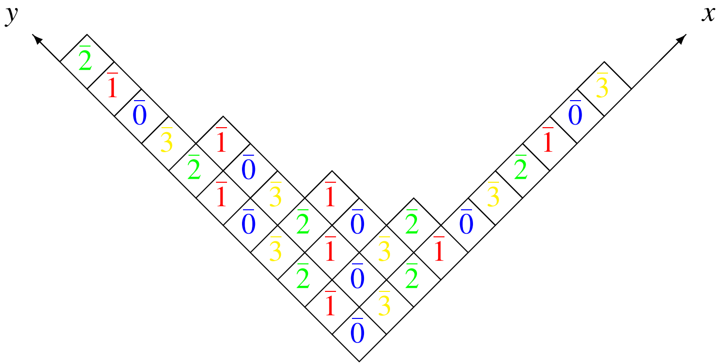


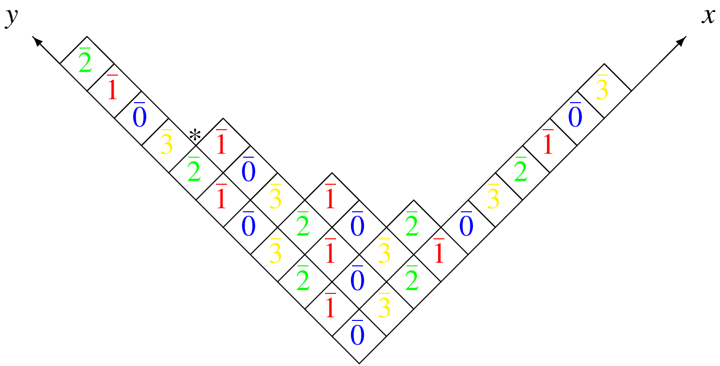
$Y_{\bar{3},11} Y_{\bar{2},12}^{-1}$

Horizontal to monomial



$$Y_{\bar{3},11} Y^{-1}_{\bar{2},12}$$

Horizontal to monomial



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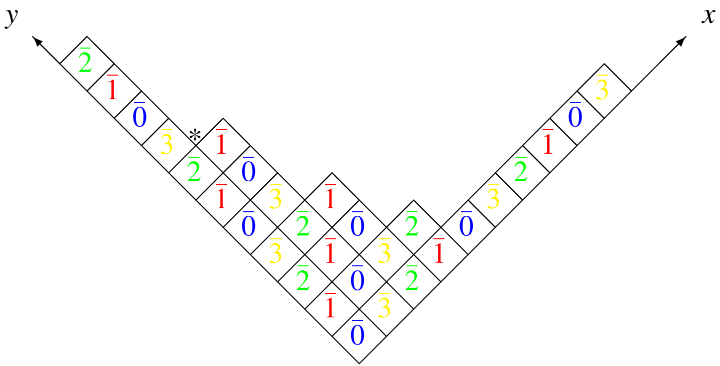
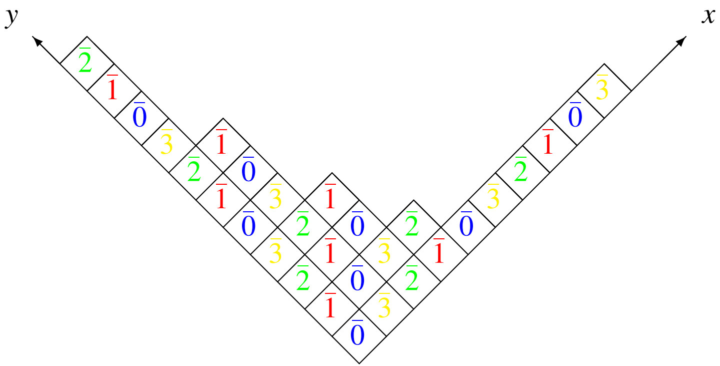
Horizontal to monomial



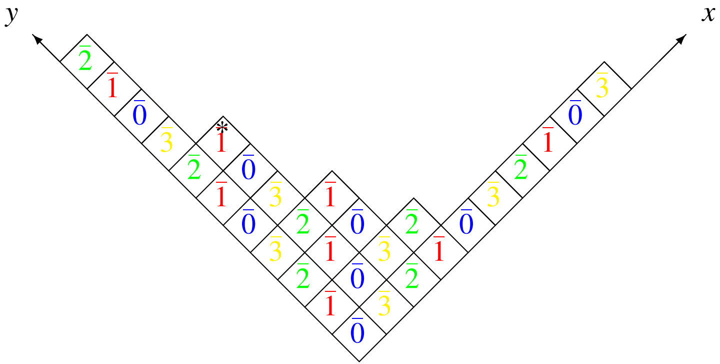
$Y_{\bar{3},11} Y_{\bar{2},12}^{-1} Y_{\bar{2},8}$

Horizontal to monomial



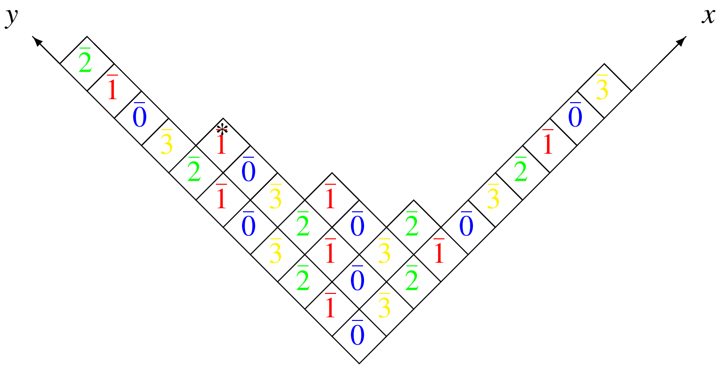
$Y_{\bar{3},11}Y_{\bar{2},12}^{-1}Y_{\bar{2},8}$

Horizontal to monomial



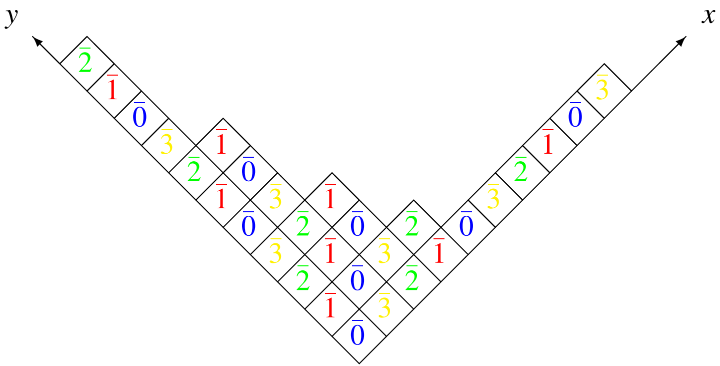
$Y_{\bar{3},11} Y^{-1}_{\bar{2},12} Y_{\bar{2},8}$

Horizontal to monomial



$Y_{\bar{3},11} Y_{\bar{2},12}^{-1} Y_{\bar{2},8} \; Y_{\bar{1},9}^{-1}$

Horizontal to monomial



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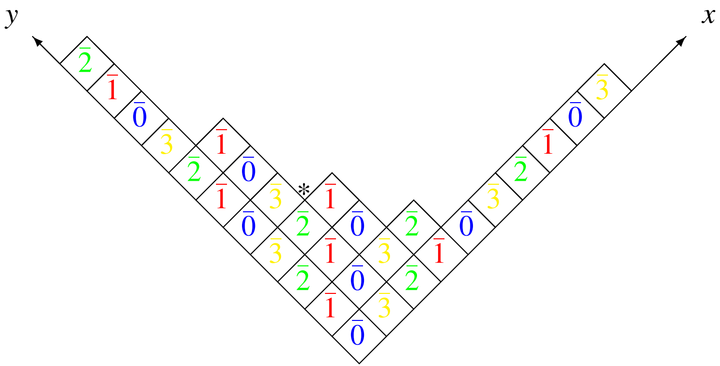

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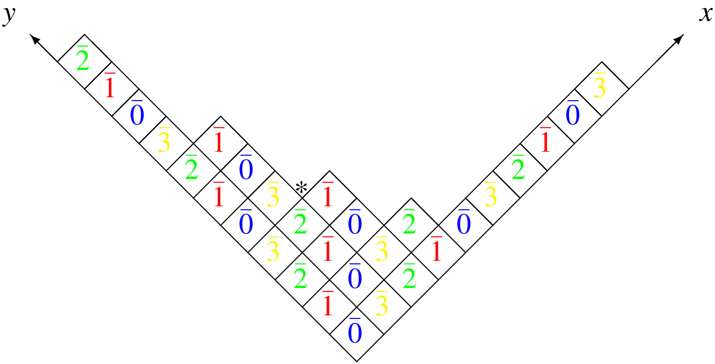
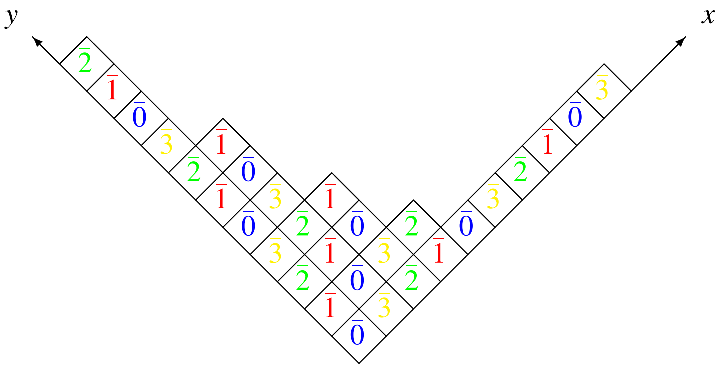
Horizontal to monomial



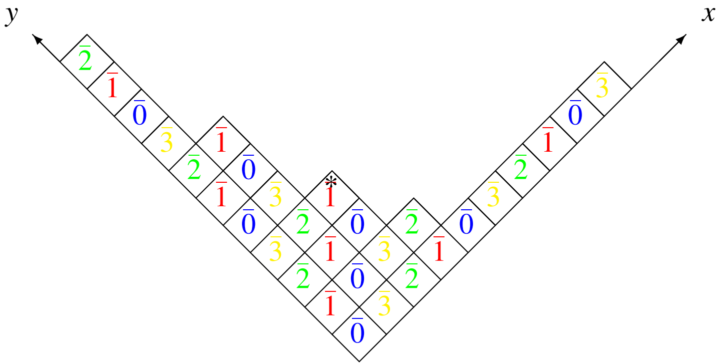
$$Y_{\bar{3},11} Y^{-1}_{\bar{2},12} Y_{\bar{2},8} \; Y^{-1}_{\bar{1},9} \; Y_{\bar{2},6}$$

Horizontal to monomial



$$Y_{\bar{3},11} Y_{\bar{2},12}^{-1} Y_{\bar{2},8} \; Y_{\bar{1},9}^{-1} \; Y_{\bar{2},6}$$

Horizontal to monomial



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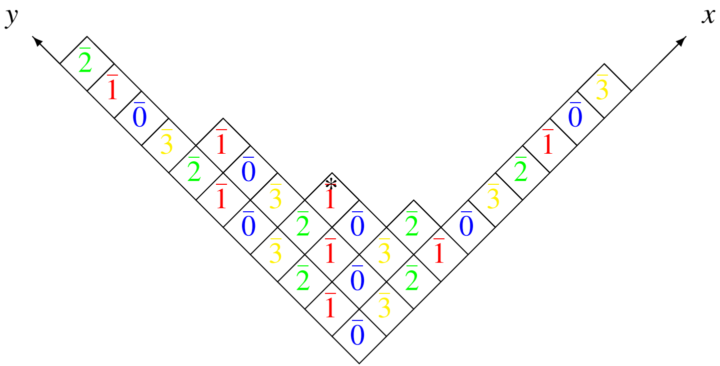
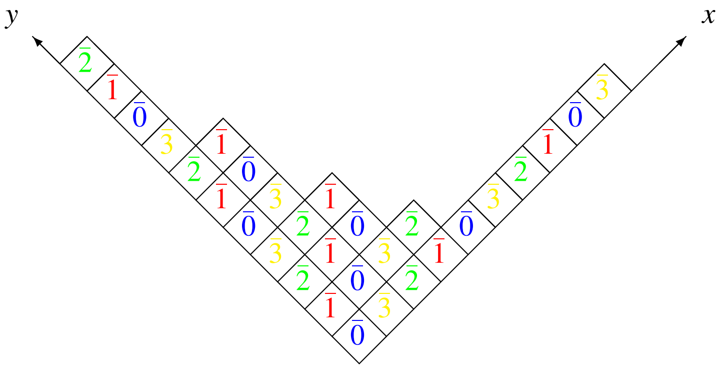
Horizontal to monomial



$$Y_{\bar{3},11} Y^{-1}_{\bar{2},12} Y_{\bar{2},8} \; Y^{-1}_{\bar{1},9} \; Y_{\bar{2},6} \; Y^{-1}_{\bar{1},7}$$

Horizontal to monomial



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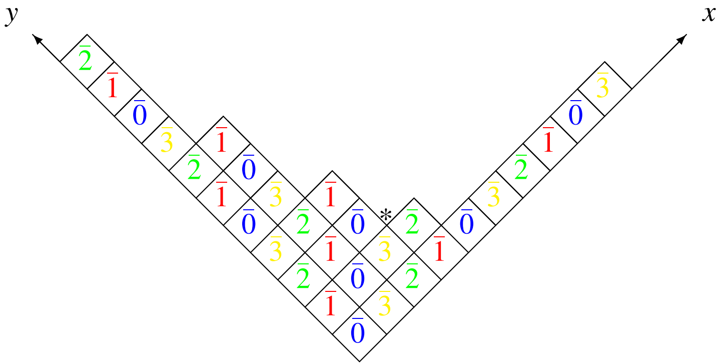
Horizontal to monomial



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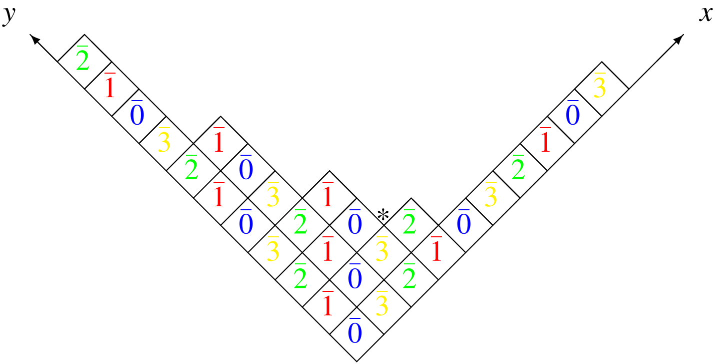
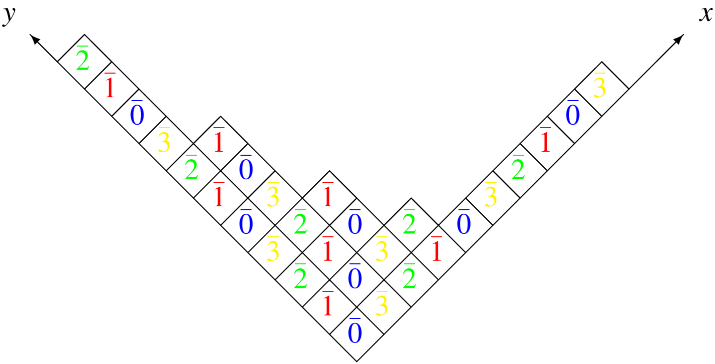
Horizontal to monomial



$$Y_{\bar{3},11} Y^{-1}_{\bar{2},12} Y_{\bar{2},8} \; Y^{-1}_{\bar{1},9} \; Y_{\bar{2},6} \; Y^{-1}_{\bar{1},7} \; Y_{\bar{3},5}$$

Horizontal to monomial



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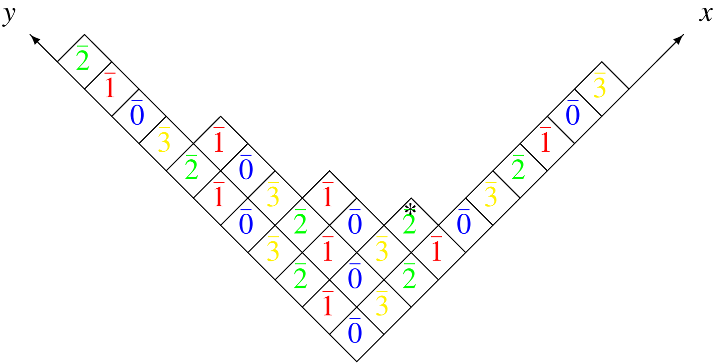

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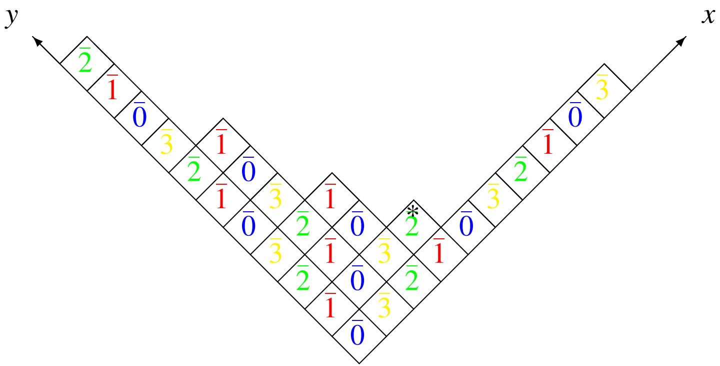
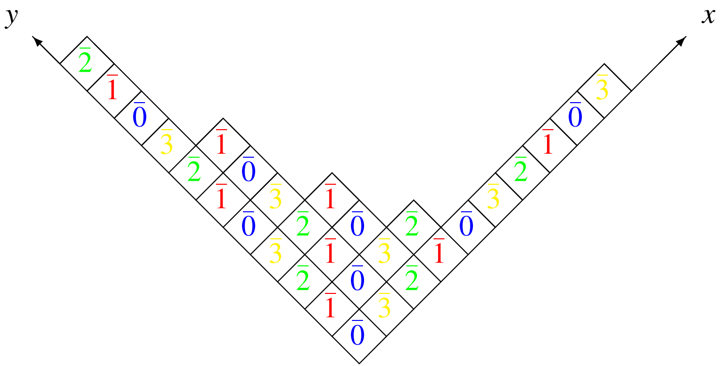


$$Y_{\bar{3},11} Y_{\bar{2},12}^{-1} Y_{\bar{2},8} \; Y_{\bar{1},9}^{-1} \; Y_{\bar{2},6} \; Y_{\bar{1},7}^{-1} \; Y_{\bar{3},5} \; Y_{\bar{2},6}^{-1}$$

Horizontal to monomial



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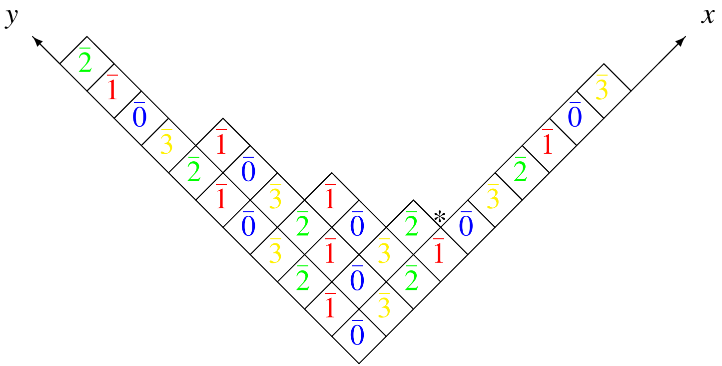

$$Y_{\bar{3},11} Y_{\bar{2},12}^{-1} Y_{\bar{2},8} \; Y_{\bar{1},9}^{-1} \; Y_{\bar{2},6} \; Y_{\bar{1},7}^{-1} \; Y_{\bar{3},5} \; Y_{\bar{2},6}^{-1}$$

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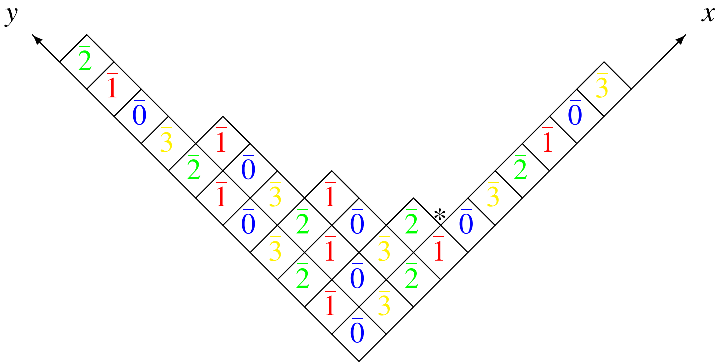

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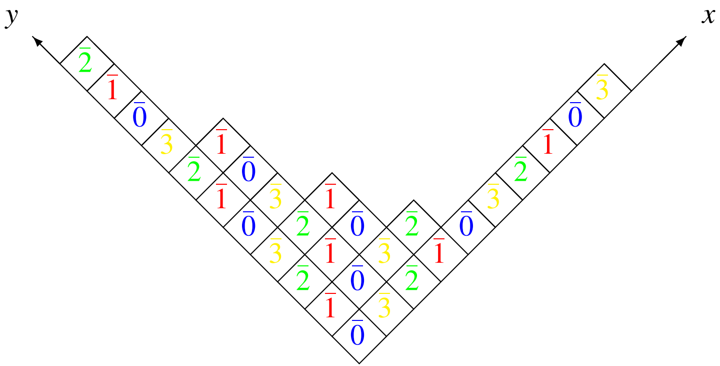
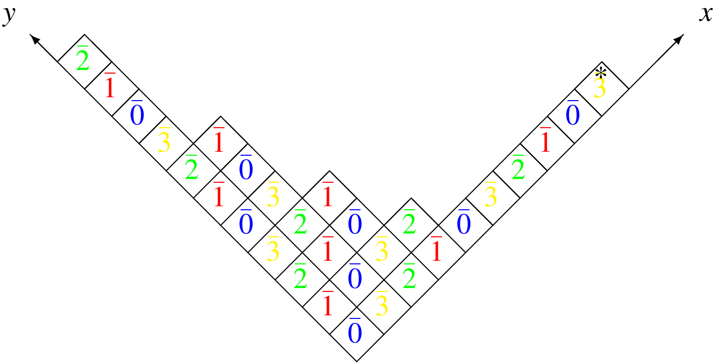


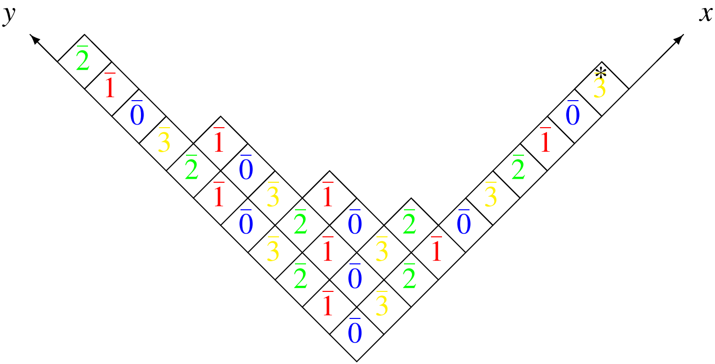
$$Y_{\bar{3},11} Y_{\bar{2},12}^{-1} Y_{\bar{2},8} \; Y_{\bar{1},9}^{-1} \; Y_{\bar{2},6} \; Y_{\bar{1},7}^{-1} \; Y_{\bar{3},5} \; Y_{\bar{2},6}^{-1} \; Y_{\bar{1},5}$$

Horizontal to monomial



$$Y_{\bar{3},11} Y^{-1}_{\bar{2},12} Y_{\bar{2},8} \; Y^{-1}_{\bar{1},9} \; Y_{\bar{2},6} \; Y^{-1}_{\bar{1},7} \; Y_{\bar{3},5} \; Y^{-1}_{\bar{2},6} \; Y_{\bar{1},5}$$

Horizontal to monomial



$$Y_{\bar{3},11} Y^{-1}_{\bar{2},12} Y_{\bar{2},8} \; Y^{-1}_{\bar{1},9} \; Y_{\bar{2},6} \; Y^{-1}_{\bar{1},7} \; Y_{\bar{3},5} \; Y^{-1}_{\bar{2},6} \; Y_{\bar{1},5} \; Y^{-1}_{\bar{3},11}$$

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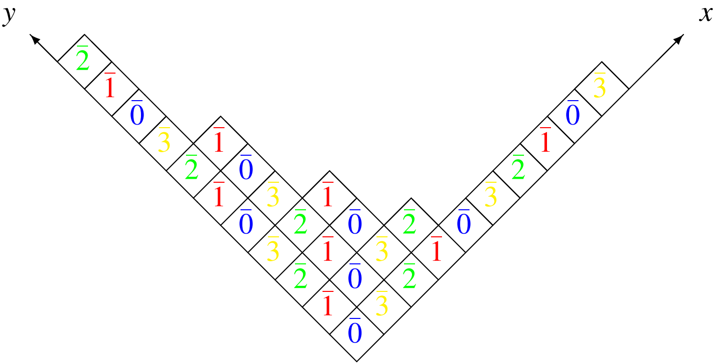

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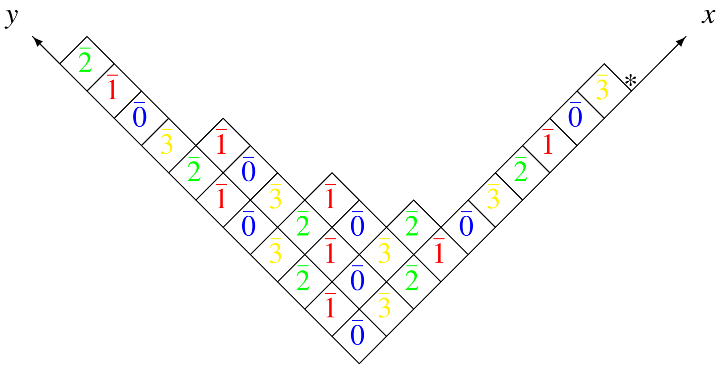

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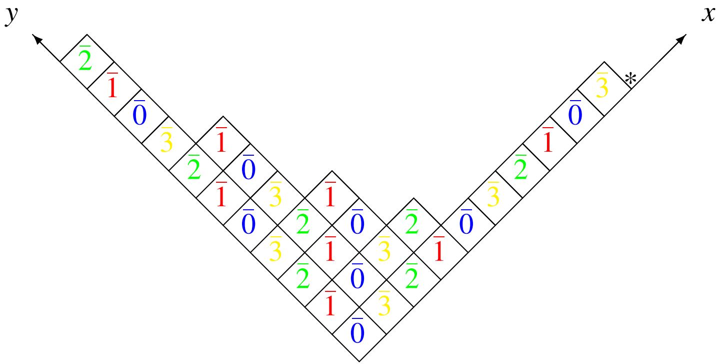

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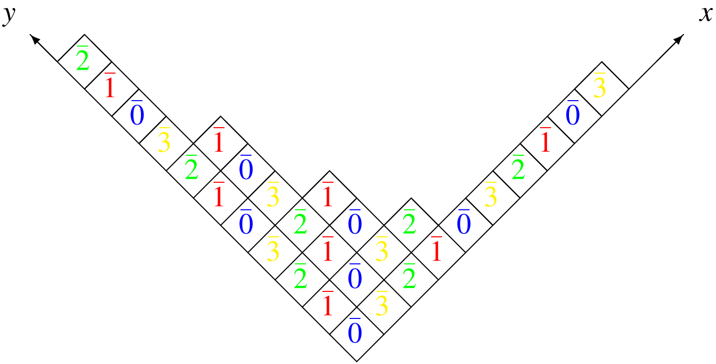

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$$||$$

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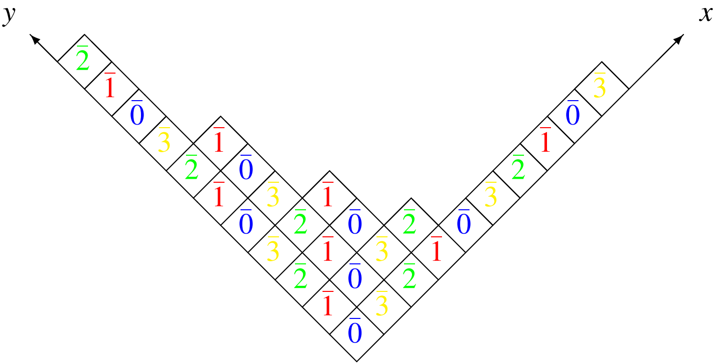

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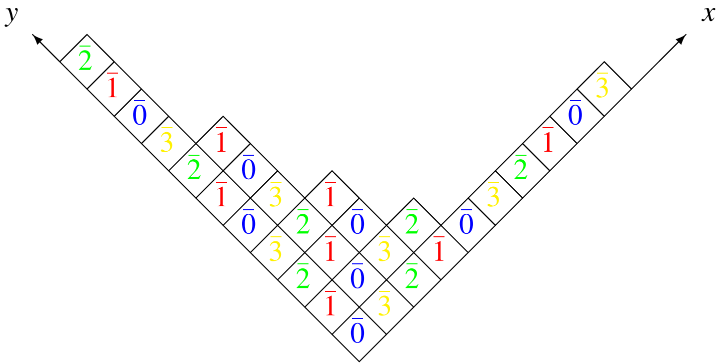

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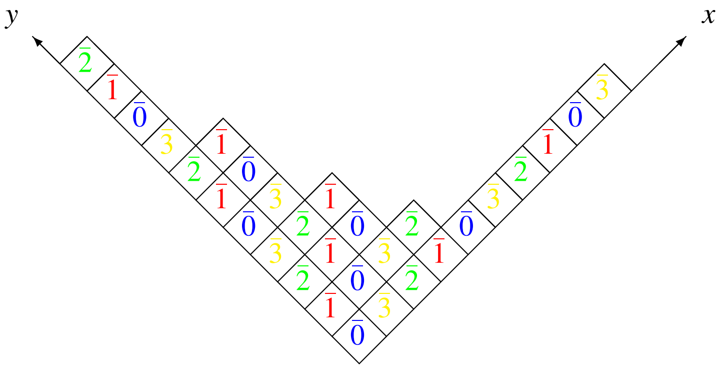

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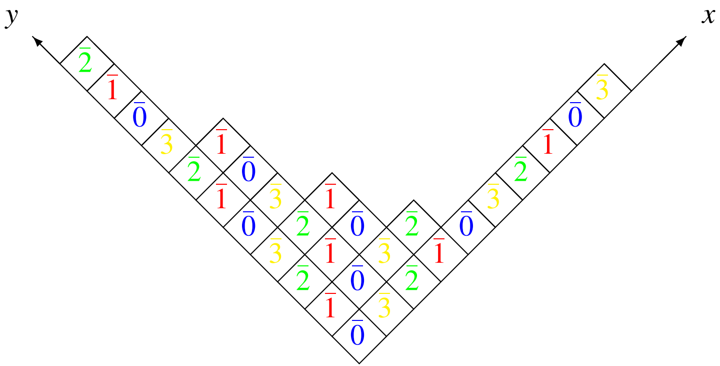

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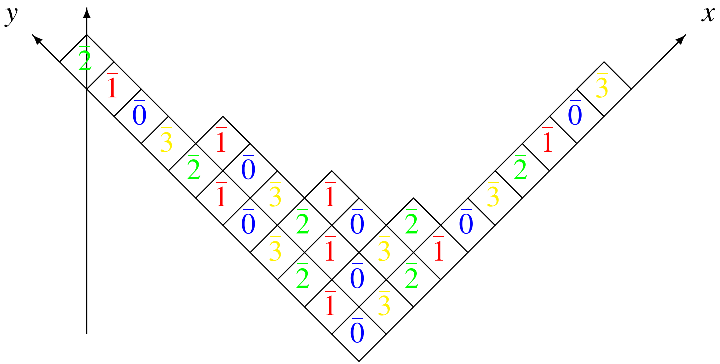

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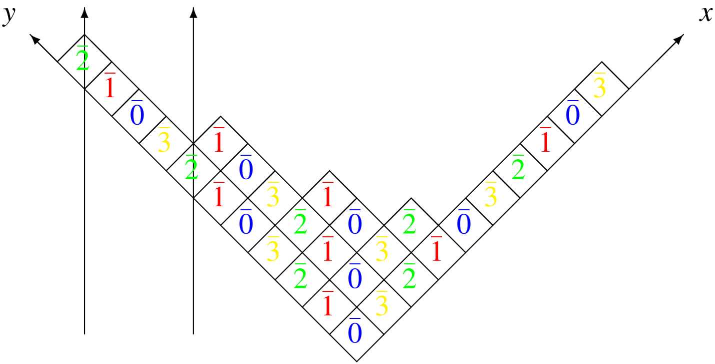

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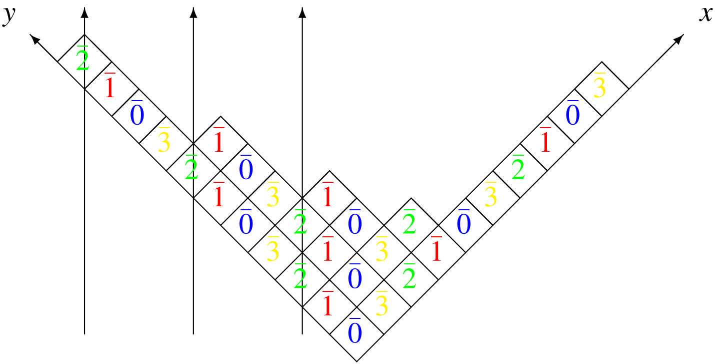

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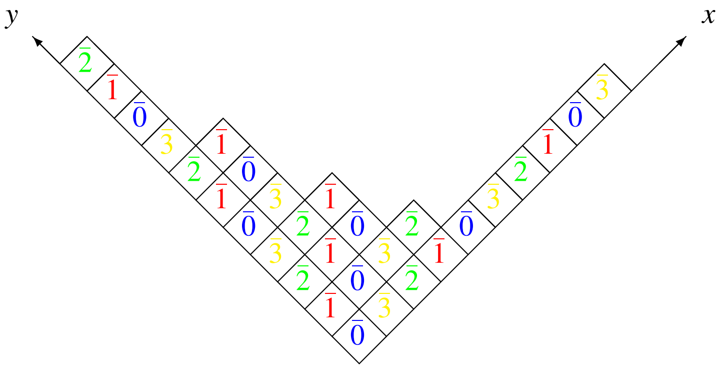

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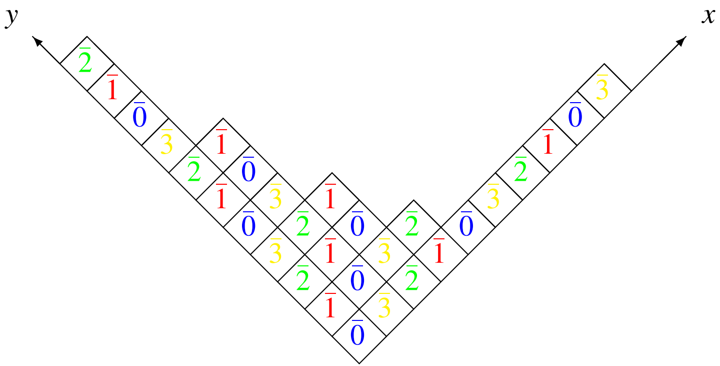

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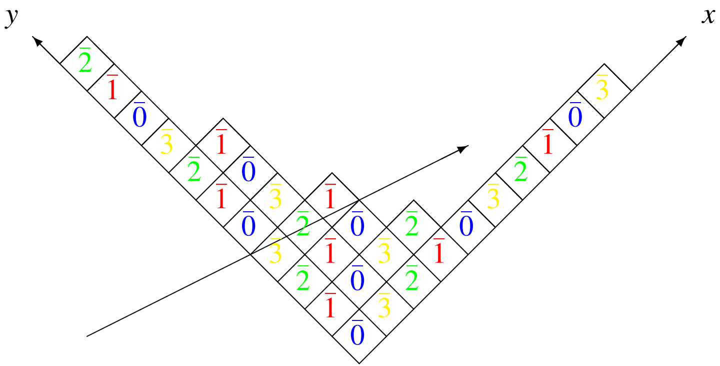

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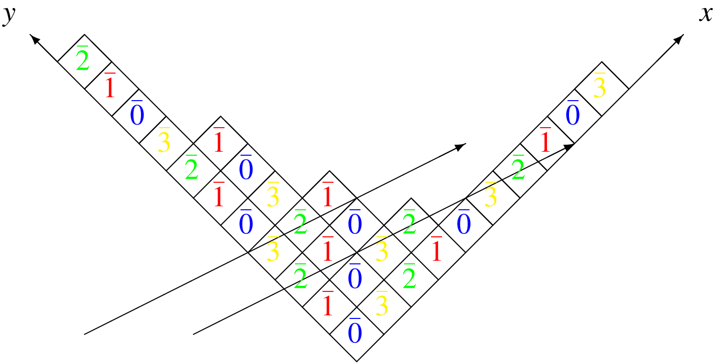

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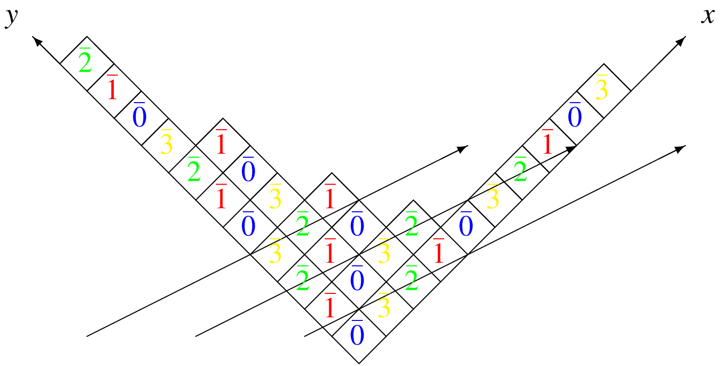

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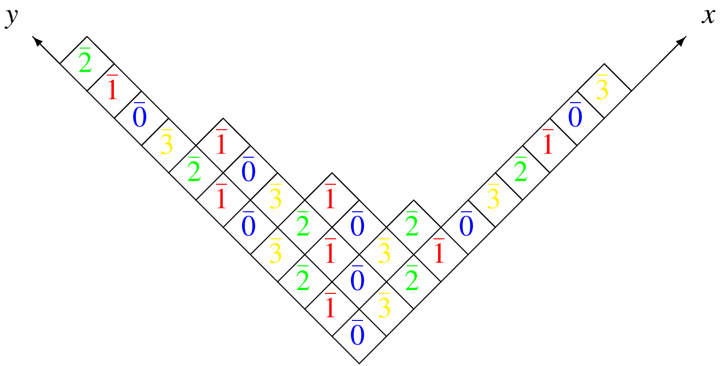

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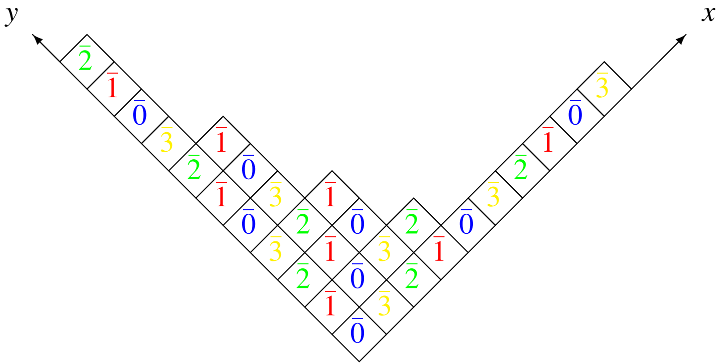

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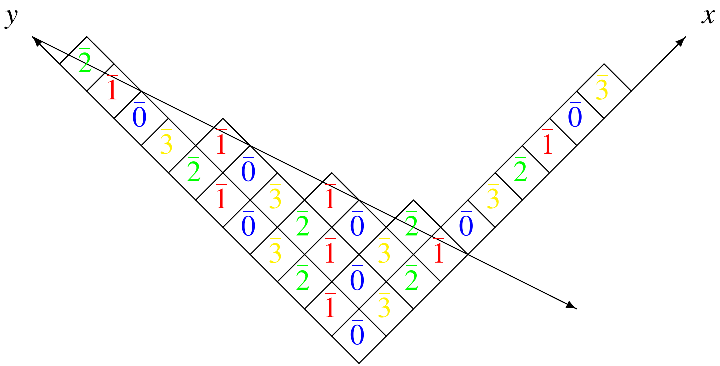

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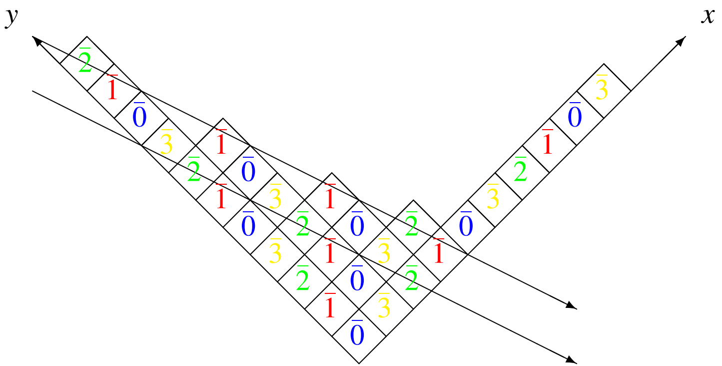

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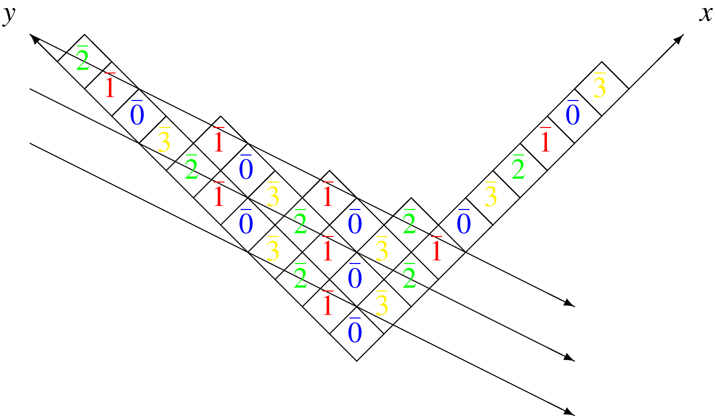

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