

Name (print): _____ Signature: _____

You have 30 minutes. Show your work. Notes not allowed! Problems are on both sides of this sheet.

Problem 1. (9 pts) Find the following derivatives

a. $\frac{d}{dx} (4 \arcsin(5x))$

Solution:

$$\frac{d}{dx} (4 \arcsin(5x)) = \frac{4}{\sqrt{1 - (5x)^2}} 5$$

b. $\frac{d}{dy} \left((\ln(3y^5))^8 \right)$

Solution:

$$\frac{d}{dy} \left((\ln(3y^5))^8 \right) = 8 (\ln(3y^5))^7 \frac{1}{3y^5} 15y^4$$

c. $\frac{dy}{dx}$, in terms of x and y , if

$$xy + \arctan y = 5$$

Solution: implicit differentiation gives

$$y + xy' + \frac{1}{1 + y^2} y' = 0$$

Solve for y' :

$$y' = \frac{-y}{x + \frac{1}{1+y^2}}$$

Problem 2. (5 pts) Use tangent line approximation to estimate $(1.003)^{18}$. (*Simplify your answer!*)

Solution: Let $f(x) = x^{18}$. Then $f(1) = 1$, $f'(x) = 18x^{17}$, $f'(1) = 18$, and, for x close to 1,

$$f(x) \approx f(1) + f'(1)(x - 1) = 1 + 18(x - 1).$$

Then

$$(1.003)^{18} = f(1.003) \approx 1 + 18(.003) = 1.054$$

So, $(1.003)^{18}$ is approximately 1.054. (*In reality, it is 1.05539928...*)

Problem 3. (6 pts) A highway patrol plane flies 3 miles above a level, straight road at a steady 120 miles per hour. The pilot sees an oncoming car and with radar determines that at the instant the line-of-sight distance from plane to car is 5 miles, the line-of-sight distance is decreasing at the rate of 160 miles per hour. Find the car's speed along the highway.

Solution: Consider a right triangle with vertical side (height) 3, horizontal side x , and hypotenuse h . We have

$$3^2 + x^2 = h^2$$

so

$$2x \frac{dx}{dt} = 2h \frac{dh}{dt}$$

When $h = 5$, $\frac{dh}{dt} = -160$, $x = \sqrt{h^2 - 3^2} = 4$, so

$$\frac{dx}{dt} = \frac{h}{x} \frac{dh}{dt} = \frac{5}{4}(-160) = -200.$$

No, the car is not going 200 miles per hour. The plane is flying 120 miles per hour in the opposite direction, so the velocity of the car is 80 mph.