Calculus Review

Homework # 7 (due Tuesday 11/6/2012 at 230pm)

$$\csc\theta = \frac{1}{\sin\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

① Using techniques similar to those we use to find $\int \sec\theta\, d\theta$ to show

→ $\int \csc\theta\, d\theta = \ln|\csc\theta - \cot\theta| + C$ ←

Do Not prove this result by differentiation – you must use integration.

② Find $\int_0^\infty e^{-x}\sin x\, dx$

③ Find the area of the region to the right of $x=3$ and between the x-axis and $y = \frac{1}{x^2-1}$

④ Find the area of the region between the x-axis, $y = \frac{x}{\sqrt{1-x^2}}$, $x=0$ and $x=1$

⑤ For the function $z = f(x,y) = \frac{x^2}{y} + \frac{y^2}{x}$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

Also find the matrix $\begin{bmatrix} \frac{\partial^2 z}{\partial x^2} & \frac{\partial^2 z}{\partial x \partial y} \\ \frac{\partial^2 z}{\partial y \partial x} & \frac{\partial^2 z}{\partial y^2} \end{bmatrix}$ of second derivatives.

⑥ Find $\int_0^1 \int_{x^2}^{x} dy\, dx = \iint_R dA$ and plot the region R

⑦ Find $\int_0^1 \int_{x^2}^{x} xy\, dy\, dx$.

⑧ Find $\int_{-1}^{2} \int_{2x^2-2}^{x^2+x} x\, dy\, dx$ and plot the region R.

⑨ Find $\int_0^1 \int_{3y}^{3} e^{x^2}\, dx\, dy$ after changing the order of integration.