The Binomial Expansion of $(x + y)^n$ Let x and y be any real numbers, then

$$(x + y)^n = \binom{n}{0} x^n y^0 + \binom{n}{1} x^{n-1} y^1 + \binom{n}{2} x^{n-2} y^2 + \cdots + \binom{n}{n} x^0 y^n$$

$$= \sum_{i=0}^{n} \binom{n}{i} x^{n-i} y^i.$$

The Sum of a Geometric Series Let r be a real number such that $|r| < 1$, and m be any integer $m \geq 1$

$$\sum_{i=0}^{\infty} r^i = \frac{1}{1-r}, \quad \sum_{i=1}^{\infty} r^i = \frac{r}{1-r}, \quad \sum_{i=0}^{m} r^i = \frac{1-r^{m+1}}{1-r}.$$

The (Taylor) Series Expansion of e^x Let x be any real number, then

$$e^x = \sum_{i=0}^{\infty} \frac{x^i}{i!}.$$

Some useful formulas for particular summations follow. The proofs (omitted) are most easily established by using mathematical induction.

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2.$$

Gamma Function Let $t > 0$, then $\Gamma(t)$ is defined by the following integral:

$$\Gamma(t) = \int_0^{\infty} y^{t-1} e^{-y} dy.$$

Using the technique of integration by parts, it follows that for any $t > 0$

$$\Gamma(t + 1) = t\Gamma(t)$$

and if $t = n$, where n is an integer,

$$\Gamma(n) = (n - 1)!.$$

Further,

$$\Gamma(1/2) = \sqrt{\pi}.$$

If $\alpha, \beta > 0$, the ***Beta function***, $B(\alpha, \beta)$, is defined by the following integral,

$$B(\alpha, \beta) = \int_0^1 y^{\alpha-1}(1 - y)^{\beta-1} dy$$

and is related to the gamma function as follows:

$$B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha + \beta)}.$$