An Application of Profile-Likelihood Based Confidence Interval to Capture–Recapture Estimators

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In recent years, more robust methods of estimating the size of a closed population (N) from capture–recapture data have been developed. However, interval estimation for N has seen few practical developments. The usual approach for constructing a confidence interval, known as a Wald confidence interval, is based on the assumption of asymptotic normality. It is well known that the small sample distribution of capture–recapture estimators are strongly asymmetric and thus deviate from normality. As a result, Wald confidence intervals are frequently unreasonable; having lower limits that extend below the number of animals known to exist, or even being negative. Two other approaches to the construction of confidence intervals, the profile-likelihood based method and the bootstrapped method, show much promise. The computational burden of the profile-likelihood confidence interval is much less than the bootstrapped confidence interval and appears to be an excellent alternative to the Wald confidence interval.

Key Words: Bootstrap; Capture–recapture; Closed population; Confidence interval; Monte Carlo simulation; Profile-likelihood; Small sample.

1. INTRODUCTION

The recent literature has presented some novel approaches to the problem of estimating the size of a closed population (N) from capture–recapture data (Huggins 1989, 1992; Alho 1990; Evans and Bonett 1993; Evans, Bonett, and McDonald 1994). These approaches promise point estimates of N that have smaller bias and are less variable than their predecessors (see Otis, Burnham, White, and Anderson 1978). As with previous methods, these recent approaches rely on asymptotic theory for variance estimates. It is thus a simple task to produce an approximate $100 \cdot (1 - 2 \cdot \alpha)\%$ Wald-type confidence interval $\hat{N} \pm Z_\alpha \cdot \hat{\sigma}_{\hat{N}}$. Here, $\hat{N}$ denotes the maximum likelihood estimate (MLE) of N; $\hat{\sigma}_{\hat{N}}$ is the estimated asymptotic standard error of the estimate; and Z_α is the $(1 - \alpha)$ standard normal quantile.

In an empirical study, Evans and Bonett (1994) found the asymptotic variance for the ML estimator of N to be positively biased. This bias is so large in situations of small

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sample size, or low-to-moderate capture probability, as to render the variance estimates useless. As if to add insult to injury, the small sample distribution of $\hat{N}$ is skewed right (White, Anderson, Burnham, and Otis 1982, p. 34). These characteristics frequently combine and result in a lower limit of a Wald confidence interval that is less than the number of individuals captured (Otis et al. 1978, p. 133–135), or even negative. Because of these properties, the utility of the symmetric Wald-based confidence interval is greatly reduced.

Bootstrapped confidence intervals (see Efron and Tibshirani 1993) provide a viable alternative to Wald confidence intervals. This approach to the construction of confidence intervals for capture–recapture estimators was suggested by Huggins (1989). More recently, Garthwaite and Buckland (1992) proposed a modified bootstrap procedure to compute confidence intervals for capture–recapture estimators. This approach was found to produce confidence intervals that had very near the anticipated nominal coverage. Because bootstrap confidence intervals are based on the empirically generated distribution of $\hat{N}$, nonnormality is no longer a concern. Also, bootstrapped confidence intervals produce limits that will fall outside the permissible range (e.g., below the number of captured individuals) only if this is possible for a given capture–recapture model (Buckland and Garthwaite 1991). Thus, the bootstrapped confidence interval appears to possess many excellent properties. However, because modeling procedures for capture–recapture data, such as log-linear and logistic regression models, are computer intensive, computing a confidence interval based on a thousand or more bootstrapped estimates could be a very time-consuming task (see Efron and Tibshirani 1993).

Many of the negative characteristics attributed to Wald confidence intervals may stem from the nonlinear nature of capture–recapture estimators. As indicated by Ratkowsky (1988), Wald confidence intervals for the parameters of nonlinear regression models suffer the same ills as those found in capture–recapture models. For nonlinear regression models, poor confidence interval coverage may be attributed to parameter effects' nonlinearity; that is, the lack of parallelism and the unequal spacing of parameter lines on the solution locus at the least-squares solution is a primary cause of poor performance in the Wald confidence interval (Ratkowsky 1988, p. 20–25). When parameter effects' nonlinearity is considerable, likelihood-based confidence intervals, not Wald, may more closely approximate the true situation (Ratkowsky 1988, p. 38). Although computationally more tedious than the Wald confidence intervals, likelihood-based confidence intervals involve far less computation than bootstrapped confidence intervals. Thus, likelihood-based confidence intervals may retain the robustness of the bootstrapped confidence interval, but present a more simplistic computational structure. Given the previous discussion, it would seem reasonable to consider likelihood-based confidence intervals for closed population capture–recapture methods. In fact, Lebreton, Burnham, Clobert, and Anderson (1992, pp. 72–73), briefly discussed the feasibility of likelihood-based confidence intervals for open population capture–recapture methods. Also, Morgan and Freeman (1989) provided examples of likelihood-based confidence intervals for band recovery data. Beyond these two notable examples, there appears to be few, if any, articles that illustrate the use of likelihood-based confidence intervals for capture–recapture methods. In particular, there appears no published work that compares the operating characteristics of Wald, bootstrap, and likelihood-based confidence intervals for closed population capture–recapture estimators. Thus, a comparative study of the three methods would seem in order.

In the following section we describe the procedure for constructing profile-likelihood based confidence intervals (Venzon and Moolgavkar 1988) for the estimator of N based on capture–recapture data. In the third section, an example of the two sample capture–recapture experiments is presented. Wald, bootstrap, and profile-likelihood confidence intervals are constructed for these data and comparisons drawn. The fourth section details a Monte Carlo simulation study that was designed to gain insight into the characteristics of the three described methods of constructing confidence.

2. PROFILE-LIKELIHOOD BASED CONFIDENCE INTERVALS

Following Venzon and Moolgavkar (1988), let $\theta_0 \in \mathbf{R}^k$ denote the parameter vector to be estimated, and $l(\theta)$ the log-likelihood for values of θ belonging to the parameter space $\Theta \subseteq \mathbf{R}^k$. If $\hat{\theta}$ denotes the MLE of θ_0, then

$$l(\widehat{\boldsymbol{\theta}}) = \max_{\theta \in \Theta} l(\boldsymbol{\theta}). \tag{2.1}$$

Suppose the jth element of θ, denoted θ_j, is the parameter of interest, with all other elements of θ to be treated as nuisance parameters. Now consider a restriction to the parameter space Θ, where θ_j is fixed at some value, say β. If the restricted space is defined as $\Theta_j(\beta) = \{\theta \in \Theta | \theta_j = \beta\}$, then

$$\bar{l}_j(\beta) = \max_{\theta \in \Theta_j(\beta)} l(\boldsymbol{\theta}) \tag{2.2}$$

is called the profile likelihood for β. Evaluation of 2.2 involves the maximization of the log-likelihood function with θ_j constrained to β (see Evans, Bonett, and McDonald 1994). An approximate $100 \cdot (1 - 2 \cdot \alpha)\%$ profile-likelihood based confidence interval for θ_{0_j} is given by

$$\{\beta \,|\, 2 \cdot [l(\widehat{\boldsymbol{\theta}}) - \bar{l}_j(\boldsymbol{\beta})]\} \leq q_{(1,\alpha)}, \tag{2.3}$$

where $q_{(1,\alpha)}$ represents the $(1 - \alpha)$ quantile of the chi-square distribution based on 1 degree of freedom.

This approach to the construction of confidence intervals is quite general. Although this article only discusses the attributes of profile-likelihood based confidence intervals for the two-period capture–recapture experiment, the method equally applies to most capture–recapture models for two or more capture periods (see Otis et al. 1978).

3. THE CAPTURE–RECAPTURE EXPERIMENT

Lincoln (1930) and Petersen (1896) independently derived an estimator for the size of a population based on a design having two trapping occasions. The first period involves the capture, tagging, and release of animals. In the second period, animals are captured and their capture status from the first period recorded. As a typical example, consider the cottontail rabbit data of Skalski, Robson, and Simmons (1983) as displayed in a 2-by-2 contingency table:

		Period 2: Captured (1)	Period 2: Not captured (2)	
Period 1	Captured (1)	$F_{11} = 7$	$F_{12} = 80$	$F_{1.} = 87$
	Not captured (2)	$F_{21} = 7$	$F_{22} = ?$	
		$F_{.1} = 14$		

It is typically assumed that the capture frequencies (F_{ij}) follow a multinomial distribution. The log-likelihood function for the capture–recapture experiment is defined as follows:

$$L(N, P_1, P_2) = \log(N!) - \sum_{ij} \log(F_{ij}!) + \sum_{ij} F_{ij} \cdot \log(P_{ij}). \qquad (3.1)$$

Here, P_{ij} represents the probability of observing the ijth outcome ($i = 1, 2$ and $j = 1, 2$). For the two-period capture–recapture experiment it is typically assumed that the observations are independent between capture periods 1 and 2, with marginal probabilities P_1 and P_2, respectively. The structure of the P_{ij} are thus simplified: $P_{11} = P_1 \cdot P_2, P_{12} = P_1 \cdot (1 - P_2), P_{21} = (1 - P_1) \cdot P_2$, and $P_{22} = (1 - P_1) \cdot (1 - P_2)$.

For the two sample capture–recapture experiment, a simple closed-form estimator, known as the Lincoln–Petersen (LP) estimate (Seber 1982, pp. 59), exists. However, this estimatordoes not have finite expectation. Several modifications to the LP estimator have been proposed. A modified form of the LP estimators was proposed by Evans and Bonett (1994) in which .5 is added to each observed frequency. This modification was shown to reduce the mean square error and, as was shown more generally by Firth (1993), for some measure of accumulated information, say n, also removes the bias of order $O(n)$. The estimator of the population size and estimated asymptotic standard error, as applied to the cottontail data, are respectively

$$\hat{N} = \frac{(F_{11} + F_{12} + 1) \cdot (F_{11} + F_{21} + 1)}{F_{11} + .5} - 1.5 = 174.5,$$

and

$$\hat{\sigma}_{\hat{N}} = \sqrt{\frac{(F_{1.} + 1) \cdot (F_{.1} + 1)(F_{12} + .5) \cdot \cdot (F_{21} + .5)}{(F_{11} + 0.5)^3}} = 43.46.$$

A symmetric 95% Wald-based confidence interval for the data of Skalski, Robson, and Simmons (1983) is 89.3 to 259.7 rabbits. Immediately, one should be concerned because the lower limit is less than the 94 captured individuals. As stated previously, this is a common trait of Wald confidence intervals for N. Skalski and Robson (1992, pp. 72–76) described a method for transformation-based Wald confidence intervals, which tends to reduce the problem of inadmissible lower confidence limits.

The simplest approach to computing profile-likelihood based confidence intervals involves a hunt-and-peck procedure (see Venzon and Moolgavkar 1988 for an iterative search method). For the lower limit, a value for β, between the MLE for N and the number of individuals captured, is selected and the log-likelihood maximized with respect to P_1 and P_2 at the fixed value of β. Using the log-likelihood of (3.1), equations (2.1) and (2.2) can be evaluated for a given value of β, followed by evaluation of (2.3). The lower limit of the confidence interval is the β such that (2.3) equals the appropriate chi-square quantile. The upper limit of the confidence interval is located in a similar manner. For the cottontail data, the profile-likelihood based 95% confidence interval is 119.1 to 350.2 rabbits. When compared to the symmetric Wald-type confidence interval two characteristics should be apparent: first, as one would hope, the lower limit is larger than the total number of individuals captured; and second, as one would expect for a positively skewed distribution, the interval is not symmetric about the estimate of N. Comparing these results with a bootstrapped 95% confidence interval of 123.8 to 320.4 rabbits (based on 10,000 bootstrapped simulations) we see that the profile-likelihood based confidence interval has far more affinity to the bootstrapped confidence interval than the Wald-based confidence interval.

4. A SIMULATION STUDY

Clearly, a single example is not definitive. Unfortunately, the small sample properties of capture–recapture estimators do not lend themselves to analytic solutions. To better understand the properties of the different methods described for confidence interval construction, a Monte Carlo simulation study was undertaken. The main purpose of this simulation was to empirically compare the three methods. Comparison of the three methods was to be based on the percentage of times that the known population size fell

Table 1. Simulated Estimate of the Average for the Modified Lincoln–Petersen Estimator of the Population Size N Based on 10,000 Simulated Realizations

Population size				*Probability set*	
25	*50*	*100*	*800*	P_1	P_2
17.6	45.5	116.3	859.9	.1	.1
25.0	60.0	128.1	825.5	.2	.1
30.6	63.9	117.5	813.2	.2	.2
32.8	60.8	110.5	808.0	.3	.2
31.9	57.5	106.6	806.0	.3	.3
30.5	54.8	104.0	804.9	.4	.3
28.8	53.0	102.8	802.3	.4	.4
27.6	52.2	102.2	801.6	.5	.4
26.9	51.8	101.7	801.2	.5	.5
26.4	51.2	101.3	801.3	.6	.5
26.0	50.9	100.8	801.2	.6	.6
25.8	50.7	100.7	800.7	.7	.6
25.6	50.5	100.5	800.7	.7	.7
25.4	50.4	100.5	800.4	.8	.7
25.3	50.3	100.3	800.4	.8	.8
25.2	50.2	100.2	800.2	.9	.8
25.1	50.1	100.1	800.1	.9	.9

Table 2. Simulated Estimate of the Average for the Asymptotic Variance Estimator (upper) and Mean Square Error (lower) for the Modified Lincoln–Petersen Based on 10,000 Simulated Realizations

Population size				*Probability set*	
25	*50*	*100*	*800*	P_1	P_2
736.0	4861.7	32761.7	232669.4	.1	.1
208.2	935.4	7135.2	159225.6		
1422.5	8386.5	37298.1	40437.8	.2	.1
291.7	1987.1	11564.9	39832.7		
2031.1	8367.5	16312.5	14890.3	.2	.2
497.2	2622.7	7042.0	14520.7		
2183.7	5319.9	5529.3	8206.6	.3	.2
638.2	1979.4	3076.9	8040.2		
1729.1	2458.6	1121.5	4666.2	.3	.3
586.5	1150.9	979.9	4632.4		
1133.4	918.8	569.2	2957.3	.4	.3
432.6	529.0	520.1	2971.6		
603.1	259.9	306.9	1861.5	.4	.4
263.2	209.8	286.4	1841.0		
246.4	134.3	193.5	1232.1	.5	.4
130.0	116.1	187.8	1267.2		
102.3	82.4	123.1	818.0	.5	.5
66.2	77.4	118.9	837.4		
49.8	48.0	80.2	544.7	.6	.5
36.3	45.2	77.0	538.7		
21.9	29.7	51.0	362.9	.6	.6
18.5	26.9	48.8	358.8		
13.5	19.0	32.7	232.6	.7	.6
11.5	17.9	31.9	234.7		
7.5	11.7	20.8	149.4	.7	.7
6.5	10.9	20.2	146.5		
4.6	6.9	12.1	87.0	.8	.7
3.6	6.1	11.3	87.5		
2.5	4.0	7.0	50.8	.8	.8
2.0	3.6	6.8	50.7		
1.2	1.8	3.2	22.6	.9	.8
0.9	1.6	3.0	22.7		
.5	.8	1.4	10.0	.9	.9
.4	.7	1.3	9.9		

above and below the simulated confidence intervals, relative to the nominal value of 2.5%. Because capture–recapture estimators and their asymptotic variance estimators areknown to be biased, the simulated mean for estimator and variance estimator, along with the mean square error, will be important when considering the simulation results.

The matrix-based language GAUSS (Aptech 1993) was used for all aspects of the simulation study. Multinomial observations for the two sample capture–recapture experiment were generated for a specific population size (25, 50, 100, and 800) and probability structure using a uniform random-number generator. Each combination of population size and probability structure was simulated 10,000 times. For each combination, the Wald and profile-likelihood 95% confidence intervals were constructed and assessed for inclusion of the known population size.

The computational burden necessary to simulate the properties of the bootstrapped confidence intervals required some downsizing. The number of primary simulations performed per population size and probability structure were reduced from 10,000 to 1,000

Table 3. Simulation Results for Wald, Profile-Likelihood, and Bootstrapped 95% Confidence Intervals on *N*

Population size								*Probability set*	
25		*50*		*100*		*800*		P_1	P_2
0.0	27.9	.0	21.7	.0	17.5	.0	8.4		
0.0	2.9	.0	3.3	.0	2.3	.8	1.6	.1	.1
0.0	15.8	.0	5.7	.0	1.8	2.1	3.2		
0.0	19.0	.0	16.4	.0	13.8	.0	7.0		
0.0	2.9	.0	2.7	.1	2.2	1.0	1.6	.2	.1
0.1	3.7	.1	2.4	1.1	2.5	2.9	1.8		
0.0	15.0	.0	12.7	.0	10.2	.2	5.4		
0.0	2.9	.2	2.7	.7	2.1	1.0	1.5	.2	.2
0.4	2.7	1.6	3.0	2.9	2.5	2.7	2.5		
0.0	13.0	.0	10.8	.0	8.6	.4	4.9		
0.3	3.2	.4	2.7	.9	2.1	1.1	1.8	.3	.2
1.0	2.3	2.1	3.2	3.4	2.5	2.0	2.7		
0.0	12.2	.0	9.0	.0	7.2	.8	4.1		
0.7	3.6	.8	2.5	1.3	2.4	1.6	1.7	.3	.3
1.4	1.4	1.6	3.0	2.4	3.1	2.3	2.3		
0.0	10.5	.0	8.2	.0	7.2	1.0	4.1		
0.9	4.2	1.2	3.1	1.2	3.1	1.9	2.3	.4	.3
2.0	1.7	2.6	1.6	2.6	2.6	2.6	2.0		
0.0	9.5	.0	7.9	.0	6.2	1.1	3.9		
1.8	4.0	2.3	3.3	2.2	2.7	2.1	2.8	.4	.4
2.4	2.5	2.5	1.5	2.7	1.5	2.4	2.3		
0.0	9.0	.0	6.8	.1	5.7	1.6	3.9		
2.3	5.2	2.5	3.9	2.9	3.5	2.9	3.5	.5	.4
2.0	2.1	2.1	2.3	2.4	2.5	3.0	2.6		
0.0	8.3	.0	6.4	.2	5.2	1.4	3.6		
2.3	6.0	3.2	5.0	3.2	4.1	2.6	1.9	.5	.5
2.0	1.6	3.0	2.0	2.6	3.0	2.6	1.9		

NOTE: Values are the percentage that the Wald (upper), profile-likelihood (middle), and bootstrapped (lower) confidence intervals did not contain the appropriate population size.

for populations of size 25, 50, 100, and 800. This procedure involved the generation of a multinomial realization for a given population size and probability structure, followed by the computation of the population size. For each simulated realization of the F_{ij}, $\hat{N}$ was computed and the cell probabilities estimated as $\hat{P}_{ij} = F_{ij}/\hat{N}$. Substituting these estimates for the parameters of the multinomial distribution, 1,000 bootstrapped realizations were generated and the estimate of N computed for each bootstrapped realization. These 1,000 estimates were ordered and a confidence interval constructed by selecting the appropriate values (the 25th and 975th for a 95% confidence interval). This approach, known as the percentile method (see Efron and Tibshirani 1993, chap. 13), was used for each of the 1,000 confidence intervals constructed for a given population size and probability structure. As with the Wald and profile-likelihood methods, each bootstrapped confidence interval was assessed for inclusion of the known population size and the results accumulated.

Table 3. Continued

Population size								*Probability set*	
25		*50*		*100*		*800*		P_1	P_2
0.0	8.3	.0	6.8	.3	5.0	1.3	3.6		
2.3	6.2	2.7	5.8	2.9	4.5	3.2	4.3	.6	.5
2.2	2.1	2.3	3.2	2.0	2.1	3.1	2.7		
0.0	7.2	.0	5.8	.3	5.1	1.7	3.3		
2.2	6.1	2.5	5.0	3.0	5.0	3.6	4.2	.6	.6
1.9	2.5	3.1	2.3	2.1	2.1	2.2	2.1		
0.0	6.7	.0	6.1	.4	5.0	1.7	3.4		
1.8	5.8	2.7	5.6	3.3	5.1	3.8	4.4	.7	.6
3.2	1.6	2.7	2.7	2.2	2.5	1.9	1.8		
0.0	7.0	.0	6.1	.4	5.0	1.6	3.0		
1.6	5.8	2.4	5.2	2.9	4.9	3.5	4.0	.7	.7
2.5	2.0	2.6	1.5	1.7	2.3	2.1	2.9		
0.0	6.9	.0	5.7	.3	4.8	1.7	3.4		
0.7	5.8	1.7	4.6	2.4	4.4	3.2	4.1	.8	.7
4.5	1.9	3.4	1.6	1.8	3.3	2.3	2.2		
0.0	6.2	.0	6.1	.2	4.8	1.6	3.4		
0.3	4.5	.9	4.3	2.1	4.2	2.9	3.5	.8	.8
9.8	2.3	4.5	2.5	2.0	2.3	2.4	2.1		
0.0	7.2	.0	6.6	.0	5.6	1.5	3.7		
0.7	3.2	.1	4.0	.5	4.0	2.2	3.2	.9	.8
24.3	1.7	8.1	2.1	3.6	2.5	2.5	2.7		
0.0	6.6	.0	6.4	.0	5.8	1.1	3.8		
2.0	1.5	.5	3.0	.0	3.4	1.4	2.9	.9	.9
42.2	.8	23.1	1.9	5.5	1.8	2.3	3.0		

NOTE: Values are the percentage that the Wald (upper), profile-likelihood (middle), and bootstrapped (lower) confidence intervals did not contain the appropriate population size.

The results of these simulations are found in Tables 1, 2, and 3. Table 1 shows the performance of the estimator for each combination of population size and probability structure. Table 2 compares the average estimated variance and the empirical mean squared error. Table 3 gives the percentage, for each method of confidence interval construction, in which the known population size fell below the lower limit and above the upper limit.

5. DISCUSSION

As expected, the simulation results indicate that the Wald-type confidence interval has very poor operating characteristics, excepting the results for the large population ($N = 800$) and moderate capture probabilities ($P_i \geq .3$). Beside the issue of nonnormality, Wald confidence intervals also suffer from the poor performance of the asymptotic variance estimator (see Table 1). Admittedly, the Wald confidence interval has simplicity as an attribute, but little more. On the other hand, the profile-likelihood confidence interval, and especially the bootstrapped confidence interval, performed well in both the small and large population cases and over nearly all ranges of capture probabilities. The bootstrapped confidence intervals were quite exceptional, except for the case of a small population ($N = 25$ or possibly 50) with small or large capture probability ($P_i \leq .2$ and $P_i \geq .8$). The small variation among adjacent values in the percent failure rate for the bootstrapped confidence interval is probably from use of 1,000 bootstrapped estimates as opposed to use of some larger value. Efron and Tibshirani (1993, chap. 13) indicated that construction of bootstrapped confidence intervals requires a minimum of 1,000 bootstrapped estimates, but that a larger number, say 10,000, may be necessary to obtain more stable estimates. Unfortunately, using such a large value would have made the simulation study unreasonably long in duration. A second source of variation among adjacent values in the percent failure rate for the bootstrapped confidence interval may be due to random variation in the simulation based percent failure estimates. The standard error for the bootstrapped percentage estimates based on 1,000 simulations is approximately .5%; all other simulated percent-failure estimates, based on 10,000 simulations, would have an approximate standard error of .2%. This latter source of variation is sufficiently large to explain the variation observed in the bootstrap percent failure rate.

The profile-likelihood confidence interval appears to be an excellent alternative to both the Wald and bootstrapped confidence intervals. It has operating characteristics that closely resemble the bootstrapped confidence interval, even surpassing the bootstrapped method for small or large capture probabilities ($P_i \leq .2$ and $P_i \geq .8$). It also is far less costly from a computational standpoint.

For the simple two period capture–recapture experiment, the computer time required to construct a bootstrapped confidence interval was nearly two orders of magnitude longer than the profile-likelihood confidence interval.

Most biologists do not rely on the simple two period capture–recapture design as the basis for collecting capture data and subsequently estimating the population size. More typical are designs with three to ten capture periods. These designs allow for more robust estimation of the population size, but require data structures, and hence models, that are far more complicated. Attempting to construct bootstrapped confidence intervals for such complicated situations would require large amounts of computer time, possibly days. The profile-likelihood approach, although more time consuming than the Wald-based method, is much less time consuming than the bootstrap approach and is a method that deserves consideration.

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