1. (2 pts) Let $A$ and $k$ be positive constants. Show that $y = A + C e^{-kt}$ is a solution to the differential equation $\frac{dy}{dt} = k(A - y)$ for any constant $C$.

Differentiating the proposed solution $y$, we get $\frac{dy}{dt} = 0 - kCe^{-kt}$. The right-hand side of the differential equation reads

$$k(A - y) = k(A - (A + C e^{-kt})) = k(-C e^{-kt}),$$

so the two sides are indeed equal.

2. (2 pts) Show that $y = e^{2x}$ is not a solution to the differential equation $y'' - 3y' - 4y = 0$.

$y'(x) = 2e^{2x}$ and $y''(x) = 4e^{2x}$. Plugging in these values, we get

$$y'' - 3y' - 4y = 0 \quad (4e^{2x}) - 3(2e^{2x}) - 4(e^{2x}) = 0 \quad -6e^{2x} \neq 0.$$  

3. (2 pts) Morphine is administered to a patient at the rate of 2.5 mg per hour. Each hour, 34.7% of the morphine is metabolized and leaves the body. Write a differential equation for $M$, the amount in mg, of morphine as a function of time $t$ measured in hours.

Constant rate in, proportional rate out:

$$\frac{dM}{dt} = 2.5 - 0.347M$$
4. (4 pts) The slope field below is for the differential equation $y' = x + y$.

(a) (2 pts) Sketch the solutions that pass through the following points:
   
   i. $(0, 0)$ included above  
   ii. $(-3, 1)$ included above  
   iii. $(-1, 0)$ included above  

(b) (2 pts) From your sketch, what is the equation of the solution to the differential equation that passes through $(-1, 0)$? Verify that your solution is correct by substituting it into the differential equation.

This solution seems to be a straight line. What is its slope? Well, it should satisfy $y' = x + y$, or $y' = -1 + 0 = -1$. So it is a line passing through $(-1, 0)$ with slope $-1$. That is,

$$y(x) = -x - 1$$

should be a solution to the differential equation.

Verifying, we take the derivative to evaluate the left-hand side of the differential equation, getting $y' = -1$. How does this compare to the right-hand side $x + y$? Well, $x + y = x + (-x - 1) = -1$. The same! So this line is indeed a solution.