1. CHOOSE ONE: (5 pts)

- The leader of a mountain climbing crew (weighing 700 N) scales a 50 m cliff. One end of a heavy rope (weighing 0.55 N/m) is attached to his belt so that, after he scales the cliff, he can secure it on top of the cliff to make the others’ climbs easier. How much work does he do in reaching the top?

We’ll use the formula \( W = \int F(y)dy \). Think of the climber as moving himself from the bottom to the top. The work needed to move changes with \( y \) because more rope is off the ground. (At height \( y \), there is \( y \cdot 0.55 \) N of rope to move in addition to 700 N for himself:

\[
\int_0^{50} (700 + 0.55y)dy = 700(50) + \frac{1}{2}50^2.
\]

So, \( W = 36,250 \).

- Find the center of mass of a thin plate of constant density \( \delta \) covering the region in the first quadrant bounded by the parabola \( x^2 \) and the line \( y = x \).

Since density is constant (more precisely, not a function of \( y \)), we use the formulas

\[
\overline{x} = \frac{M_y}{M} = \frac{\int x\delta(x-x^2)dx}{\int \delta(x-x^2)dx} = \frac{\int x\delta(x-x^2)dx}{\int \delta(x^2-x)dx}
\]

\[
\overline{y} = \frac{M_x}{M} = \frac{\int y\delta(x-x^2)dx}{\int \delta(x-x^2)dx} = \frac{\int \frac{1}{2}(x+x^2)\delta(x-x^2)dx}{\int \delta(x-x^2)dx}
\]

We see that \( M = \delta((1/3)x^3 -(1/2)x^2) \big|_0^1 = \delta/6 \).

The other integral-ingredients are computed similarly \( M_y = 1/12 \) and \( M_x = 1/24 \).
2. CHOOSE ONE: (5 pts)

- Find parametric equations and a parameter interval for the motion of a particle starting at the point \((-2, 0)\), tracing the top half of the circle \(x^2 + y^2 = 4\) clockwise, then retracing the top-right quarter of the circle counterclockwise. (Ending at the point \((0, 2)\).)

Clockwise circle movement is parametrized by

\[
(x(t), y(t)) = (2 \cos(-t), 2 \sin(-t)) \quad \text{for } t \geq 0,
\]

\[
= (2 \cos t, -2 \sin t) \quad \text{for } t \geq 0.
\]

Now, since we are starting at \((-2, 0)\), we should put \(t = \pi\). Also, since the \(y\) value doubles back to cover where it came from (going back up to \((0, 2)\) instead of going down to \((0, -2)\) as \(2\pi < t < 5\pi/2\)), we may simply take \(|2 \sin t|\) for \(y(t)\).

Solution:

\[
\begin{cases}
  x(t) = 2 \cos t \\
y(t) = |2 \sin t| 
\end{cases}
\quad \text{for } \pi \leq t \leq 5\pi/2.
\]

- Determine the length of one arc of the cycloid parametrized by:

\[
x(t) = \frac{1}{2} (t - \sin t) \quad \text{and} \quad y(t) = \frac{1}{2} (1 - \cos t).
\]

Hint: the identity \(\sin^2 \theta = \frac{1}{2} \left(1 - \cos 2\theta\right)\) may be useful.

If \(x(t) = f\) and \(y(t) = g\), we use the formula \(\int \sqrt{(f')^2 + (g')^2} \, dt\).

\[
f'(t) = \frac{1}{2} (1 - \cos t) \quad \text{and} \quad g'(t) = \frac{1}{2} (\sin t),
\]

so we must compute \(\int_a^b \sqrt{\frac{1}{4} \left(1 - \cos t\right)^2 + \frac{1}{4} \sin^2 t} \, dt\). Using the identity, this reduces to \(\int_a^b \sqrt{\sin^2 (t/2)} \, dt = \int_a^b \sin(t/2) \, dt\). Finally, we must determine the interval \([a, b]\).

Since the cycloid is the motion of a dot on the rim of a wheel, we need the wheel to make one revolution. A quick glance at \(y(t)\) shows that this happens when \(t = 2\pi\).

\[
\int_0^{2\pi} \sin(t/2) \, dt = -2 \cos(t/2) \bigg|_0^{2\pi} = 2 - (-2) = 4.
\]