Problem 1. (6 pts) Find the derivatives:
\[
\frac{d}{dx} \left( \ln(\tan(3x^7)) \right)
\]

Solution:
\[
\frac{d}{dx} \left( \ln(\tan(3x^7)) \right) = \frac{1}{\tan(3x^7)} \frac{1}{\cos^2(3x^7)} 21x^6
\]

\[
\frac{d}{dz} \left( (\arctan(7z))^4 \right)
\]

Solution:
\[
\frac{d}{dz} \left( (\arctan(7z))^4 \right) = 4 (\arctan(7z))^3 \frac{1}{1 + (7z)^2}
\]

Problem 2. (5 pts) Use linear approximation to approximate \(\sqrt[3]{25}\).

Solution: Notice that \(\sqrt[3]{27} = 3\). Let \(f(x) = \sqrt[3]{x}\). Let \(a = 27\). Then \(f(a) = 3\), \(f'(x) = \frac{1}{3}x^{-2/3}\), \(f'(a) = \frac{1}{3}(27)^{-2/3} = \frac{1}{27}\). The linear approximation of \(f(x)\) at \(x = a\) is then

\[
L_a(x) = f(a) + f'(a)(x - a) = 3 + \frac{1}{27}(x - 27)
\]

Plug in \(x = 25\), get

\[
3 + \frac{1}{27}(-2) = \frac{25}{27}
\]

By the way: \(25/27 \approx 2.9259\) and the real value is \(\sqrt[3]{25} \approx 2.9240\).
**Problem 3.** (4 pts) Simplify \(\tan(\arccos x)\) to an expression not involving trig functions.

*Solution:* Consider a right triangle with one acute angle \(\alpha\), hypotenuse 1, adjacent side \(x\). Then \(\cos \alpha = x\), so \(\alpha = \arccos x\). Then the opposite side is \(\sqrt{1 - x^2}\), and so

\[
\tan(\arccos x) = \tan \alpha = \frac{\sqrt{1 - x^2}}{x}
\]

**Problem 4.** (5 pts) Water is flowing at the rate of 50 m\(^3\)/min from a shallow concrete conical reservoir (vertex down) of base radius 45 m and height 6 m. How fast is the water level falling when the water is 5 m deep?

*Solution:* Know \(\frac{dV}{dt} = -50\), need \(\frac{dh}{dt}\), where \(V = \frac{1}{3} \pi r^2 h\), and similar triangles suggest that \(\frac{r}{h} = \frac{45}{6}\).

Then \(r = \frac{45}{6} h\), and

\[
V = \frac{1}{3} \pi \left(\frac{45}{6} h\right)^2 h = \frac{45^2}{36^2} \pi h^3
\]

Then

\[
\frac{dV}{dt} = \frac{45^2}{36^2} \pi 3h^2 \frac{dh}{dt}
\]

We are interested in the moment when \(h = 5\), so

\[
-50 = \frac{45^2}{36^2} \pi 3 5^2 \frac{dh}{dt}
\]

Consequently

\[
\frac{dh}{dt} = -\frac{50 \cdot 6^2}{\pi \cdot 45^2 \cdot 5^2}
\]