Math 115 — First Midterm

February 8, 2011

Name: ______________________________

Instructor: ______________________ Section: ____________

1. **Do not open this exam until you are told to do so.**
2. This exam has 10 pages including this cover. There are 9 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it. Include units in your answer where that is appropriate.
6. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
8. **Turn off all cell phones and pagers**, and remove all headphones.
9. You must use the methods learned in this course to solve all problems.
10. Note that problems 6–9 will be graded giving very little partial credit.

Problem	Points	Score
1	12	
2	12	
3	12	
4	10	
5	12	
6	12	
7	12	
8	12	
9	6	
Total	100	

1. [12 points] Cyanide is used in solution to isolate elemental gold in gold mines. This unfortunately may result the groundwater near mines being contaminated with cyanide, which then must be removed. Suppose that at a certain mine site cyanide is removed from the groundwater starting in 2005. The concentration, c (in ppm), of cyanide found in the groundwater at the site t years after the year 2005 is given in the following table.

t (years)	0	1	2
c (ppm)	25.0	21.8	18.9

(Values for c are rounded.)

a. [4 points] Find the equation of an exponential model for $c(t)$. Show your work.

b. [4 points] Using your equation from (a), how many years will it take for the concentration of cyanide to be reduced to 10 ppm? Show all of your work.

c. [2 points] The cyanide removal process involves pumping groundwater through a filtering system. Suppose that the speed of this pumping process is doubled from the pumping speed which produced the data given above. Call the resulting concentration function $c_1(t)$. Use your expression for $c(t)$ from (a) to write an equation for $c_1(t)$.

d. [2 points] Now instead suppose that the groundwater cleaning started 3 years earlier. Call the resulting concentration function $c_2(t)$. Use your expression for $c(t)$ from (a) to write an equation for $c_2(t)$.

2. [12 points] Consider the following table giving values, rounded to three decimal places, of a function $f(x)$.

x	0	0.5	1
$f(x)$	0	0.247	0.841

a. [3 points] Estimate $f'(1)$. Be sure it is clear how you obtain your answer.

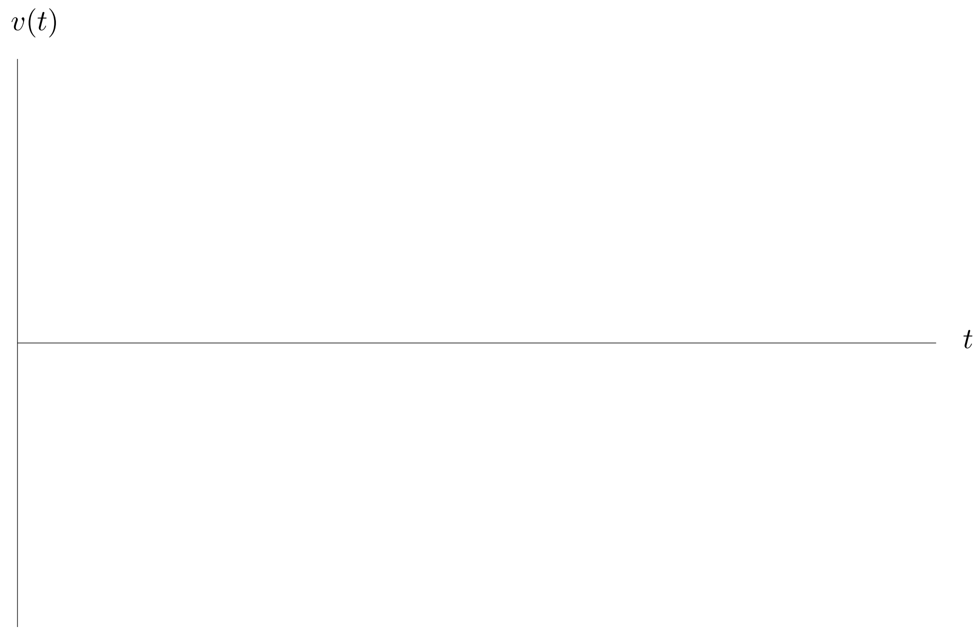
b. [4 points] Estimate $f''(1)$. Again, be sure that it is clear how you obtain your answer.

c. [3 points] Estimate $f(1.25)$, being sure your work is clear.

d. [2 points] Based on your work in (a) and (b), is your estimate in (c) an over- or under-estimate? Explain.

3. [12 points] Suppose that when you merge onto the highway the blue car in front of you is moving at 55 mph. Immediately after you merge, the driver of the blue car speeds up until, after five minutes, it is going 85 mph. Then, during the next five minutes it slows down to 55 mph again. This process then repeats over the following 10 minutes, with the blue car speeding up to 85 mph and then decreasing to 55 mph again.

a. [6 points] Assuming the speed of the blue car follows a sinusoidal pattern, on the axes below draw a well-labeled sketch of two periods of a function $v(t)$ which outputs the speed of the car t minutes after you merge onto the highway.



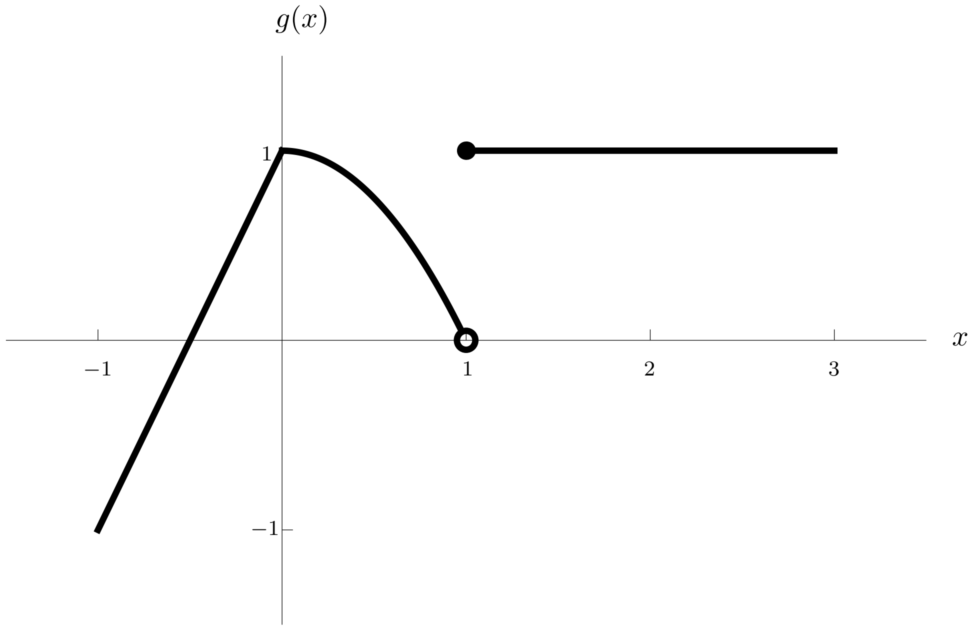
b. [6 points] Find a possible formula for $v(t)$ from part (a). What are the period and amplitude of $v(t)$?

$v(t) =$ ______________________________

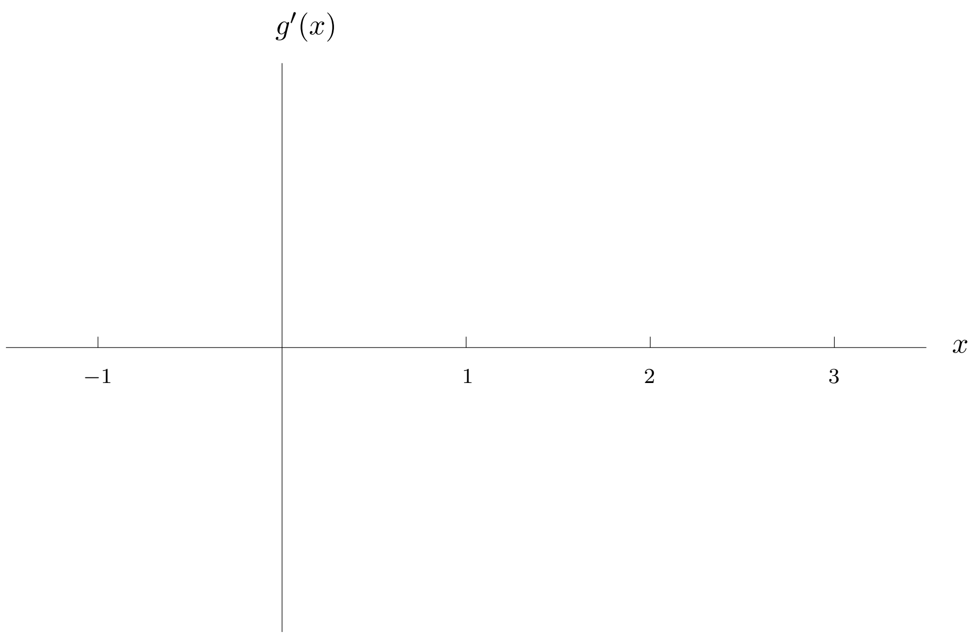
period = __________

amplitude = __________

4. [10 points] The graph of a function $g(x)$ is given below.



Accurately sketch a graph of $g'(x)$ on the axes below. Be sure to label the vertical axis.



5. [12 points] A paperback book (definitely not a valuable calculus textbook, of course) is dropped from the top of Dennison hall (which is 40 m high) towards a very large, upward pointing fan. The average velocity of the book between time $t = 0$ and later times is shown in the table of data below (in which t is in seconds and the velocities are in m/s).

between $t = 0$ seconds and $t =$	1	2	3	4	5
average velocity is	-5	-10	-11.67	-9	-7.2

a. [8 points] Fill in the following table of values for the height $h(t)$ of the book (measured in meters). Show how you obtain your values.

t	0	1	2	3	4	5
$h(t)$	40	______	______	______	______	______

b. [4 points] Based on your work from (a), is $h''(1) > 0$, < 0, or $= 0$? Is $h''(3) > 0$, < 0, or $= 0$? Explain.

6. [12 points] For the graph $y = f(x)$ in the figure below, arrange the following numbers from smallest to largest:

A. The slope of the graph at A.

B. The slope of the graph at B.

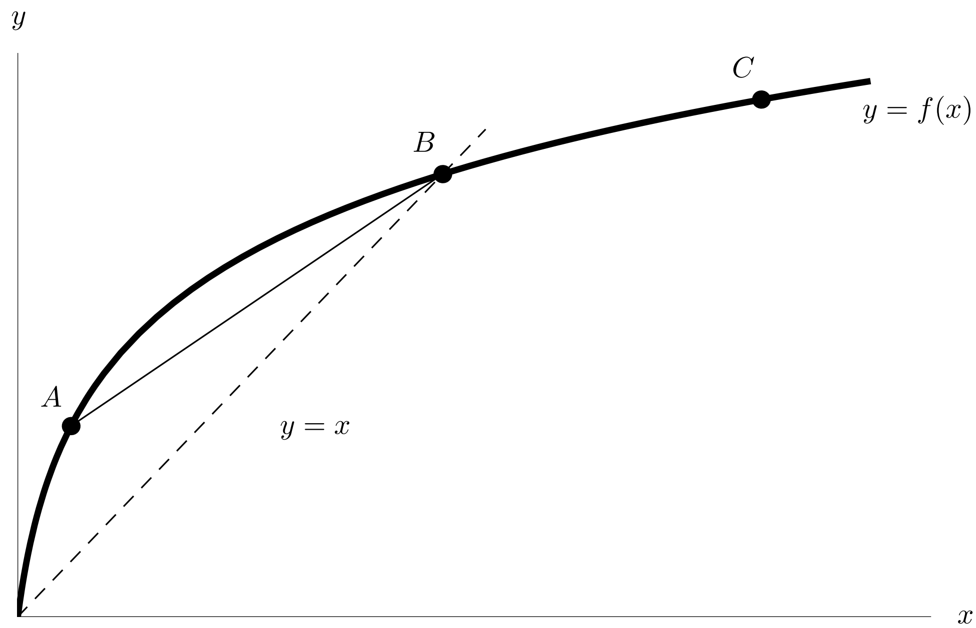
C. The slope of the graph at C.

AB. The slope of the line AB.

0. The number 0.

1. The number 1.

Explain the positions of the number 0 and the number 1 in your ordering. Any unclear answers will be counted as incorrect.



______ < ______ < ______ < ______ < ______ < ______

7. [12 points] For each of the descriptions of a function f that follow, indicate which of the graphs below match the description. For each description there may be no, one, or several graphs that match; write **none** if no graphs match the description. You may need to use a graph more than once. In each case you should assume that f is defined only on the domain $[0, 2]$.

- $f''(x) < 0$ for $x < 1$ and $f''(x) > 0$ for $x > 1$; $f'(x) < 0$ for $x < 1$ and $f'(x) > 0$ for $x > 1$; and $f(x)$ is continuous everywhere except at $x = 1$.

  matching graph(s): ____________________

- $f''(x) > 0$ for all $x \neq 1$; $f(x) < 0$ for all $x \neq 1$; and $f(x)$ is differentiable everywhere except at $x = 1$.

  matching graph(s): ____________________

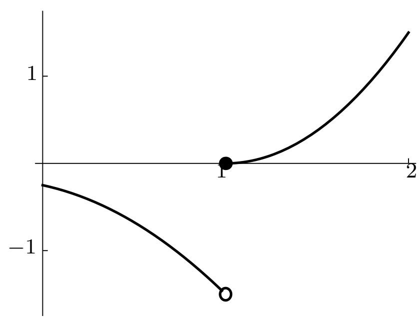
- $f''(x) < 0$ for all $x \neq 1$; $f'(x) < 0$ for $x < 1$ and $f'(x) > 0$ for $x > 1$; and $f(x) < 0$ for all $x \neq 1$.

  matching graph(s): ____________________

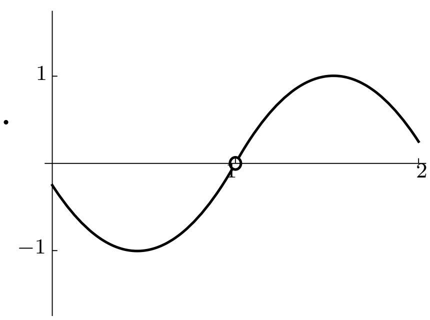
- $f''(x) < 0$ for $x < 1$ and $f''(x) > 0$ for $x > 1$; $f'(x) < 0$ for $x < 1$ and $f'(x) > 0$ for $x > 1$; and $f(x)$ is differentiable everywhere except at $x = 1$.

  matching graph(s): ____________________

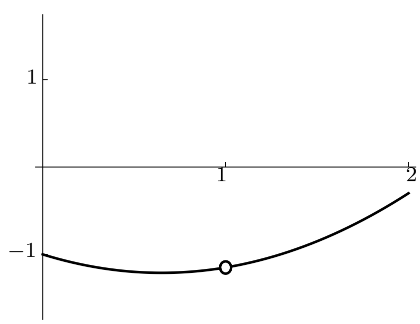
A.



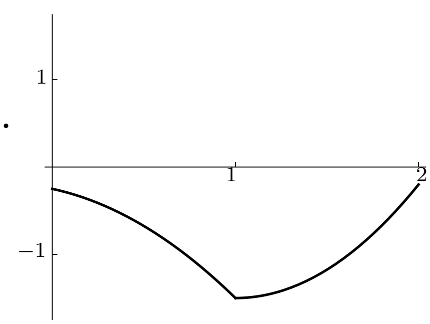
B.



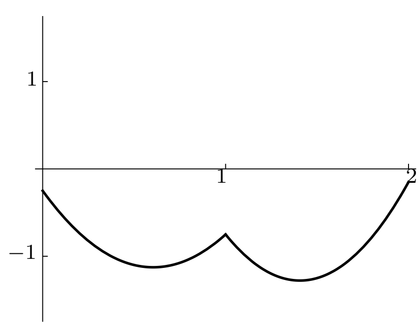
C.



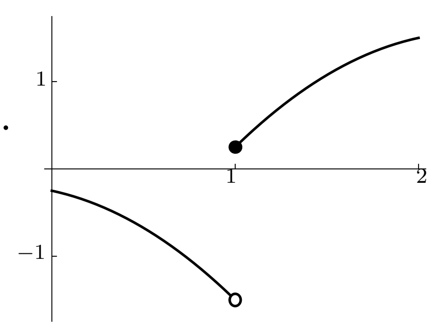
D.



E.



F.



8. [12 points] Let $P(d)$ be a function giving the total electricity that a solar array has generated, in kWH, between the start of the year and the end of the dth day of the year. Each of the following centences (a)–(d) expresses a mathematical equality in practical terms. For each, give a **single** mathematical equality involving P (and, as needed, its inverse and derivatives) that corresponds to the sentence.

a. [3 points] The end of the day on which the array had generated 3500 kWH of electricity was the end of the 4th of January.

b. [3 points] At the end of January 4th, the array was generating electricity at a rate of 1000 kWH per day.

c. [3 points] When the array had generated 5000 kWH of electricity, it would take approximately half a day to generate an additional 1000 kWH of electricity.

d. [3 points] At the end of January 30th, it would take approximately one day to generate an additional 2500 kWH of electricity.

9. [6 points] The population, $P(t)$, of China, in billions, can be approximated by

$$P(t) = 1.267(1.007)^t,$$

where t is the number of years since the start of 2000.

a. [2 points] Calculate the *continuous growth rate* of $P(t)$.

b. [4 points] Using the limit definition of the derivative, write an explicit expression for the derivative of $P(t)$ at the beginning of 2011. You do not need to simplify your expression.