Chapter 1 exercises

1. **(a)** Suppose x and y are the coordinates of a point on the circle $x^2 + y^2 = 1$. Is y a function of x? Why or why not?

(b) Suppose x and y are the coordinates of a point on the part of the circle $x^2 + y^2 = 1$ that is above the x-axis. Is y a function of x? Why or why not?

2. **(a)** Is the area, A, of a square a function of the length of one of its sides, s?

(b) Is the area, A, of a rectangle a function of the length of one of its sides, s?

3. **(a)** Which of the graphs in Figure 1.63 represent y as a function of x? (Note that an open circle indicates a point that is not included in the graph; a solid dot indicates a point that is included in the graph.)
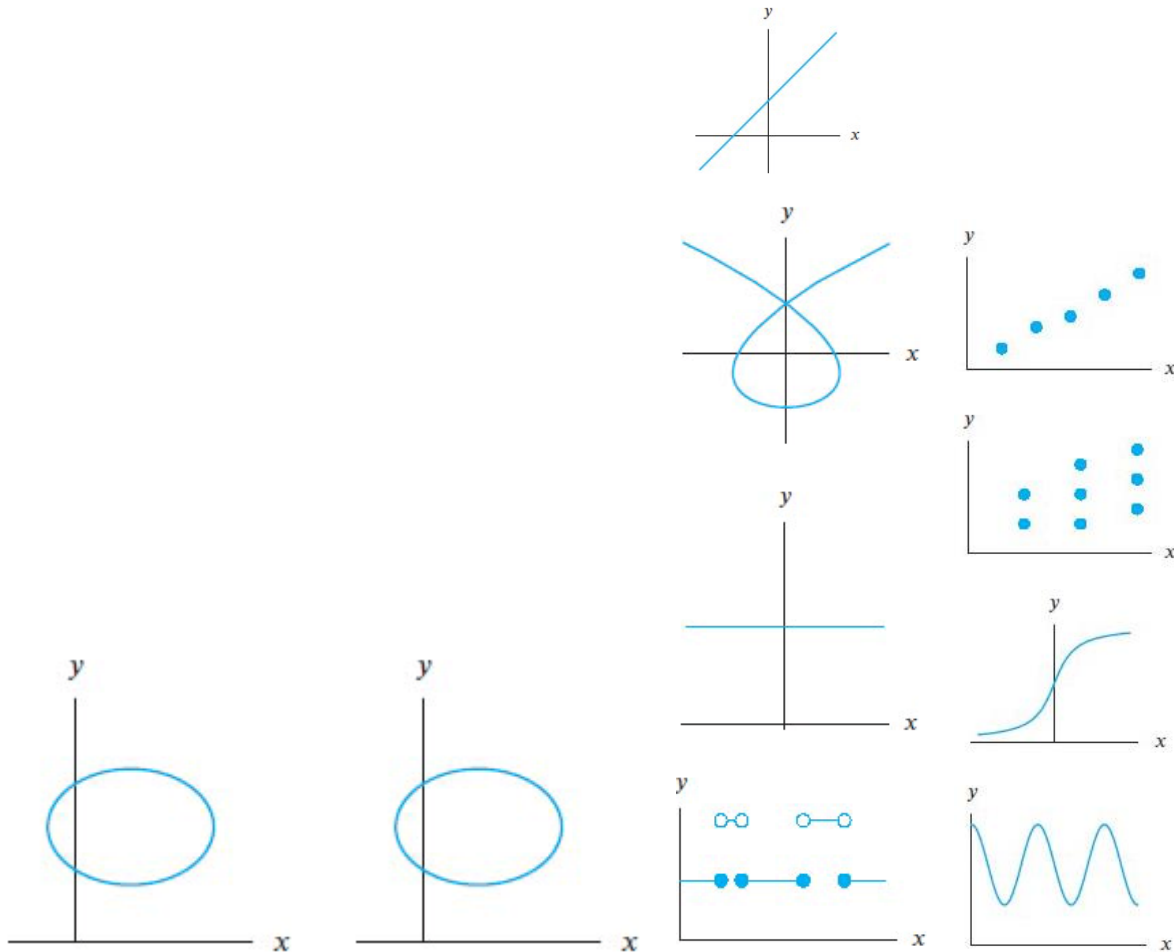
(b) Which of the graphs in Figure 1.63 could represent the following situations? Give reasons.

(i) SAT Math score versus SAT Verbal score for a small number of students.

(ii) Total number of daylight hours as a function of the day of the year, shown over a period of several years.

(c) Among graphs (I)-(IX) in Figure 1.63, find two that could give the cost of train fare as a function of the time of day. Explain the relationship between cost and time for both choices.

4. (a) Make a table of values for $f(x) = 10/(1 + x^2)$ for $x = 0, 1, 2, 3$.

(b) What x-value gives the largest $f(x)$ value in your table? How could you have predicted this before doing any calculations?

5. Table 1.44 shows data for two populations (in hundreds) for five different years. Find the average rate of change per year of each population over the following intervals.

Table 1.44

Year	1995	1997	2000	2005	2012
P_1	53	64	75	86	97
P_2	85	81	77	73	69

(a) 1995 to 2005

(b) 2000 to 2012   (c) 1995 to 2012

6. The following tables represent the relationship between the button number, N, that you push, and the snack, S, delivered by three different vending machines.

(a) One of these vending machines is not a good one to use, because S is not a function of N. Which one?

Vending Machine #1

N	S
1	M&Ms
2	pretzels
3	dried fruit
4	Hershey's
5	fat-free cookies
6	Snickers

(b) For which vending machine(s) is S a function of N?

Vending Machine #2

| N	S
1	M&Ms or dried fruit
2	pretzels or Hershey's
3	Snickers or fat-free cookies

(c) For which of the vending machines is N not a function of S?

Vending Machine #3

N	S
1	M&Ms
2	M&Ms
3	pretzels
4	dried fruit
5	Hershey's
6	Hershey's
7	fat-free cookies
8	Snickers
9	Snickers

8. In 1947, Jesse Owens, the US gold medal track star of the 1930s and 1940s, ran a 100-yard race against a horse. The race, "staged" in Havana, Cuba, is filled with controversy; some say Owens received a head start, others claim the horse was drugged. Owens himself revealed some years later that the starting gun was placed next to the horse's ear, causing the animal to rear and remain at the gate for a few seconds. Figure 1.65 depicts speeds measured against time for the race.

Figure 1.65
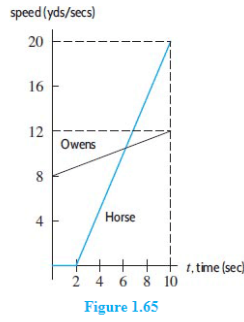
(a) How fast were Owens and the horse going at the end of the race?

(b) When were the participants both traveling at the same speed?

Exercises (11)-(13) give data from a linear function. Find a formula for the function.

11.

x	200	230	300	320	400
$g(x)$	70	68.5	65	64	60

12.

t	1.2	1.3	1.4	1.5
$f(t)$	0.736	0.614	0.492	0.37

13.

t	5.2	5.3	5.4	5.5
$f(t)$	73.6	61.4	49.2	37

In Problems 21–23, use Table 1.45, which gives values of $v = r(s)$, the eyewall wind profile of a typical hurricane.[33] The eyewall of a hurricane is the band of clouds that surrounds the eye of the storm. The eyewall wind speed v (in mph) is a function of the height above the ground s (in meters).

Table 1.45

s	0	100	200	300	400	500
v	90	110	116	120	121	122
s	600	700	800	900	1000	1100
v	121	119	118	117	116	115

21. Evaluate and interpret $r(300)$.

22. At what altitudes does the eyewall wind speed appear to equal or exceed 116 mph?

23. At what height is the eyewall wind speed greatest?

24. You are looking at the graph of y, a function of x.

   (a) What is the maximum number of times that the graph can intersect the y-axis? Explain.
   (b) Can the graph intersect the x-axis an infinite number of times? Explain.

25. A bug starts out ten feet from a light, flies closer to the light, then farther away, then closer than before, then farther away. Finally the bug hits the bulb and flies off. Sketch the distance of the bug from the light as a function of time.

26. Although there were 20 women in the Senate in 2013, the first woman ever elected to the Senate was Hattie Wyatt Caraway of Arkansas. She was appointed to fill the vacancy caused by the death of her husband, then won election in 1932, was reelected in 1938, and served until 1945. Table 1.46 shows the number of female senators at the beginning of the first session of each Congress.[34]

   (a) Is the number of female senators a function of the Congress's number, c? Explain.
   (b) Is the Congress's number a function of the number of female senators? Explain.
   (c) Let $S(c)$ represent the number of female senators serving in the cth Congress. What does the statement $S(110) = 16$ mean?
   (d) Evaluate and interpret $S(112)$.

Table 1.46 *Female senators, S, in Congress c*

c	98	100	102	104	106	108	110	112	113
S	2	2	2	8	9	14	16	17	20

27. A light is turned off for several hours. It is then turned on. After a few hours it is turned off again. Sketch the light bulb's temperature as a function of time.

28. According to Charles Osgood, CBS news commentator, it takes about one minute to read 15 double-spaced typewritten lines on the air.[35]

   (a) Construct a table showing the time Charles Osgood is reading on the air in seconds as a function of the number of double-spaced lines read for $0, 1, 2, \ldots, 10$ lines. From your table, how long does it take Charles Osgood to read 9 lines?
   (b) Plot this data on a graph with the number of lines on the horizontal axis.
   (c) From your graph, estimate how long it takes Charles Osgood to read 9 lines. Estimate how many lines Charles Osgood can read in 30 seconds.
   (d) Construct a formula which relates the time T to n, the number of lines read.

29. The distance between Cambridge and Wellesley is 10 miles. A person walks part of the way at 5 miles per hour, then jogs the rest of the way at 8 mph. Find a formula that expresses the total amount of time for the trip, $T(d)$, as a function of d, the distance walked.

30. A cylindrical can is closed at both ends and its height is twice its radius. Express its surface area, S, as a function of its radius, r. [Hint: The surface of a can consists of a rectangle plus two circular disks.]

31. A lawyer does nothing but sleep and work during a day. There are 1440 minutes in a day. Write a linear function relating minutes of sleep, s, to minutes of work, w.

For the functions in Problems 32–34:

   (a) Find the average rate of change between the points
      (i) $(-1, f(-1))$ and $(3, f(3))$
      (ii) $(a, f(a))$ and $(b, f(b))$
      (iii) $(x, f(x))$ and $(x + h, f(x + h))$
   (b) What pattern do you see in the average rate of change between the three pairs of points?

32. $f(x) = 5x - 4$

33. $f(x) = \frac{1}{2}x + \frac{5}{2}$

34. $f(x) = x^2 + 1$