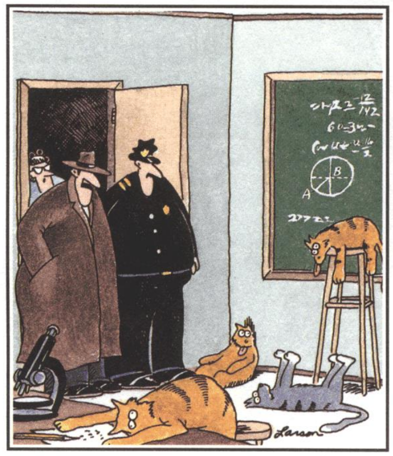
CLASS DISCUSSION: MATH 117

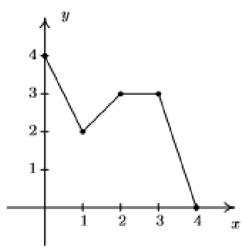
12 March 2019   30th anniversary of www



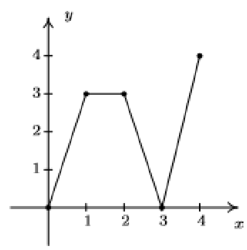
"Notice all the computations, theoretical scribblings, and lab equipment, Norm. ... Yes, curiosity killed these cats."

Review

In Exercises 78 - 85, use the graphs of $y = f(x)$ and $y = g(x)$ below to find the function value.



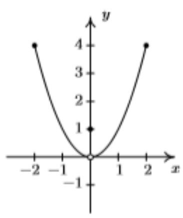
$y = f(x)$



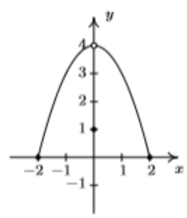
$y = g(x)$

78. $(f+g)(0)$
79. $(f+g)(1)$
80. $(f-g)(1)$
81. $(g-f)(2)$
82. $(fg)(2)$
83. $(fg)(1)$
84. $\left(\frac{f}{g}\right)(4)$
85. $\left(\frac{g}{f}\right)(2)$

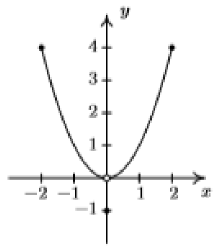
We said earlier in the section that it is not good enough to say local extrema exist where a function changes from increasing to decreasing or vice versa. As a previous exercise showed, we could have local extrema when a function is constant so now we need to examine some functions whose graphs do indeed change direction. Consider the functions graphed below. Notice that all four of them change direction at an open circle on the graph. Examine each for local extrema. What is the effect of placing the "dot" on the y-axis above or below the open circle? What could you say if no function value were assigned to $x = 0$?



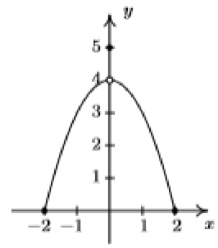
(a) Function I



(b) Function II



(c) Function III



(d) Function IV

Transformations: Shifts

1. Using Table 2.14, complete the tables for g, h, k, m, where:

(a) $g(x)=f(x-1)$ (b) $h(x)=f(x+1)$ (c) $k(x)=f(x)+3$ (d) $m(x)=f(x-1)+3$

Explain how the graph of each function relates to the graph of f(x).

Table2.14

x	−2	−1	0	1	2
$f(x)$	−3	0	2	1	−1

x	−1	0	1	2	3
$g(x)$					

x	−3	−2	−1	0	1
$h(x)$					

x	−2	−1	0	1	2
$k(x)$					

x	−1	0	1	2	3
$m(x)$					

2. Figure 2.27 shows f(x). Graph $y=f(x+3)+3$. Label all important features.

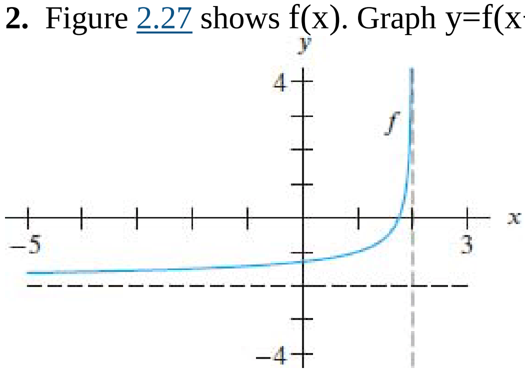


Figure 2.27

In Exercises (3)-(6), use Figure 2.28 to graph the transformation of f.

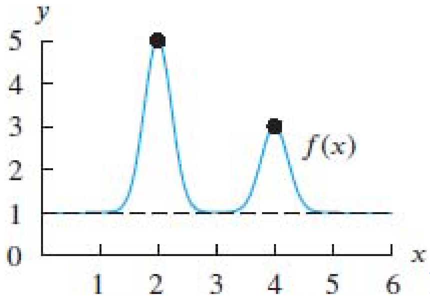


Figure 2.28

3. $y=f(x+2)$ **4.** $y=f(x)+2$ **5.** $y=f(x-1)-5$ **6.** $y=f(x+6)-4$

7. The graph of f(x) contains the point (3,−4). What point must be on the graph of
(a) $f(x)+5$? **(b)** $f(x+5)$? **(c)** $f(x-3)-2$?

8. The domain of the function g(x) is $-2<x<7$. What is the domain of $g(x-2)$?

9. The range of the function R(s) is $100\leq R(s)\leq 200$. What is the range of $R(s)-150$?

10. **(a)** Using Table 2.15, evaluate

Table2.15

x	0	1	2	3	4	5	6	7	8	9
$f(x)$	−10	−7	4	29	74	145	248	389	574	809
$g(x)$	−6	−7	6	33	74	129	198	281	378	489

(i) $f(x)$ for $x=6$ **(ii)** $f(5)-3$ **(iii)** $f(5-3)$ **(iv)** $g(x)+6$ for $x=2$
(v) $g(x+6)$ for $x=2$ **(vi)** $3g(x)$ for $x=0$ **(vii)** $f(3x)$ for $x=2$ **(viii)** $f(x)-f(2)$ for $x=8$
(ix) $g(x+1)-g(x)$ for $x=1$

(b) Using the values in the table, solve
(i) $g(x)=6$ **(ii)** $f(x)=574$ **(iii)** $g(x)=281$

(c) The values in the table were obtained using the formulas $f(x) = x^3+x^2+x-10$ and $g(x) = 7x^2-8x-6$. Use the table to find two solutions to the equation $x^3+x^2+x-10 = 7x^2-8x-6$.

11. Figure 2.29 shows $y=f(x)$. Give a formula in terms of f for the function in Figure 2.30. Your formula should be of the form $y=f(x-h)+k$ for appropriate constants h and k.

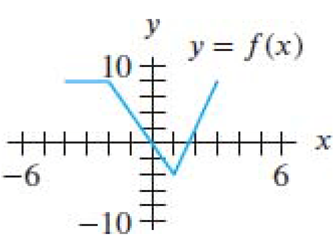


Figure 2.29

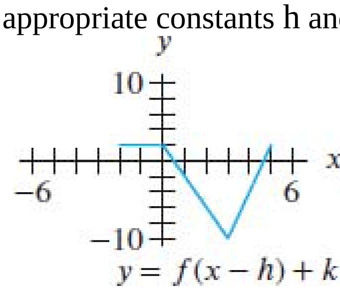


Figure 2.30

12. Figure 2.31 shows $y=m(r)$. Each graph in parts (a)-(d) is a translation of the graph of $y=m(r)$. Give a formula for each of these functions in terms of m.

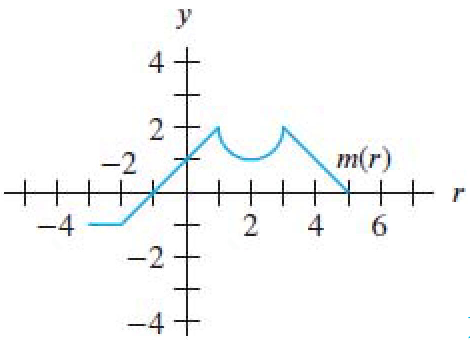
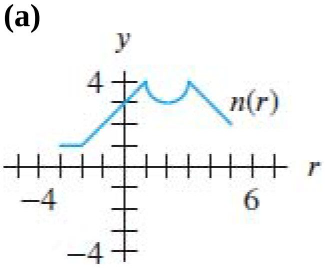
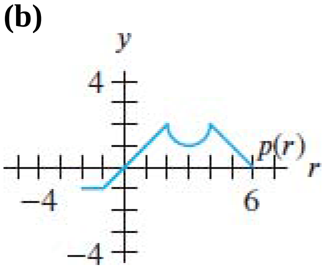


Figure 2.31

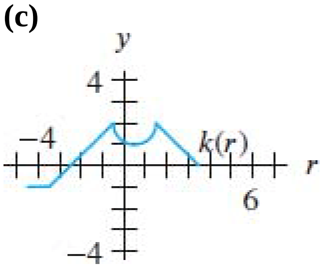
(a)



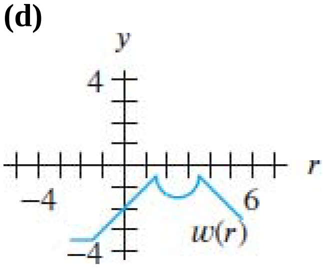
(b)



(c)



(d)



13. The graph of $g(x)$ contains the point $(-2,5)$. Write a formula for a translation of g whose graph contains the point

- **(a)** $(-2,8)$
- **(b)** $(0, 5)$

14. Table 2.16 contains values of $f(x)$. Each function in parts (a)-(c) is a translation of $f(x)$. Find a possible formula for each of these functions in terms of f. For example, given the data in Table 2.17, you could say that $k(x)=f(x)+1$.

Table 2.16

x	0	1	2	3	4	5	6	7
$f(x)$	0	0.5	2	4.5	8	12.5	18	24.5

Table 2.17

x	0	1	2	3	4	5	6	7
$k(x)$	1	1.5	3	5.5	9	13.5	19	25.5

15. Tables 2.18 and 2.19 give values of functions v and w. Given that w(x) = v(x−h)+k, find the constants h and k.

Table 2.18

x	$v(x)$
−2	11
−1	17
0	20
1	17
2	11

Table 2.19

x	$w(x)$
3	4
4	10
5	13
6	10
7	4

(a)

x	0	1	2	3	4	5	6	7
$h(x)$	−2	−1.5	0	2.5	6	10.5	16	22.5

(b)

x	0	1	2	3	4	5	6	7
$g(x)$	0.5	2	4.5	8	12.5	18	24.5	32

(c)

x	0	1	2	3	4	5	6	7
$i(x)$	−1.5	0	2.5	6	10.5	16	22.5	30

In Problems (16)-(17), let s(t) denote the average weight (in pounds) of a baby at age *t* months.

16. The weight, V, of a particular baby named Albertine is related to the average weight function s(t) by the equation

$$V = s(t)+2.$$

Find Albertine's weight at ages t = 3 and t = 6 months. What can you say about Albertine's weight in general?

17. The weight, W, of another baby named Tex is related to s(t) by the equation

$$W = s(t+4).$$

What can you say about Tex's weight at age t=3 months? At t=6 months? Assuming that babies increase in weight over the first year of life, decide if Tex is of average weight for his age, above average, or below average.

18. The function P(t) gives the number of people in a certain population in year t. Interpret in terms of population:

(a) $P(t)+100$

(b) $P(t+100)$

In Problems (19) - (24), *explain in words* the effect of the transformation on the graph of q(z) for positive constants a, b.

19. $q(z)+3$ **20.** $q(z)-a$ **21.** $q(z+4)$ **22.** $q(z-a)$ **23.** $q(z+b)-a$ **24.** $q(z-2b)+ab$

25. Let T(d) give the average temperature in your hometown on the d^{th} day of last year (so d = 1 is January 1, etc).

(a) Graph T(d) for $1 \le d \le 365$.

(b) Give a possible value for each of the following: T(6); T(100); T(215); T(371).

(c) What is the relationship between T(d) and T(d+365)? Explain.

(d) If you graph w(d)=T(d+365) on the same axes as T(d), how would the two graphs compare?

(e) Do you think the function T(d)+365 has any practical significance? Explain.

26. Let S(d) be the height of high tide in Seattle on a specific day, *d*, of the year. Use shifts of the function S(d) to find formulas for each of the following functions:

(a) T(d), the height of high tide in Tacoma on day d, if we assume that high tide in Tacoma is always one foot higher than high tide in Seattle.

(b) A(d), the height of high tide in Astoria on day d, if we assume that high tide in Astoria is the same height as the previous day's high tide in Seattle.

27. Let H(t) be the thermometer reading (Celsius) of a person t hours after onset of an illness. Normal body temperature is 37°C. Find a formula for the fever, the number f(t) of degrees greater than normal, after t hours.