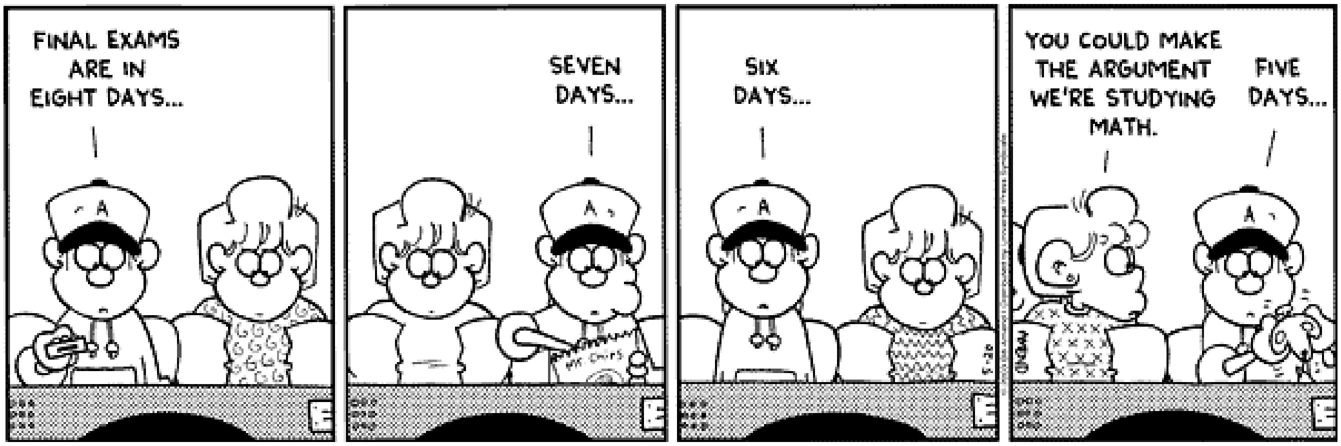
MATH 117 PRACTICE PROBLEMS FOR FINAL

25 APRIL 2019



1. Let $f(x) = 3x^2 + x + 4$, and let $g(x) = 4x - 3$.

   (a) Calculate $f(2x) - 3g(x^2)$. Simplify your answer.

   (b) Calculate $\frac{g(x+h)-g(x)}{h}$. Simplify your answer as much as possible *h*

   (c) Calculate $\frac{f(x+h)-f(x-h)}{2h}$. Simplify your answer as much as possible.

   (d) Compute $g \circ f(x)$.

   (e) Compute $f \circ g(x)$.

   (f) Compute $g \circ g(x)$.

2. Find an equation of a straight line that has x-intercept 11 and is *perpendicular* to the line that passes through the points P = (5, 4) and Q = (7, -1).

3. Factor fully each of the following:

   (a) $16x^4 - 1$

   (b) $2x^5 - x^4 - 15x^3$

   (c) $x^2 - 16y^2$

   (d) $a^2b^3c^4 - 7a^4b^4c^7 + 9(a^2bc)^3$

4. Simplify by removing brackets:

$$a - 2b - [4a - 6b - \{3a - c + (5a - 2b - (3a - c + 2b))\}]$$

5. Solve for *x* in each equation below:

(a) $84 + (x + 4)(x - 3)(x + 5) = (x + 1)(x + 2)(x + 3)$

(b) $3(x - 1)^2 - 3(x^2 - 1) = x - 15$

6. Find the *domain* of each of the following functions:

(a) $y = 4x^3 + x^2 + 7x - 11$

(b) $y = \sqrt{2x + 4} - \sqrt[3]{3 - x}$

(c) $y = \frac{x(x-1)(x-3)}{(x-4)(x+9)}$

7. Using the method of *completing the square*, determine the maximum or minimum value achieved by each of the following:

(a) $f(x) = 7 + 4x + 2x^2$

(b) $g(x) = -4x^2 + x + 1$

(c) $h(x) = x^2 - 7x + 5$

8. Solve the following equation for *x*:

$$(2x - 1)(3x + 5) + 7x + 5 = (6x + 5)(x - 3) - 10(x + 4)$$

9. In the year 2019, Alphaville had 4011 inhabitants and Betaville had only 1880. The population of Alphaville is declining by 79 persons each year. The population of Betaville is growing by 333 persons every year. When will the population sizes of Alphaville and Betaville coincide? (Express your answer to the nearest year.)

10. Find an equation of the *inverse* of the function:

$$y = \frac{3x - 5}{1 - 2x}$$

11. *True* or *False?* (You need not provide an explanation.)

(i) $(a + b + c)^2 = a^2 + b^2 + c^2$ for all real numbers *a*, *b* and *c*.

(ii) $\sqrt{a + b} = \sqrt{a} + \sqrt{b}$ for all non-negative *a* and *b*

(iii) $\sqrt{0} = 0$

(iv) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ for all positive b and all non-negative a.

(v) The ratio of the circumference of a circle to its diameter is π.

(vi) If the radius of a circle is halved, then its *circumference* is halved.

(vii) If the radius of a circle is tripled, then its *area* is tripled.

(viii) $(-55)^0 = 1$

(ix) $(81/10000)^{3/2} = 729/1000000$

(x) $0^0 = 1$

12. (a) Find the *distance* between the points $A = (7, 11)$ and $B = (17, -13)$. (Simplify your answer.)

(b) Find the *midpoint* of the line segment joining points A and B.

13. Suppose that a rope is just long enough to cover the equator of the Earth. *About how much longer* would the rope need to be so that it could be suspended 3 feet above the entire equator.

14. Find the *largest value* of the expression $1 - 4x - 2x^2$. (Hint: Complete the square.)

15. Find all values of x satisfying the equation:

$$\frac{2x+1}{4x-1} = \frac{4x-7}{8x+3}$$

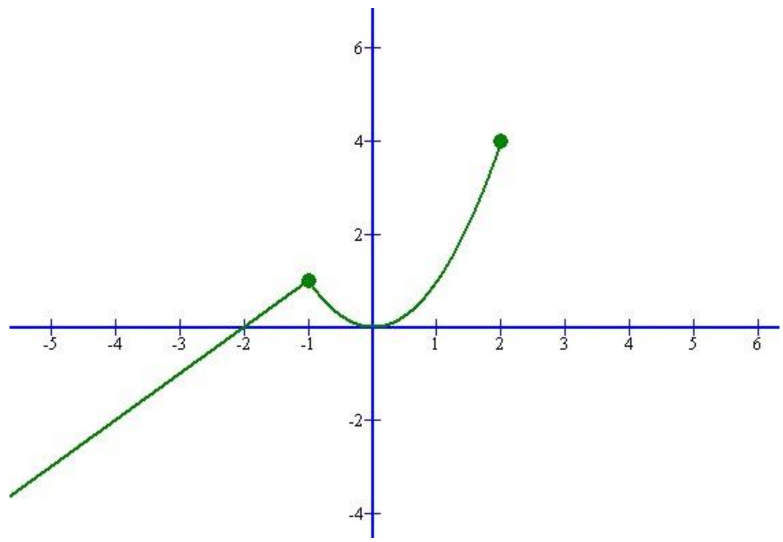
16. Suppose that the domain of a function $y = f(x)$ is the interval $[-2, 7]$ and that the range of f is $[-5, 11]$. Define a new function, h, as follows: $h(x) = 13 + 5f(3x - 9)$

(a) Determine the *domain* of the function h.

(b) Determine the *range* of the function h.

17. Let $f(x) = x^3 + x + 1$. Find the value of $\frac{f(x+3) - f(x)}{3}$ and simplify as much as possible.

18. Let $F(x)$ be a function with domain $[-6, 2]$ and range $[-4, 4]$. The graph of F is displayed below:



Sketch the graph of each of the following:

a. $F(2x)$ b. $F(x + 3)$

c. $F(x - 2)$ d. $3F(-x)$ e. $-2F(x)$ f. $2 + 2F(x/2)$

19. Simplify fully the following expression: $\frac{x+1}{x-3} - \frac{x^2+2}{x^2-9}$ (Hint: Find common denominator.)

20. Let $f(x) = 1 + \frac{1}{x^3}$

(a) Evaluate $f^{-1}(2)$.

(b) Evaluate $(f(2))^{-1}$.

(c) Evaluate $f(2^{-1})$.

21. Let $H(x) = \sqrt[5]{2019 + x^2}$. Find functions, F and G, each simpler than H, such that $H(x) = F \circ G(x)$.

22. Let f(x) = 2x – b and g(x) = 3x + 5. Find b for which $f \circ g(x) = g \circ f(x)$.

23. For which value(s) of the constant b will the following equation have *only one root?* $3x^2 + bx + 1 = 0$

24. Consider the graph of the polynomial $y = f(x) = -x^2(x-2)^4(x-3)^5(x-5)(x^2+x+1)$

(a) Is f(x) a polynomial?
(b) The domain of *f* is:
(b) Find the zeroes of f(x).
(c) What happens to y as $x \to \infty$?
(d) What happens to y as $x \to -\infty$?

25. Suppose that f is an *odd function* and that $f(x) = \frac{x}{1+2x^2}$ *when* $x \geq 0$. What is the value of f(-2) ?

26. Find the vertex of the parabola $y = 4x^2 + x + 2$.

27. Find the *largest value* of $f(x) = 5 - (3x^4 + 1)^{2018}$.

28. Find the *domain* of the function $f(x) = \sqrt{x-5} + x^4 + 2019$

29. Factor and simplify each of the following. (Caution: Do not multiply out!)

a. $(a + 2b)^2 - 16x^2$ (Think difference of two squares.)

b. $(2x + a - 3)^2 - (3 - 2x)^2$

30. Multiply the two polynomials and express in standard form:

$$(x^3 - 2x^2 + x - 3)(2x^3 - x^2 + 3x + 2)$$

31. How many (real) roots?

a) $y = (x-2)^3(x+4)^8(x^2+x+1)$

b) $y = 4x^2 - 144x + 9$

c) $y = (x^2 - 3x + 4)^3$

d) $y = 6x^2 - 4x + 5$

32. Using the quadratic formula, solve each of the following:

(a) $y = 4x^2 - x - 1$

(b) $y = 3 - 9x^2 + 5x$

33. (a) Find an equation of the parabola that has roots x = -9 and x = -4 and passes through the point P = (-3, 1).

(b) Find an equation of a cubic polynomial that has roots x = 0, x = 1 and x = -4 and passes through the point P = (2, -

34. Let P(t) be the price of a house (in thousands of dollars) t years after it was built.

The function P(t) is given by $P(t) = 5t^2 - 18t + 225$.

a. What is the price of the house five years after it was built? Include units.
b. Find the vertical intercept of the function P(t) and provide a practical interpretation for it. Include units.

c. Use the method of completing the square to put the formula for P(t) in vertex form. Show all your algebraic work step-by-step.

d. What is the highest price of the house during the first 5 years after it was built?

In what year was the highest price attained?

e. What is the lowest price of the house during the first 5 years after it was built?

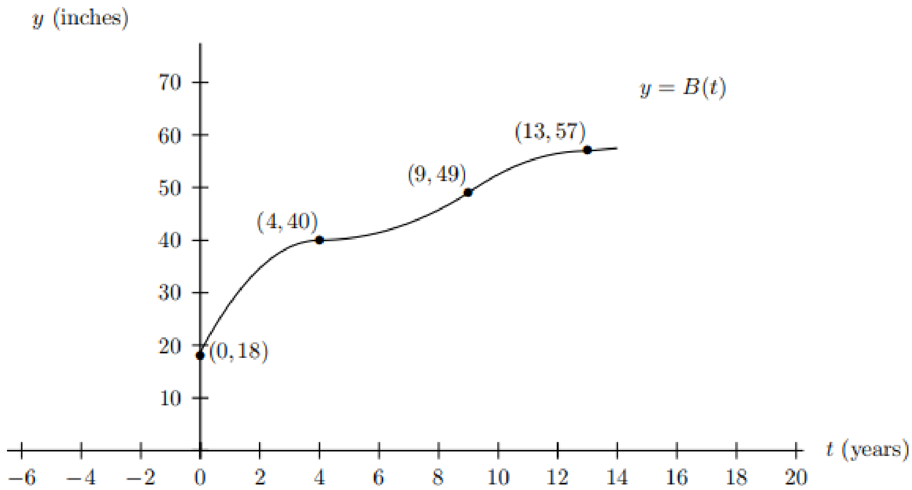
In what year *w*as the lowest price attained?

35. Albertine and Swann have 4 children: Alana, Brentley, Clarissa, and Donovan. Let A(t), B(t), C(t), and D(t) denote the height, in inches, of Alana, Brentley, Clarissa, and Donovan, respectively, at time t, measured in years since January 1, 1990. Alana was born on January 1, 1990.

(a) Alana and Brentley are twins (i.e. they were born at the same time), but Brentley is shorter. He is always 5% shorter than Alana. Write a formula for B(t) in terms of A(t).

(b) Clarissa was born exactly 4 years after Alana. Clarissa is always the same height as Alana was when she was the same age. Write a formula for C(t) in terms of A(t).

(c) Donovan was born exactly 6 years after Brentley. However, Donovan has a larger build, and is always 4 inches taller than Brentley was at the same age. Below, you are given a portion of the graph of y = B(t). The coordinates of four points on the graph are labeled. Using this information, sketch as much as possible of the graph of y = D(t) on the same axes. Label four points on your graph.



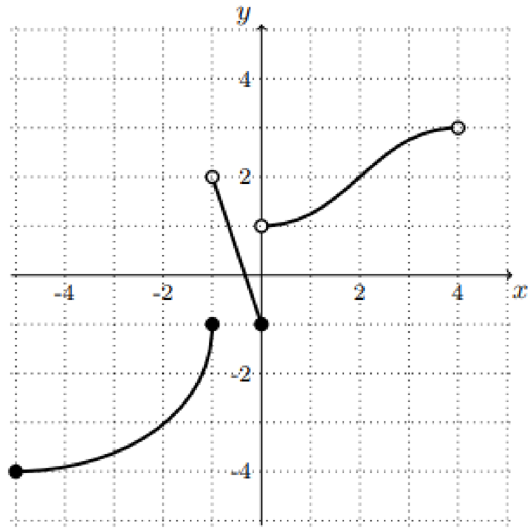
36.

Let $j(x)$ be a function with domain $[-10, 13]$; some values of $j(x)$ are given in the table below.

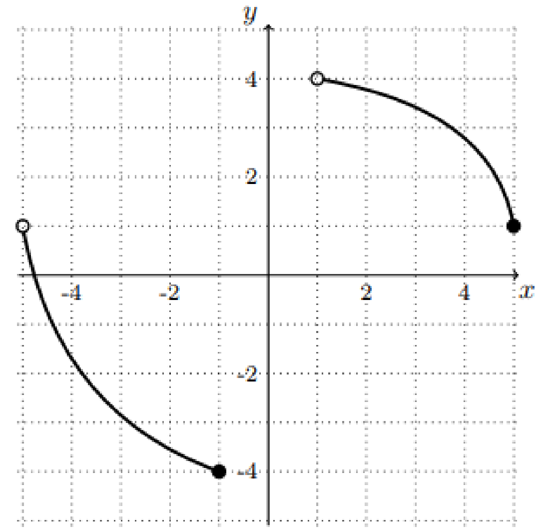
x	-10	-4	0	1	7	13
$j(x)$	-4	-2	1	1.5	1.8	1.9

The graphs of $y = k(x)$ and $y = \ell(x)$ are given below. Note that $\ell(x)$ is an invertible function.

GRAPH OF $y = k(x)$



GRAPH OF $y = \ell(x)$



a. Based on the information above, which of the following statements could be true about the function j(x) on [−10, 13]? Circle all that apply.

j(x) is concave up. j(x) is concave down.

j(x) is neither concave up nor concave down. j(x) is an increasing function.

j(x) is a decreasing function. j(x) is a quadratic function with vertex (0, 1).

j(x) is a linear function. j(x) is a vertical shift of k.

b. What is the range of the function $\ell(x)$?

c. Evaluate the following expressions, writing your answers in the space provided. If the expression cannot be evaluated based on the information given, write undefined.

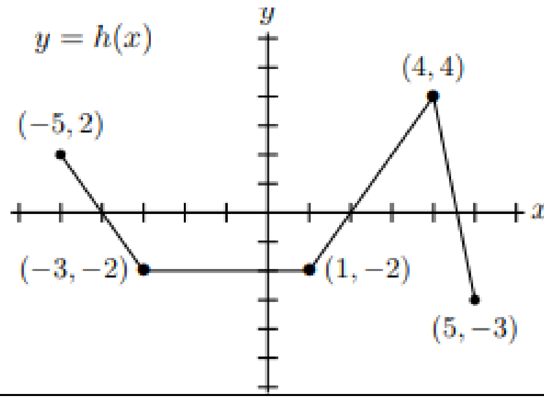
$\ell^{-1}(5)$ ________________ $\ell^{-1}(1)$ ________________

$j(-4)^{-1}$ ________________ $j(k(-5))$ ________________

d. Find all values of x for which $\ell(k(x)) = -4$. Show your work and write your answer in the space provided. Write none if there are no such values of x.

37.

The graph of a function $h(x)$ is shown on the right. Below are the graphs of several transformations of $h(x)$. For each of these graphs, write the letter of the ONE function from the list on the right of the page whose graph is shown. (**Clearly** write the capital letter of your choice on the answer blank provided.) *No work or explanation is required.*



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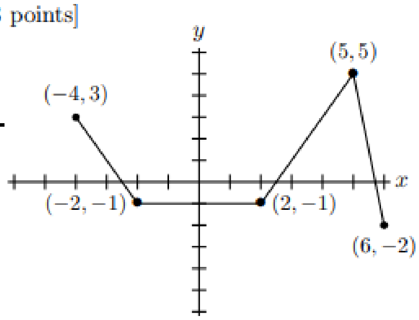
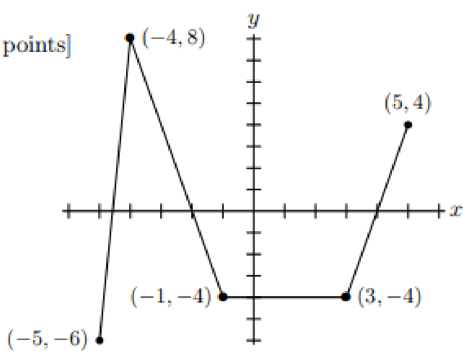
a. [3 points]

Answer: ______



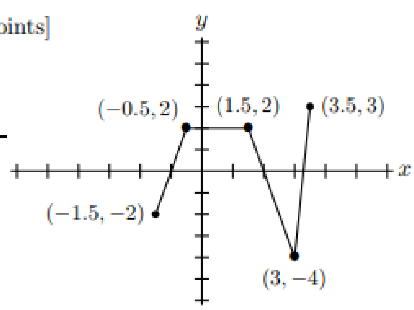
b. [3 points]

Answer: ______



c. [3 points]

Answer: ______



Answer Choices

A. $h(x+1)+1$
B. $h(x-1)+1$
C. $h(x+1)-1$
D. $h(x-1)-1$
E. $h(-x)+1$
F. $h(-x)-1$
G. $-h(x)+1$
H. $-h(x)-1$
I. $-h(x+1)$
J. $-h(x-1)$
K. $h(-x)$
L. $-h(-x)$
M. $2h(x)$
N. $2h(-x)$
O. $-2h(x)$
P. $\frac{1}{2}h(x)$
Q. $\frac{1}{2}h(-x)$
R. $-\frac{1}{2}h(x)-1$
S. $\frac{1}{2}h(x-1)$
T. $h(-2(x-1))$
U. $-h(2x-1)$
V. $-h(2(x-1))$
W. $-h(\frac{1}{2}x-1)$
X. $h(-\frac{1}{2}(x+1))$
Y. $-h(\frac{1}{2}(x-1))$
Z. NONE OF THESE

38. (a) Let $f(x) = \frac{x}{5+x^2}$. Find $\lim_{n\to\infty} f(x)$.

(b) Let $g(x) = \frac{9+x^2}{5+x^2}$. Find $\lim_{n\to\infty} g(x)$.