MATH 117 PRACTICE PROBLEMS FOR TEST I

Study sections 1.1 – 1.5 and 2.1 – 2.3.

1. Without using a calculator, match the following functions to the lines in Figure 1.49.

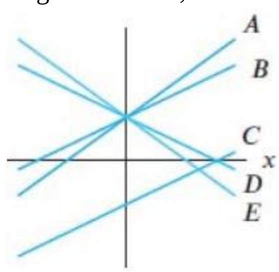


Figure 1.49

(a) $f(x) = 5 + 2x$

(b) $g(x) = -5 + 2x$

(c) $h(x) = 5 + 3x$ (d) $j(x) = 5 - 2x$

(e) $k(x) = 5 - 3x$

(f) For each part above, explain how you were able to determine the correct match.

2. The cost, in dollars, of renting a car for a day from three different rental agencies and driving it d miles is given by the following functions:

$$C_1 = 0.50d$$

$$C_2 = 30 + 0.20d$$

$$C_3 = 50 + 0.10d$$

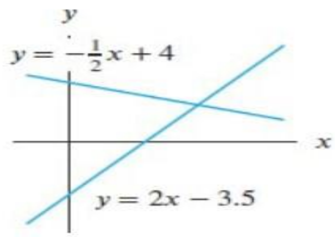
(a) For each agency, describe the rental agreement in words.

(b) Graph the cost function for each agency on one set of axes.

(c) For what driving distance does Agency 1 cost the same as Agency 2?

(d) Determine the different circumstances for which each agency is cheapest. Explain how you know.

3. Determine the point of intersection.



4. Line *l* in Figure 1.52 below is parallel to the line $y = 2x + 1$. Find the coordinates of the point *P*.

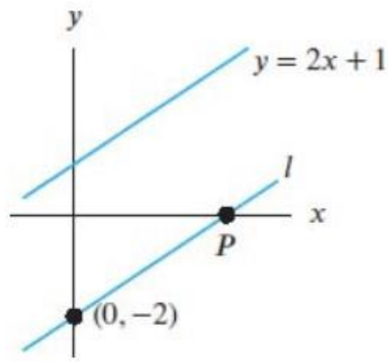


Figure 1.52

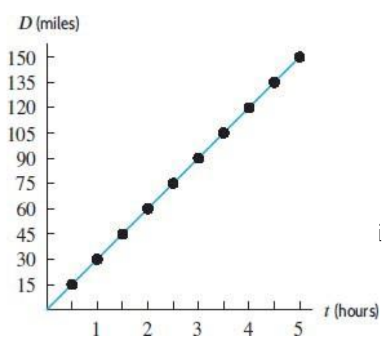
5. In economics, the demand for a product is the amount of that product that consumers are willing to buy at a given price. The quantity demanded of a product usually decreases if the price of that product increases. Suppose that a company believes there is a linear relationship between the demand for its product and its price. The company knows that when the price of its product was \$3 per unit, the quantity demanded weekly was 500 units, and that when the unit price was raised to \$4, the quantity demanded weekly dropped to 300 units. Let D = D(p) represent the quantity demanded weekly at a unit price of p dollars.

(a) Calculate D(5). Use appropriate units. Interpret your result.
(b) Find a formula for D as a function of p.
(c) The company raises the price of the good and the new quantity demanded weekly is 50 units. Predict the new price.
(d) Give an economic interpretation of the slope of the function that you found in part (b).
(e) Find D when p = 0. Find p when D = 0. Give economic interpretations of both.

6. In 2007, you had 40 songs in your favorite iTunes playlist. In 2010, you had 120 songs. In 2014, you had 40. What was the average rate of change per year in the number of songs in your favorite iTunes playlist between

(a) 2007 and 2010?
(b) 2010 and 2014? (c) 2007 and 2014?

7. The graph below shows distance traveled as a function of time.



(a) Find ΔD and Δt, and then compute the rate of change, $\Delta D/\Delta t$, over each of the intervals:

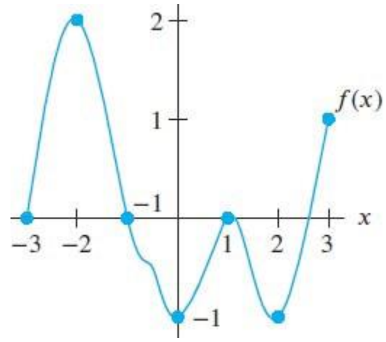
i. $t = 2$ and $t = 5$

ii. $t = 0.5$ and $t = 2.5$

iii. $t = 1.5$ and $t = 3$

(b) Interpret the results of part (a).

8. Use the graph below to answer the following questions.



(a) Find the average rate of change of f for $1 \le x \le 3$.

(b) Find the average rate of change of f for $-3 \le x \le -2$.

(c) Find the average rate of change of f for $-3 \le x \le 1$.

(d) In general, when is a function considered to be increasing? Decreasing?

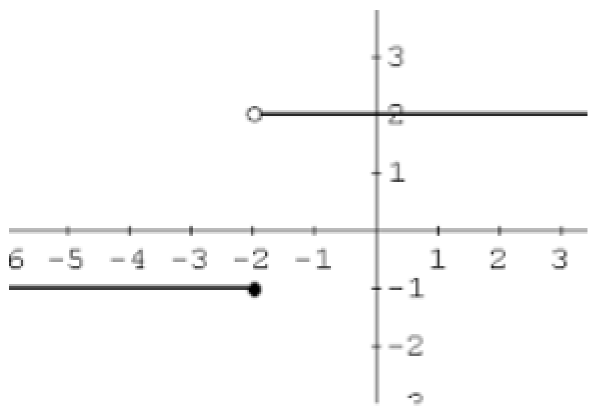
(e) For this function, on which intervals is the function increasing? Decreasing?

9. Let $f(x) = 4 - x^2$.

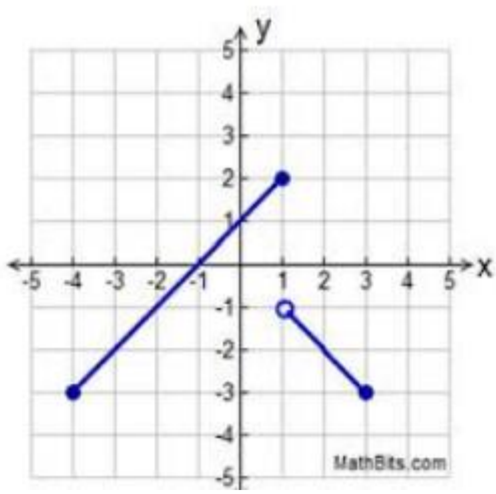
(a) Find $f(0)$ and $f(2)$. What is the average rate of change of $f(x)$ on the interval $0 \le x \le 2$?
(b) Find the average rate of change of $f(x)$ on the interval $2 \le x \le 4$.

(c) Find the average rate of change of $f(x)$ on the interval $b \le x \le 2b$, where b is a positive constant.

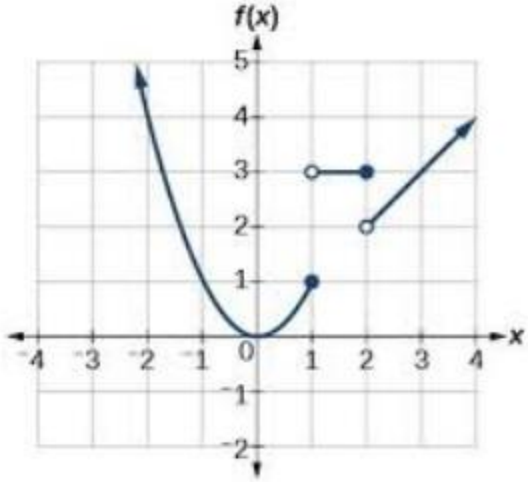
10. Create a Piecewise Function from the following graph



11. Create a Piecewise Function from the following graph



12. Create a Piecewise Function from the following graph



13. Table 1.16 shows the times, *t* in sec, achieved every 10 meters by Carl Lewis in the 100-meter final of the World Championship in Rome in 1987. Distance, *d*, is in meters.

Table 1.16

t	0.00	1.94	2.96	3.91	4.78	5.64
d	0	10	20	30	40	50
t	6.50	7.36	8.22	9.07	9.93	
d	60	70	80	90	100	

(a) For each successive time interval, calculate the average rate of change of distance. What is a common name for the average rate of change of distance?

(b) Where did Carl Lewis attain his maximum speed during this race? Some runners are running their fastest as they cross the finish line. Does that seem to be true in this case?

14. A tech company finds that there is a linear relationship between the amount of money that it spends on advertising and the number of computers it sells. If the company spends no money on advertising, it sells 300 computers. For each additional $5000 spent, an additional 20 computers are sold.

(a) If x is the amount of money that the company spends on advertising, find a formula for y, the number of computers sold as a function of x. Be sure to express your answer in simplified form.

(b) How many computers does the company sell if it spends $25,000 on advertising? $50,000? Use your formula from part (a) to show your work.

(c) How much advertising money must be spent to sell 700 computers?

(d) What is the slope of the line you found in part (a)? Give an interpretation of the slope that relates computers sold and advertising costs.

15. Sri Lanka is an island that experienced approximately linear population growth from 1950 to 2000. On the other hand, Afghanistan was torn by warfare in the 1980s and did not experience linear or near-linear growth.

(a) Table 1.28 gives the population of these two countries, in millions. Which of these two countries is A and which is B? Explain.

Table 1.28

Year	1950	1960	1970	1980	1990	2000
Population of country A	8.2	9.8	12.4	15.1	14.7	23.9
Population of country B	7.5	9.9	12.5	14.9	17.2	19.2

(b) What is the approximate rate of change of the linear function? What does the rate of change represent in practical terms?

(c) Estimate the population of Sri Lanka in 1988.

16. A stalactite is an icicle-shaped formation that hangs from the ceiling of a cave and is produced by precipitation of minerals from water dripping through the cave ceiling. Most stalactites have pointed tips. It grows according to the formula $L(t) = 17.75 + \frac{1}{250}\ t, where\ L(t)$, where $L(t)$ represents the length of the stalactite, in inches, and t represents the time, in years, since the stalactite was first measured.

(a) What is the vertical intercept?

(b) Explain the meaning of the vertical intercept in practical terms. Include units in your answer.

(c) What is the slope?

(d) Explain the meaning of the slope in practical terms. Include units in your answer.

17. Let $h(x) = x^2 - 3x + 5$. Evaluate and simplify the following expressions.

(a) $h(2)$

(b) $h(a - 2)$

(c) $h(a) - 2$

(d) $h(a) - h(2)$

18. If $h(x) = ax^2 + bx + c$, find $h(0)$.

19. Let $f(t) = t^2 - 4$.

(a) Find $f(0)$.

(b) Solve $f(t) = 0$.

(c) Solve $f(t) = -2$.

20. Table 2.4 shows $N(s)$, the number of sections of Economics 101, as a function of s, the number of students in the course. If s is between two numbers listed in the table, then $N(s)$ is the higher number of sections.

Table 2.4

s	50	75	100	125	150	175	200
$N(s)$	4	4	5	5	6	6	7

(a) Evaluate and interpret:

i. $N(150)$

ii. $N(80)$

(b) Solve for s and interpret:

i. $N(s) = 7$

ii. $N(s) = 4$

21. Let $v = f(t)$ be the speed of a braking car, in feet per second, t seconds after the brakes are first applied. A graph of f is given in Figure 2.6.

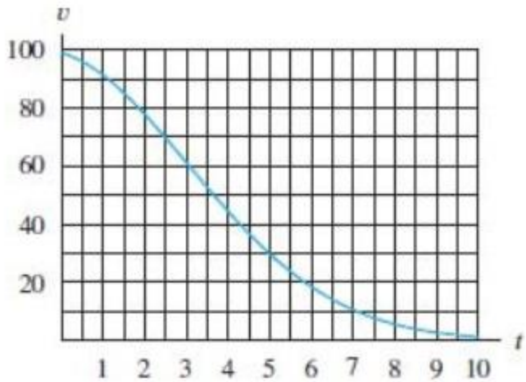


Figure 2.6

Use Figure 2.6 to complete the following:

(a) Evaluate $f(1)$. Explain the meaning of your answer in terms of the car.

(b) Evaluate $f(7) - f(5)$. Explain the meaning of your answer in terms of the car.

(c) Solve for $f(t) = 60$. Explain the meaning of your answer in terms of the car.

22. Let $f(x) = x^2 + 2x - 2$ and $g(x) = 2x + 2$.

(a) Sketch a graph of $g(x)$.

(b) Find the value(s) of x such that $g(x) = 2$. Describe how you can find this value both algebraically and graphically.

(c) Using your graph from part (a), find all values of x such that $g(x) < 2$. How can you use the graph to find these values? How can you find these values algebraically? Demonstrate both methods in your notebook.

(d) Sketch a graph of $f(x)$.

(e) Using your graph from part (d), find all values of x such that $f(x) < 1$. How can you use the graph to find these values? How can you find these values algebraically? Demonstrate both methods in your notebook.

(f) Sketch a graph of $f(x)$ and $g(x)$ on the same axis.

(g) Using your graph from part (f), find all values of x such that $f(x) < g(x)$. How can you use the graph to find these values? How can you find these values algebraically?
Demonstrate both methods in your notebook.

23. The profit, in dollars, made by a theater when n tickets are sold is $P(n) = 20n - 500$.

(a) Calculate $P(0)$, and explain what this number means for the theater.

(b) Under what circumstances will the profit equal 0?

(c) What is the meaning of the quantity $P(100)$? What are its units?

24. For what value(s), if any, is each function *undefined*? How does this help us determine the domain? State the domain for each function as well.

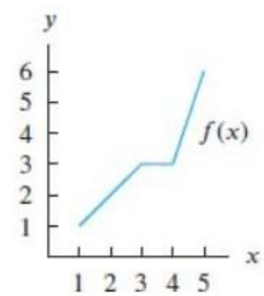
(a) $f(x) = \dfrac{x-2}{x-3}$

(b) $h(x) = \sqrt{x-15}$

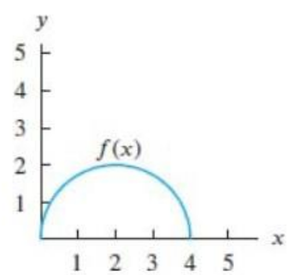
(c) $g(x) = \dfrac{1}{\sqrt{x-15}}$

(d) Explain why the functions in (b) and (c) do not have the same domain.

25. Estimate the domain and range of the function. Write your answer as an inequality. Assume the entire graph is shown.



(a)



(b)

26. Find the domain of each function algebraically.

(a) $f(x) = \dfrac{1}{x+3}$

(b) $g(t) = \dfrac{1}{t^2-4}$

(c) $h(s) = \dfrac{1}{s^4+2}$

(d) $p(r) = \sqrt{r-16}$

(e) $q(a) = \dfrac{a-2}{\sqrt{a-5}}$

27. A movie theater seats 200 people. For any particular show, the amount of money the theater makes is a function of the number of people, n, in attendance. If the theater makes \$4.00 per ticket, find the domain and range of this function. Sketch its graph.

28. In month $t = 0$, a small group of rabbits escapes from a ship onto an island where there are no rabbits. The island rabbit population, $p(t)$, in month t is given by

$$p(t) = \frac{1000}{1+19(0.9)^t} \quad , t \geq 0$$

(a) Evaluate $p(0)$, $p(10)$, $p(50)$, $p(100)$, and $p(500)$ and explain their meaning in terms of rabbits.

(b) Graph the points that you just found. Then graph the function using a calculator or WolframAlpa. What viewing window is appropriate to understand how the function is behaving?

(c) Does the graph suggest the growth in population you would expect among rabbits on an island? Why or why not?

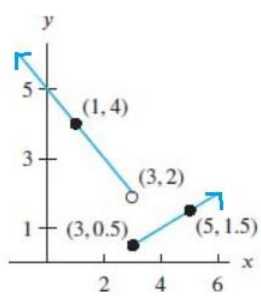
(d) Using your graph and your knowledge of the scenario, estimate the range of $p(t)$. What does this tell you about the rabbit population?

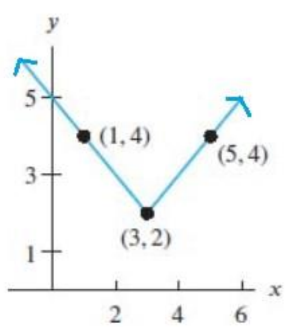
29. Sketch a graph of each piecewise defined function given below.

$$f(x) = \begin{cases} x+1, & -2 \le x < 0 \\ x-1, & 0 \le x < 2 \\ x-3, & 2 \le x < 4 \end{cases}$$

$$f(x) = \begin{cases} x+4, & x \le -2 \\ 2, & -2 < x < 2 \\ 4-x, & x \ge 2 \end{cases}$$

30. Write formulas for each of the following functions.





31. Given that

$$f(x) = \begin{cases} 3x & \text{for } -1 \le x \le 1 \\ -x+4 & \text{for } 1 < x \le 5 \end{cases}$$

(a) Find $f(0)$ and $f(3)$.

(b) Sketch a graph of $f(x)$.

(c) Find the domain and range of $f(x)$.

32. Let $f(x) = 6|x+5| - 7$

(a) What are the domain and range of $f(x)$?

(b) Find all values of x such that $f(x) = 11$.

33. A floor-refinishing company charges \$1.83 per square foot to strip and refinish a tile floor for up to 1000 square feet. There is an additional charge of \$350 for toxic waste disposal for any job that includes more than 150 square feet of tile.

(a) Express the cost, y, of refinishing a floor as a piecewise-defined function of the number of square feet, x, to be refinished.

(b) Sketch a graph of the function. Give the domain and range.

34. The Ironman Triathlon is a race that consists of three parts: a 2.4-mile swim followed by a 112-mile bike race and then a 26.2-mile marathon. Albertine swims steadily at 2 mph, cycles steadily at 20 mph, and then runs steadily at 9 mph. Assuming that no time is lost during the transition from one stage to the next, write a piecewise-defined formula for the distance covered, d, in miles, as a function of the elapsed time t in hours, from the beginning of the race. Sketch a graph of the function.

35. Figure 1.64 shows the average monthly temperature in Albany, New York, over a twelve-month period. (January is month 1.)

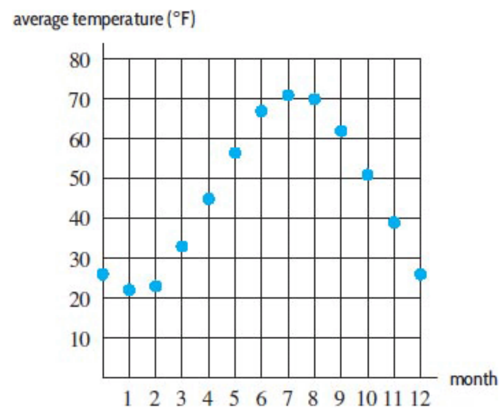


Figure 1.64

(a) Make a table showing average temperature as a function of the month of the year.

(b) What is the warmest month in Albany?

(c) Over what interval of months is the temperature increasing? decreasing?