MATH 117 SOLUTIONS: TEST II 28 MARCH 2019

INSTRUCTIONS: Answer any 9 of the following 12 problems. You may answer more than 9 to earn extra credit.

1. Suppose $C(x) = 50x^2 - 140x + 101$ represents the cost, in hundreds of euros, to produce x thousand 60 watts LED bulbs.

(a) Using a complete sentence, explain what C(2) = 21 means. *(Include units!)*

Answer: To produce 2,000 LED bulbs, it costs 2,100 euros.

(b) Using a complete sentence, explain what $C^{-1}(78)$ means. *(Include units!)*

Answer: $C^{-1}(78)$ represents the number (in thousands) of LED bulbs that can be purchased for 7800 euros.

2. The domain and range of the function y = f(x) are $[-2, 6)$ and $(-\infty, -10]$, respectively.

(a) Let $g(x) = 1 - f(x + 8)$.

Find the domain of g.

Solution: $-2 \leq x + 8 \leq 6$; so $-10 \leq x \leq -2$

So the domain of g is [-10, -2].

Find the range of g.

Solution: The range of f is $(-\infty, -10]$. $Since\ f\ is\ multiplied\ by -1,$

$the\ range\ of\ -f\ is\ [10, \infty).$

$Finally, the\ range\ of\ g(x) = 1 - f(x+8)\ is\ (-\infty, -9]$

(b) If $f^{-1}(x)$ exists, then

Find the domain of f^{-1}.

$Answer:\ (-\infty, -10]$

Find the range of f^{-1}.

$Answer:\ [-2, 6]$

3. For each of the following equations, find any and all real roots.

(a) $27 - 3x^2 = 0$

Solution: Factoring, we find $0 = 3(9 - x^2) = (3 - x)(3 + x)$

Hence the roots are x = 3 and x = -3.

(b) $2x^2 - 5x - 3 = 0$

Solution:

Factoring yields $0 = 2x^2 - 5x - 3 = (2x - 3)(x - 1)$.

Hence the roots are x = 3/2 and x = 1.

(c) $z^3 - 7z^2 + 12z = 0$

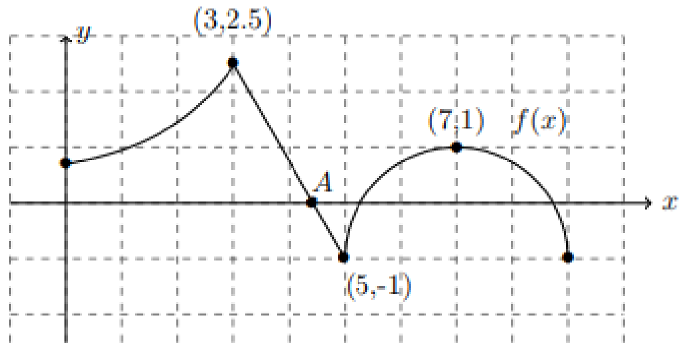
Solution: Factoring yields $z^3 - 7z^2 + 12z = z(z^2 - 7z + 12) = z(z-4)(z-3)$

Hence the roots are z = 0, z = 3, and z = 4.

(d) $x^2 + 5 = 0$

Solution: There are no roots since $x^2 + 5 \geq 5$.

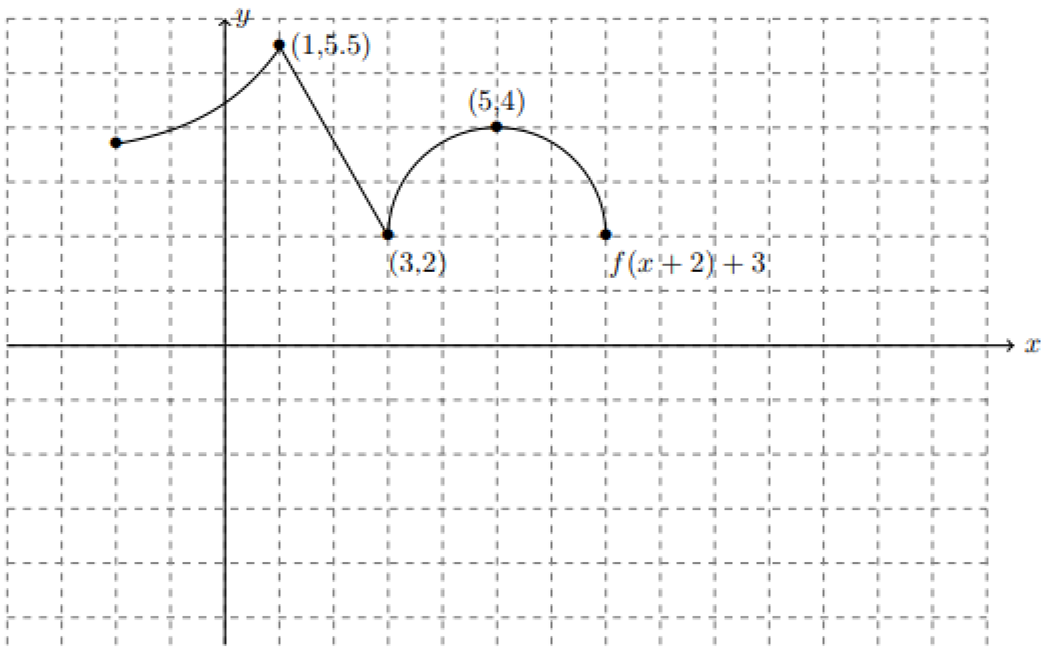
4. Given below is the graph of a function y = f(x) with domain [0, 9]. Assume that f is linear between x = 3 and x = 5.



(a) Using the graph, fill in the following blanks. Where applicable, you can use either interval or inequality notation.

- f is increasing on $[0, 3] \cup [5, 7]$
- f is concave down on $[5, 9]$
- The range of *f* is $[5, 9]$
- The exact coordinates of the point A are $x = \frac{31}{7}, y = 0$

(b) Carefully sketch a graph of f (x + 2) + 3 on the axes below. **Label the points** on your sketch corresponding to the labeled points on the graph of f.



5. Odette is a student at a nearby college. Let C(h) be the total tuition, in thousands of dollars; the college charges her if she takes *h* credit hours, and let *a* be the average number of credit hours students take at the college. For each of the following, choose the one expression from the list of "Answer Choices" that best represents the described quantity. Clearly write the capital letter of your choice on the answer blank provided.

A. $C(3)$	F. $C(a)+3$	K. $3C^{-1}(a)$	P. $C^{-1}(3a)$
B. $C^{-1}(3)$	G. $C(a-3)$	L. $C^{-1}(a)/3$	Q. $3C(C(a))$
C. $C(a)$	H. $C(a)-3$	M. $C(a)/3$	R. $C(3C^{-1}(a))$
D. $C^{-1}(a)$	I. $3C(a)$	N. $C(a/3)$	S. $C^{-1}(3C(a))$
E. $C(a+3)$	J. $C(3a)$	O. $C^{-1}(a/3)$	T. $C^{-1}(C(3a))$

(a) Odette's tuition (in thousands of dollars) if she takes a total of 3 credit hours

Answer: **A.** C(3)

(b) Odette's total tuition (in thousands of dollars) if she takes 3 credit hours more than average

Answer: **E.** C(a+3)

(c) Odette's tuition (in thousands of dollars) if she takes one third the average number of credit hours.

Answer: **N.** C(a/3)

(d) The amount (in thousands of dollars) that Odette pays for tuition if she takes the average number of credit hours but has a scholarship that covers three thousand dollars of her tuition.

Answer: **H.** C(a)

(e) The number of credit hours Odette takes if her total tuition is three times as much as the tuition for taking the average number of credit hours.

Answer: **S.** $C^{-1}(3C(a))$

6. In 1999, a type of deer was set free on a previously uninhabited island in Lake Superior, in an attempt to establish a permanent population of deer on the island. The population of deer grew over time. Population measurements were made each year, as shown in the following table.

Year	1999	2000	2001	2002
Population	20	23	27	31

Let P(t) be a function that gives the population of deer on the island as a function of time, t, measured in years since 1999.

(a) In the context of this problem, give a practical interpretation of $P(40)$

Answer: P(40) represents the deer population in the year 2039.

(b) In the context of this problem, give a practical interpretation of $P^{-1}(40)$.

Answer: $P^{-1}(40)$ represents the number of years after 1999 when the deer population reaches 40.

(c) What is the average rate of change of the deer population over the interval t = 1999 to t = 2002? Use appropriate units.

Answer: $\frac{\Delta P}{\Delta t} = \frac{P(3)-P(0)}{3-0} = \frac{31-20}{3} = \frac{11}{3} \approx \mathbf{3.7\ deer/year}$

7. Let $y = F(x) = \frac{x}{1+x}$.

i. What is the domain of F?

Answer: all real numbers except for x = 0.

ii. Find the average rate of change of *F* over the interval $0 \le x \le 4$.

Solution: $\frac{\Delta F}{\Delta x} = \frac{F(4)-F(0)}{4-0} = \frac{\frac{4}{5}-0}{4-0} = \frac{1}{5}$

iii. Find the average rate of change of F over the interval $1 \le x \le 5$.

Solution: $\frac{\Delta F}{\Delta x} = \frac{F(5)-F(1)}{5-1} = \frac{\frac{5}{6}-\frac{1}{2}}{5-1} = \frac{\frac{1}{3}}{4} = \frac{1}{12}$

iv. Find the average rate of change of F over the interval $2 \le x \le 2 + h$.

Solution: $\frac{\Delta F}{\Delta x} = \frac{F(2+h)-F(2)}{(2+h)-2} = \frac{\frac{2+h}{3+h}-\frac{2}{3}}{h} = \frac{3(2+h)-2(3+h)}{h(3+h)(3)} = \frac{h}{3h(3+h)} = \frac{1}{3(3+h)}$

v. Find $F^{-1}(2)$

Solution: $Set\ F^{-1}(2) = x.\ Then\ F(x) = 2, and\ so\ \frac{x}{x+1} = 2.\ \ This\ implies\ that\ x = 2(x+1).$ Solving for x, we obtain x = -2.

vi. Find $(F(2))^{-1}$

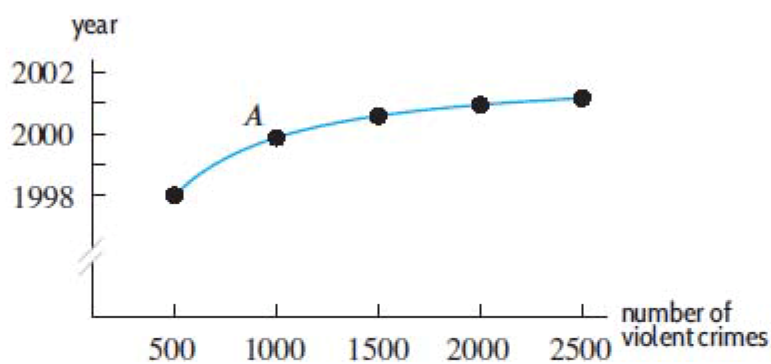
Solution: Solution: $(F(2))^{-1} = \frac{1}{F(2)} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$

vii. Find $F(2^{-1})$

Solution: $F(2^{-1}) = F\left(\frac{1}{2}\right) = \frac{\frac{1}{2}}{1+\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{3}$

8.

a) An incumbent politician running for reelection declared that the number of violent crimes is no longer rising and is presently under control. Does the graph shown below support this claim? Why or why not?



Solution: No, not at all. Note that the free variable, year, is on the y-axis, not the x-axis! Now in 1998, there were 500 violent crimes; in 2000, there were 1000 violent crimes, and in 2002, there were 2500 violent crimes. Indeed the number of violent crimes is skyrocketing!

b) Use the table below to decide whether you expect the graph of R(t) to be

t	1.5	2.4	3.6	4.8
$R(t)$	−5.7	−3.1	−1.4	0

concave up or concave down. Explain.

Solution: We will compute the average rate of change of R(t) over three intervals:

Over $[1.5, 2.4]$the average rate of change was

$$\frac{\Delta R}{\Delta t} = \frac{R(2.4)-R(1.5)}{2.4-1.5} = \frac{-3.1-(-5.7)}{2.4-1.5} = \frac{2.6}{0.9} = 2.89$$

Over $[2.4, 3.6]$ the average rate of change was

$$\frac{\Delta R}{\Delta t} = \frac{R(3.6)-R(2.4)}{3.6-2.4} = \frac{-1.4-(-3.1)}{1.2} = \frac{1.7}{1.2} = 1.42$$

Over $[3.6, 4.8]$ the average rate of change was

$$\frac{\Delta R}{\Delta t} = \frac{R(4.8)-R(3.6)}{4.8-3.6} = \frac{0-(-1.4)}{1.2} = \frac{1.4}{1.2} = 1.17$$

Since the average rate of change is decreasing, it must follow that the graph of R(t) is concave down.

9. (A) Let $f(x) = x^2 + 3 \quad and \ g(x) = \frac{1}{x+3}$

(i) Find $f \circ g(x)$

Solution: $f \circ g(x) = f(g(x)) = f\left(\frac{1}{x+3}\right) = \left(\frac{1}{x+3}\right)^2 + 3$

(ii) Find $g \circ f(x)$

Solution: $g \circ f(x) = g(x^2 + 3) = \frac{1}{(x^2+3)+3} = \frac{1}{x^2+6}$

(iii) Find $f \circ f(x)$

Solution: $f \circ f(x) = f(x^2 + 3) = (x^2 + 3)^2 + 3$

(B) Note: The following questions are independent of part (A).

(i) Let $h(x) = (3x+1)^{99}$. Find simpler functions $f(x)$ *and* $g(x)$ *such that* $h(x) = f \circ g(x)$

Solution: Let $\mathrm{g(x)} = 3\mathrm{x} + 1$ and $\mathrm{f(x)} = \mathrm{x}^{99}$

(ii) Let $v(x) = \frac{1}{1+9x^4}$.

Find simpler functions $f(x)$ *and* $g(x)$ *such that* $v(x) = f \circ g(x)$

Solution: Let $\mathrm{g(x)} = 1 + 9x^4$ and $\mathrm{f(x)} = \frac{1}{x}$.

10. On the planet Melancholia, it is known that 8 degrees Alpha corresponds to 14 degrees Beta and that 30 degrees Alpha corresponds to 80 degrees Beta. Find a linear relation between Alpha degrees and Beta degrees. Express *degrees Alpha as a function of degrees Beta.*

Solution:

Let us think of Alpha as the dependent variable and Beta the independent variable.

So Alpha = F(Beta).

Now when the Beta temperature is 14, Alpha is 8 deg So F(14) = 8.

Similarly, when the Beta temperature is 80 deg, the Alpha temperature is 30 deg. So F(80) = 30.

Since the relation between Alpha and Beta temperatures is linear, we compute the slope of the line: $m = \frac{\Delta\, Alpha}{\Delta\, Beta} = \frac{30-8}{80-14} = \frac{22}{66} = \frac{1}{3}$.

Using point-slope form:

$$Alpha - 8 = \frac{1}{3}(Beta - 14)$$

11. Information about two different functions, C(w) and G(t), is given below.

w	0	1	3
$C(w)$	6	4	1

$$G(t) = \frac{t^2+3t-1}{(t+2)(t-3)}$$

a) Find $C^{-1}(1)$ **12.**

Answer: $C^{-1}(1) = 3$

b) Find G(C(3))

Solution:

$$G(C(3)) = G(1.3) = \frac{1.3^2+3(1.3)-1}{(3.3)(-1.7)} = \frac{3.29}{-5.61} = -5.86$$

c) Give the domain of G using interval notation.

Answer: $(-\infty,-2) \cup (-2,3) \cup (3,\infty)$

d) Find the average rate of change of G between t = -1 and t = 2. Simplify your answer.

Solution: First noting that $G(2) = \frac{2^2+3(2)-1}{(2+2)(2-3)} = \frac{9}{-4} = \frac{9}{4}$ and

$G(-1) = \frac{1-3-1}{(1)(-4)} = \frac{-3}{-4} = \frac{3}{4}$ we find that

$$\frac{\Delta G}{\Delta t} = \frac{G(2)-G(-1)}{2-(-1)} = \frac{\frac{9}{4}-\frac{3}{4}}{3} = \frac{\frac{6}{4}}{3} = \frac{1}{2}$$

12. On a particularly cold winter day, Albertine decides to turn on her gas-powered heater at 5:00 pm. Over the next few hours, she records the temperature of her house and the amount of gas that the heater has used.
She also notices that the temperature of her house seems to affect how loudly her dog barks.

Albertine uses the following three functions to model her observations:

- T(t) represents the temperature (in degrees Fahrenheit) of Albertine's house *t* minutes after 5:00 pm.
- G(t) represents the amount of gas (in cubic feet) that the heater used during the first *t* minutes after 5:00 pm.
- B(x) represents the sound intensity (in decibels) of the dog's barks when the temperature of her house is x degrees Fahrenheit.

You may assume that each of T, G, and B has an inverse.

Find a mathematical expression for each of the quantities below using the functions T, G, B, and/or their inverses.

(a) The number of minutes the heater had been on when it had used 2 cubic feet of gas.

Answer: $G^{-1}(2)$

(b) The sound intensity (in decibels) of the dog's barks at 6:20 pm

Answer: $B(T(80))$

(c) The amount of gas (in cubic feet) used by the heater between 5:30 pm and 7:00 pm.

Answer: $G(T(120)) - G(T(30))$

(d) The temperature (in degrees Fahrenheit) of Albertine's house *k* minutes before 7:00 pm.

Answer: $T(120 - k)$

(e) Albertine's heater has used 3 cubic feet of gas at 6:12 pm.

Answer: $G(42) = 3$

(f) The heater uses 3 cubic feet of gas every 72 minutes.

Answer: $G(k + 72) - G(k) = 3$ for all $k > 0$