MATH 117 SOLUTIONS: TEST III 16 APRIL 2019

NOTE: ONLY AN INEXPENSIVE NON-GRAPHING CALCULATOR IS PERMITTED!

Instructions: Answer any *10 of the 12* problems. You may answer more than 10 to obtain extra credit.

Unless otherwise stated, be certain to show your work in each problem that you select.

1. TRUE OR FALSE: Are the statements in Problems (a) − (q) true or false? You needn't justify your answers. Write in full either "True" or "False" rather than abbreviating.

(a) The quadratic function f(x) = x(x+2) is in factored form. **True**

(b) If g(x) = (x +1)(x+2), then the zeros of *g* are 1 and 2. **False**

(c) A quadratic function whose graph is concave up has a maximum. **False**

(d) All quadratic equations have the form f(x) = ax^2. **False**

(e) If the height above the ground of an object at time *t* is given by s(t) = $at^2 + bt + c$, then s(0) tells us *when* the object hits the ground. **False**

(f) To find the zeros of f(x) = $ax^2 + bx + c$, solve the equation $ax^2 + bx + c = 0$ for x. **True**

(g) Every quadratic equation has two real solutions. **False**

(h) There is only one quadratic function with zeros at x = −2 and x = 2. **False**

(i) A quadratic function has exactly two zeros. **False**

(j) The graph of every quadratic function is a parabola. **True**

(k) The maximum or minimum point of a parabola is called its vertex. **True**

(l) If a parabola is concave up its vertex is a maximum point. **False**

(m) If the equation of a parabola is written as $y = a(x – h)^2 + k$, then the vertex is located at the point (−h, k). **False**

(n) If the equation of a parabola is written as $y = a(x – h)^2 + k$, then the axis of symmetry is found at x = h. **True**

(o) If the equation of a parabola is $y = ax^2 + bx + c$ and a < 0, then the parabola opens downward. **True**

(p) A parabola cannot intersect the x-axis three times. **True**

(q) A parabola has one and only one y-intercept. **True**

2. Solve for *x* by *completing the square:*

(a) $x^2 – 7x + 4 = 0$

Solution: $0 = x^2 – 7x + 4 = (x - \frac{7}{2})^2 - \frac{49}{2} + 4 = (x - \frac{7}{2})^2 - \frac{41}{2}$

Hence $(x - \frac{7}{2})^2 - \frac{41}{2} = 0 \quad \Rightarrow \left(x - \frac{7}{2}\right)^2 = \frac{41}{2} \quad \Rightarrow x - \frac{7}{2} = \pm\sqrt{\frac{41}{2}}$

Thus $x = \frac{7}{2} \pm \sqrt{\frac{41}{2}}$

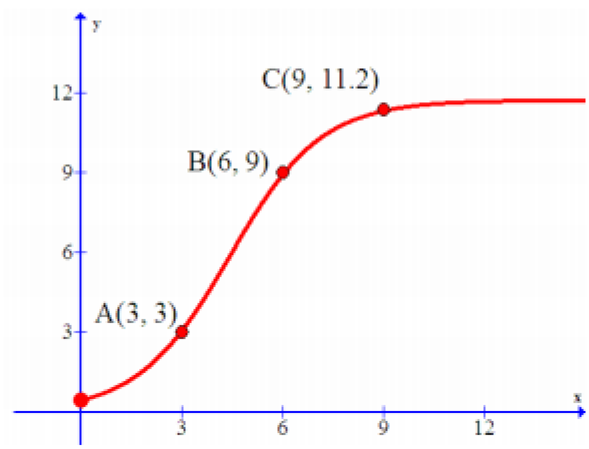
(b) $\frac{x}{x+3} = \frac{3x+2}{x+1}$

Solution: Cross-multiplying: $x(x+1) = (3x+2)(x+3) \Rightarrow \; x^2 + x = 3x^2 + 11x + 6 \;\Rightarrow 2x^2 + 10x + 6 = 0 \;\Rightarrow\; x^2 + 5x + 3 = 0 \;\Rightarrow$

Completing the square: $(x + \frac{5}{2})^2 - \frac{25}{4} + 3 = 0 \;\Rightarrow \left(x + \frac{5}{2}\right)^2 = \frac{13}{4} \;\Rightarrow x + \frac{5}{2} = \pm\sqrt{\frac{13}{4}} \;\Rightarrow\; x = -\frac{5}{2} \pm \sqrt{\frac{13}{4}} \;\Rightarrow x = \frac{5}{2} \pm \frac{\sqrt{13}}{2} = \frac{5 \pm \sqrt{13}}{2}$

3. (a) Compute the *average rate of change* from A to B, from B to C and from A to C.

Which one gives the largest average rate of change?



Solution: From A to B: $\frac{\Delta y}{\Delta x} = \frac{9-3}{6-3} = 2$

From B to C: $\frac{\Delta y}{\Delta x} = \frac{11.2-9}{9-6} = \frac{2.2}{3} = 0.73$

From A to C: $\frac{\Delta y}{\Delta x} = \frac{11.2-3}{9-3} = \frac{8.2}{6} = 1.37$

The largest rate of change occurs from A to B.

(b) Find the equation of a quadratic function that has roots x = -1 and x = 7, and passes through the point Q = (3, 1).

Solution: Any quadratic with the given roots must have the form y = k(x + 1)(x – 7) for a constant, k.

Since the parabola psses thourhg the point Q = (3, 1), we use this fact to obtain k, viz.

$1 = k(3 + 1)(3 - 7) = -16k$.

Thus $k = -1/16$. Hence the quadratic function is:

$$\boldsymbol{f(x) = -\frac{1}{16}(x+1)(x-7)}$$

4. Let $y = g(x)$ be a function with domain $[-29, 19]$ and range of $[1, 5]$.

Let $f(x) = 5 + 3\,g(4x - 9)$.

(a) Find the *domain* of $y = f(x)$.

Solution: We require that $-29 \leq 4x - 9 \leq 19$

Thus $-20 \leq 4x \leq 28 \Rightarrow \boldsymbol{-5 \leq x \leq 7}$.

Alternatively, in interval form: $[-5, 7]$

(b) Find the *range* of $y = f(x)$

Solution: The range of $f(x)$ is the range of g multiplied by 3 and then translated 5 units up.

So $[1,5] \Rightarrow [3,15] \Rightarrow [8,20]$

Hence the new range is $[8,20]$.

5. *Without solving*, determine the *number of roots* that each of the following polynomials has. Show your work.

(a) $y = 2x^2 - 3x + 11$

Solution:

The discriminant $= b^2 - 4ac = 9 - 4(22) < 0$. Thus, this quadratic has 0 roots.

(b) $y = x^2 - x - 5$

Solution: The discriminant $= b^2 - 4ac = 1 - 4(1)(-5) > 0$. Thus, this quadratic has 2 roots.

(c) $y = 318 - 30x - x^2$

Solution: The discriminant $= b^2 - 4ac = 900 - 4(318)(-1) = 96 + 4(318) > 0$. Thus, this quadratic has 2 roots.

(d) $y = x^2 - 100x + 2500$

Solution: The discriminant $= b^2 - 4ac = 10000 - 4(2500) = 0$. Thus, this quadratic has exactly 1 root.

6. (a) Use the quadratic formula to find the roots of $f(x) = -x^2 + 3x + 1$.

Solution $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{9 - (4)(-1)}}{2(-1)} = \frac{-3 \pm \sqrt{13}}{-2} = \frac{3 \pm \sqrt{13}}{2}$$

(b) Use the quadratic formula to find the roots of $f(x) = 2x^2 - 8x - 3$

Solution: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{8 \pm \sqrt{64-(4)(-6)}}{2(2)} = \frac{8 \pm \sqrt{88}}{4} = \frac{4 \pm \sqrt{22}}{2}$

7. Note: You do not have to show any work for this problem.

If (2, −6) is a point on the graph of y = h(x), find a point on the graph of each of the functions below.

(i) **(1, -6)** is a point on the graph of y = h(2x).

Solution:

To obtain the graph of y = h(2x) from the graph of y = h(x), we compress horizontally towards the y-axis by a factor of 1/2 , moving (2, -6) to the point (1, -6).

(ii) **(-2, -5)** is a point on the graph of y = h(−x) + 1.

Solution:

To obtain the graph of y = h(-x) + 1 from the graph of y = h(x), we first reflect the graph about the y-axis (moving the point (2, -6) to the point (-2, -6)) and then shift the resulting graph up by one unit (moving the point (-2, -6) to the point (-2, -5)).

(iii) **(3, 18)** is a point on the graph of y = −3h(x − 1).

Solution: To obtain the graph of y = -3h(x – 1) from the graph of y = h(x), we first stretch the graph vertically away from the x-axis by a factor of 3 (moving the point (2, -6) to the point (2, -18)). Then we reflect the graph about the x-axis, moving (2, -18) to the point (2, 18). Finally, we shift the resulting graph to the right by one unit (moving the point (2, 18) to the point (3, 18).

(iv) Now assume that y = h(x) is an even function. Then (-2, -6) is also a point on the graph of y = h(x).

Solution: Since y is an even function, h(-2) = h(2). Hence (-2, -6) lies on the graph of y = h(x).

(v) Now assume that y = h(x) is an odd function. Then (-2, 6) is also a point on the graph of y = h(x).

Solution: Since y is a odd function, h(-2) = -h(2). Thus (-2, 6) lies on the graph of y = h(x).

8. The temperature *T*, in degrees Fahrenheit, *t* hours after 8 am is given by:

$$T(t) = -\frac{1}{2}t^2 + 12\,t + 38.$$

(Use appropriate units in each of the following.)

(a) What is the temperature, in degrees Fahrenheit, at 2 pm?

Solution: At 2 pm, t = 6. T(t) = 92 deg F.

(b) What is the temperature, in degrees Fahrenheit, at 8 am the next day?

Solution: At 8 am, the next day, t = 24, and T(t) = 38 deg F.

(c) When is the temperature the greatest? (Use am. or pm.)

Solution: This parabola is concave down. The axis of symmetry is x = - b /(2a) = 12.

Hence the temperature is maximum at 8 pm.

(d) What is the *warmest* temperature of the day? (Use appropriate units.)

Solution: The warmest temperature is given by the y-value of the vertex.

This is T(12) = 110 deg F.

9. Tristan was a giraffe. He was six feet tall when he was born, and from that moment, he grew at a constant rate of three inches per month until he was twenty feet tall, at which point he stopped growing. He remained twenty feet tall for the rest of his life. Recall that there are 12 inches in a foot and 12 months in a year. (Use appropriate units in each of the following.)

(a) Let *m* be Tristan's age, in *months*, and let *h* be Tristan's height, in feet. Find a formula for *h* in terms of *m* that is valid during the time he was growing, that is, from the time Tristan was born until the time he reached his full-grown height of 20 feet.

Solution: Since h has a constant average rate of change, it is a linear function of m. The constant average rate of change is0.25 feet per month, and the initial value is 6 feet. This means a formula for *h* is given by h(m) = 6 + 0.25m feet. (This is valid only for the time that Tristan was growing.)

(b) How old was Tristan when he stopped growing, that is, when he reached his full-grown height? *Include units.*

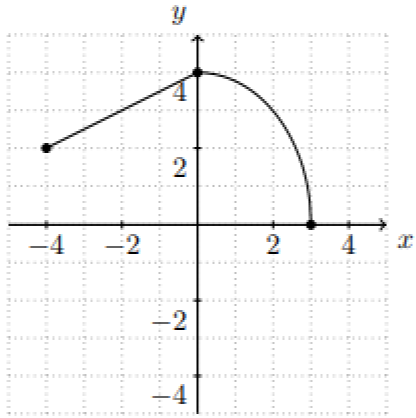
Solution: Since h = 6 + 0.25m, we solve the equation 20 = 6 + 0.25m to obtain m = 56 months.
Thus Tristan stopped growing when he was **56 months old**.

(c) Let j(m) be Tristan's height in feet when he was *m* months old. So h = j(m).
Note that j(m) is defined only while Tristan is alive. Tristan died at the age of 400 months. What are the domain and range of j(m) in the context of this problem?
Use either interval notation or inequalities to give your answers.

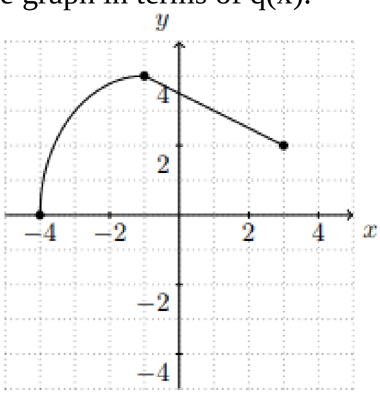
Solution: Tristan was alive from the age of 0 to the age of 400 months, so the domain is the interval **[0, 400].**

The range consists of all the values of Tristan's height during his life. He was born 6 feet tall and grew to a height of 20 feet. The range is thus the interval **[6, 20]**.

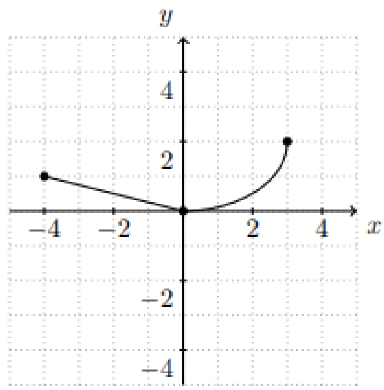
10. Consider the following graph of a function y = q(x) defined on [−4, 3]



For each of the following graphs, if the graph is not a combination of shifts, stretches, compressions and reflections of the graph of y = q(x), write *NOT a transformation*. Otherwise, write a formula for the function corresponding to the graph in terms of q(x).

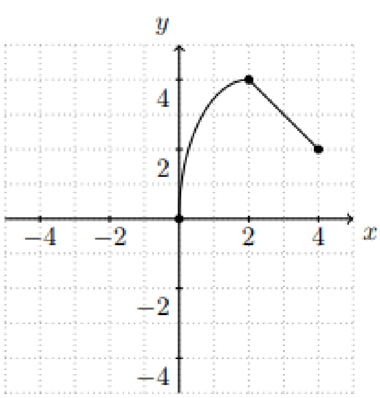


This is the graph of

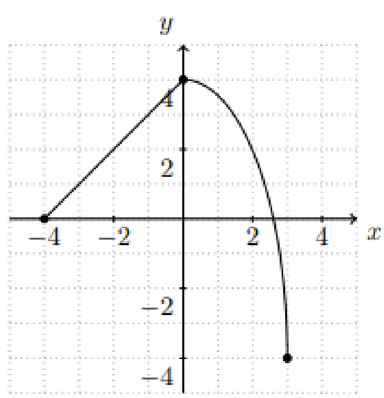


This is the graph of

y = q(-(x+1))
y = - ½ q(x) + 2



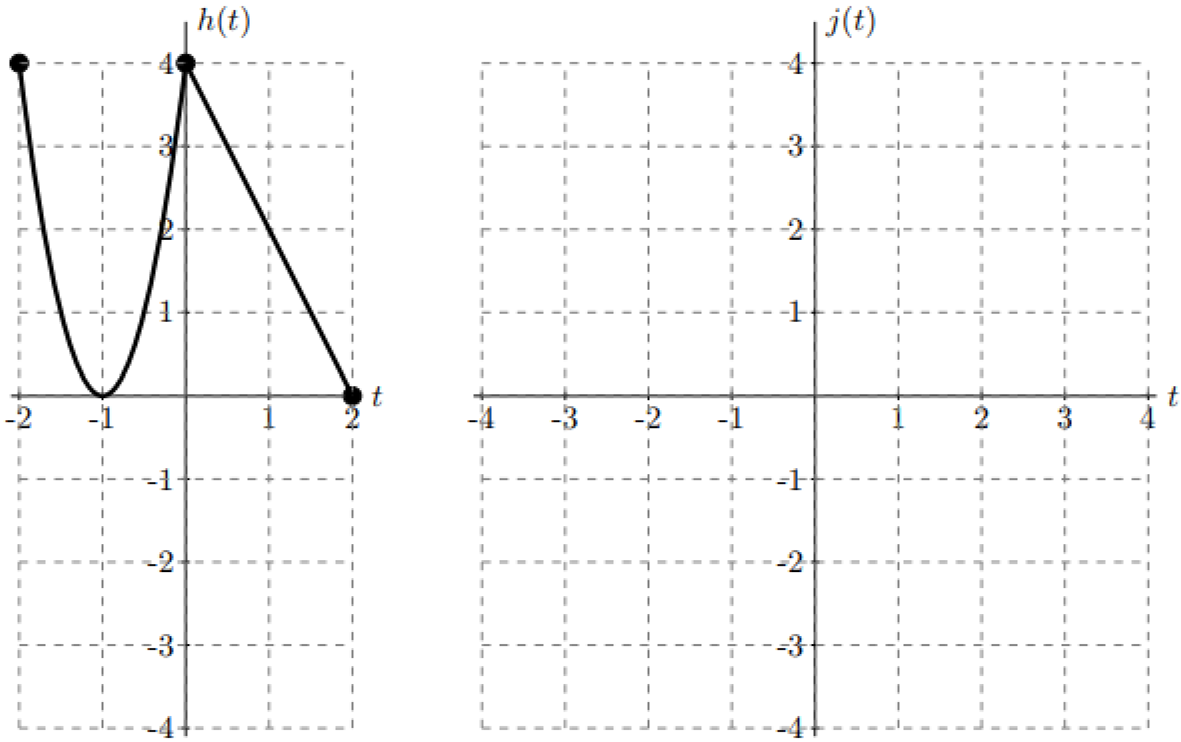
This is the graph of



This is the graph of

Not a Transformation
y = 2q(x) – 4

11. A graph of the function h(t) is given below. On the empty set of axes, carefully sketch a well-labeled graph of $j(t) = -\frac{1}{2}h(t-2) - 1$.



Solution: First, shift the graph 2 units to the right. Then compress vertically by a factor of ½. Next, reflect about the x-axis and finally lower the graph vertically by 1 unit.

12. In the following, compute the *average rate of change* of the given function over the interval [1+ h, 3]. Simplify your answers.

(a) F(x) = 2x – 3

Solution: $\frac{\Delta y}{\Delta x} = \frac{F(3)-F(1+h)}{3-(1+h)} = \frac{3-(2(1+h)-3)}{2-h} = \frac{3-(2+2h-3)}{2-h} = \frac{3-(2h-1)}{2-h} = \frac{3-2h+1}{2-h} = \frac{4-2\text{h}}{2-\text{h}}$

(b) $G(x) = x^2 + 3x$

Solution: $\frac{\Delta y}{\Delta x} = \frac{F(3)-F(1+h)}{3-(1+h)} = \frac{18-((1+h)^2+3(1+h))}{2-h} = \frac{18-(1+2h+h^2+3+3h)}{2-h} = \frac{18-(4+5h+h^2)}{2-h} =$

$$\frac{18-4-5h-h^2}{2-h} = \frac{14-5h-h^2}{2-h} = \frac{h^2+5h-14}{2-h}$$

Bonus riddles

1. David's father has three sons: Snap, Crackle and _________?

Answer: David

2. What comes once in a minute, twice in a moment, but never in a thousand years?

Answer: The letter “m”.

3. When does Christmas come before Thanksgiving?

Answer: In a dictionary.