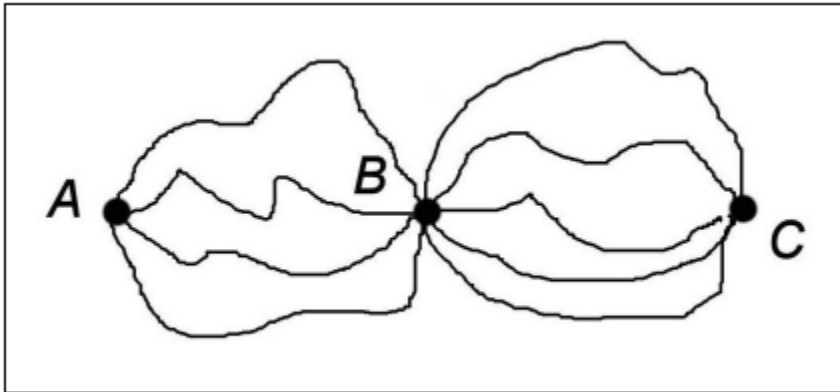


CLASS DISCUSSION: 4 FEBRUARY 2019

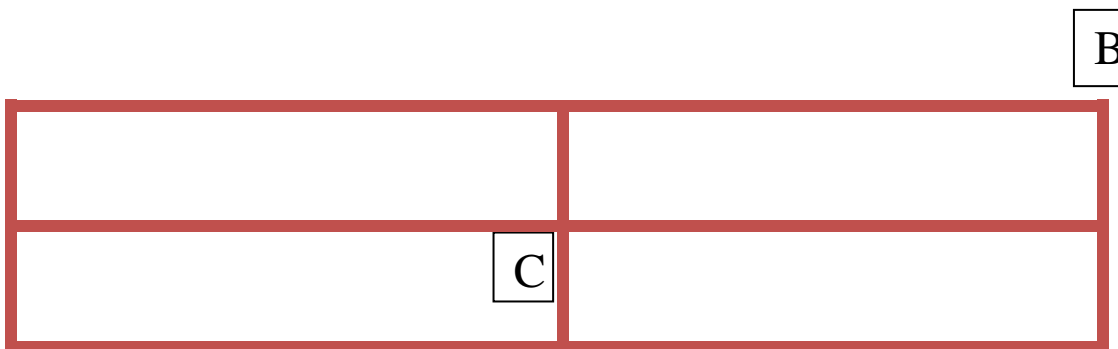
COUNTING: AN INTRODUCTION

1. What is the multiplication principle? How many 3 character license plates can be manufactured if the first character is any upper-case letter, the second character is a vowel, and the third character is one-digit integer?



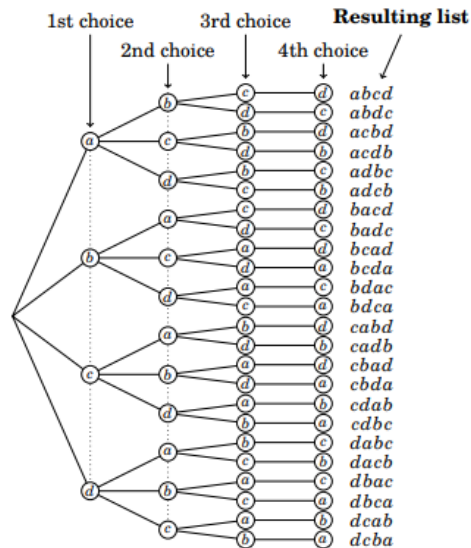
How Many Paths from A to C?

2. In the figure below, how many shortest paths exist from vertex A to vertex B? What if the path must pass through C?



A

3. State the addition and subtraction principles.
4. (a) How many words of length 4 can you create using the letters a, b, c, and d?
(b) What is no letter in the word can be repeated?



Constructing lists from letters in $\{a, b, c, d\}$, without repetition.

Exercises for Section 3.3

- Five cards are dealt off of a standard 52-card deck and lined up in a row. How many such lineups are there that have at least one red card? How many such lineups are there in which the cards are either all black or all hearts?
- Five cards are dealt off of a standard 52-card deck and lined up in a row. How many such lineups are there in which all 5 cards are of the same suit?
- Five cards are dealt off of a standard 52-card deck and lined up in a row. How many such lineups are there in which all 5 cards are of the same color (i.e., all black or all red)?
- Five cards are dealt off of a standard 52-card deck and lined up in a row. How many such lineups are there in which exactly one of the 5 cards is a queen?
- How many integers between 1 and 9999 have no repeated digits? How many have at least one repeated digit?
- Consider lists made from the symbols A, B, C, D, E , with repetition allowed.
 - How many such length-5 lists have at least one letter repeated?
 - How many such length-6 lists have at least one letter repeated?
- A password on a certain site must be five characters long, made from letters of the alphabet, and have at least one upper case letter. How many different passwords are there? What if there must be a mix of upper and lower case?
- This problem concerns lists made from the letters $A, B, C, D, E, F, G, H, I, J$.
 - How many length-5 lists can be made from these letters if repetition is not allowed and the list must begin with a vowel?
 - How many length-5 lists can be made from these letters if repetition is not allowed and the list must begin and end with a vowel?
 - How many length-5 lists can be made from these letters if repetition is not allowed and the list must contain exactly one A ?
- Consider lists of length 6 made from the letters A, B, C, D, E, F, G, H . How many such lists are possible if repetition is not allowed and the list contains two consecutive vowels?
- Consider the lists of length six made with the symbols P, R, O, F, S , where repetition is allowed. (For example, the following is such a list: (P, R, O, O, F, S) .) How many such lists can be made if the list must end in an S and the symbol O is used more than once?
- How many integers between 1 and 1000 are divisible by 5? How many are not divisible by 5?

5. Define: $P(n, r)$ and $\binom{n}{r}$.
6. Derive a formula for $P(n, r)$.
7. Using the result from (2), derive a formula for $\binom{n}{r}$.

How many really basic mathematical objects are there? One is surely the 'miraculous' jar of the positive integers 1, 2, 3 . . . Another is the concept of a fair coin. Though gambling was rife in the ancient world and although prominent Greeks and Romans sacrificed to Tyche, the goddess of luck, her coin did not arrive on the mathematical scene until the Renaissance. Perhaps one of the things that had delayed this was a metaphysical position which held that God speaks to humans through the action of chance. . . . The modern theory begins with the expulsion of Tyche from the Pantheon. There emerges the vision of the fair coin, the biased coin. This coin exists in some mental universe and all modern writers on probability theory have access to it. They toss it regularly and they speculate about what they 'observe.'

- Philip Davis & Reuben Hersh, **The Mathematical Experience**

