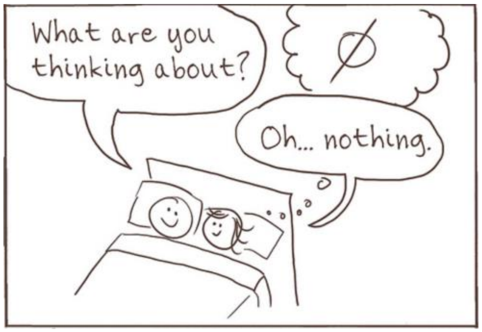
MATH 201 CLASS DISCUSSION 18 JANUARY 2019

SET NOTATION; CARDINALITY; POWER SET *(revised)*



1. Suppose $A = \{1,2,3,4\}$ and $B = \{a,c\}$.

(a) $A \times B$ (b) $B \times A$ (c) $A \times A$ (d) $B \times B$ (e) $\varnothing \times B$ (f) $(A \times B) \times B$ (g) $A \times (B \times B)$ (h) B^3

2. Suppose $A = \{\pi, e, 0\}$ and $B = \{0,1\}$.

(a) $A \times B$ (b) $B \times A$ (c) $A \times A$ (d) $B \times B$ (e) $A \times \varnothing$ (f) $(A \times B) \times B$ (g) $A \times (B \times B)$ (h) $A \times B \times B$

3. $\{x \in \mathbb{R} : x^2 = 2\} \times \{a,c,e\}$

4. $\{n \in \mathbb{Z} : 2 < n < 5\} \times \{n \in \mathbb{Z} : |n| = 5\}$

5. $\{x \in \mathbb{R} : x^2 = 2\} \times \{x \in \mathbb{R} : |x| = 2\}$

6. $\{x \in \mathbb{R} : x^2 = x\} \times \{x \in \mathbb{N} : x^2 = x\}$

7. $\{\varnothing\} \times \{0, \varnothing\} \times \{0,1\}$

8. $\{0,1\}^4$

Sketch these Cartesian products on the x-y plane $\mathbb{R}^2$ (or $\mathbb{R}^3$ for the last two).

9. $\{1,2,3\} \times \{-1,0,1\}$

10. $\{-1,0,1\} \times \{1,2,3\}$

11. $[0,1] \times [0,1]$

12. $[-1,1] \times [1,2]$

13. $\{1,1.5,2\} \times [1,2]$

14. $[1,2] \times \{1,1.5,2\}$

15. $\{1\} \times [0,1]$

16. $[0,1] \times \{1\}$

17. $\mathbb{N} \times \mathbb{Z}$

18. $\mathbb{Z} \times \mathbb{Z}$

19. $[0,1] \times [0,1] \times [0,1]$

20. $\{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \le 1\} \times [0,1]$

(A) Find the power set of each of the following sets:

1. $\{1,2,3,4\}$

2. $\{1,2,\varnothing\}$

3. $\{\{\mathbb{R}\}\}$

4. $\varnothing$

5. $\{\varnothing\}$

6. $\{\mathbb{R},\mathbb{Q},\mathbb{N}\}$

7. $\{\mathbb{R},\{\mathbb{Q},\mathbb{N}\}\}$

8. $\{\{0,1\},\{0,1,\{2\}\},\{0\}\}$

(B) Write out the following sets by listing their elements between braces.

9. $\{X : X \subseteq \{3,2,a\} \text{ and } |X| = 2\}$

10. $\{X \subseteq \mathbb{N} : |X| \le 1\}$

11. $\{X : X \subseteq \{3,2,a\} \text{ and } |X| = 4\}$

12. $\{X : X \subseteq \{3,2,a\} \text{ and } |X| = 1\}$

Decide if the following statements are true or false. Explain.

13. $\mathbb{R}^3 \subseteq \mathbb{R}^3$

14. $\mathbb{R}^2 \subseteq \mathbb{R}^3$

15. $\{(x,y) : x - 1 = 0\} \subseteq \{(x,y) : x^2 - x = 0\}$

16. $\{(x,y) : x^2 - x = 0\} \subseteq \{(x,y) : x - 1 = 0\}$

(C) Find the indicated sets.

1. $\mathscr{P}(\{\{a,b\},\{c\}\})$
2. $\mathscr{P}(\{1,2,3,4\})$
3. $\mathscr{P}(\{\{\emptyset\},5\})$
4. $\mathscr{P}(\{\mathbb{R},\mathbb{Q}\})$
5. $\mathscr{P}(\mathscr{P}(\{2\}))$
6. $\mathscr{P}(\{1,2\})\times\mathscr{P}(\{3\})$
7. $\mathscr{P}(\{a,b\})\times\mathscr{P}(\{0,1\})$
8. $\mathscr{P}(\{1,2\}\times\{3\})$
9. $\mathscr{P}(\{a,b\}\times\{0\})$
10. $\{X\in\mathscr{P}(\{1,2,3\}):|X|\leq 1\}$
11. $\{X\subseteq\mathscr{P}(\{1,2,3\}):|X|\leq 1\}$
12. $\{X\in\mathscr{P}(\{1,2,3\}):2\in X\}$

Suppose that $|A|=m$ and $|B|=n$. Find the following cardinalities.

13. $|\mathscr{P}(\mathscr{P}(\mathscr{P}(A)))|$
14. $|\mathscr{P}(\mathscr{P}(A))|$
15. $|\mathscr{P}(A\times B)|$
16. $|\mathscr{P}(A)\times\mathscr{P}(B)|$
17. $|\{X\in\mathscr{P}(A):|X|\leq 1\}|$
18. $|\mathscr{P}(A\times\mathscr{P}(B))|$
19. $|\mathscr{P}(\mathscr{P}(\mathscr{P}(A\times\emptyset)))|$
20. $|\{X\subseteq\mathscr{P}(A):|X|\leq 1\}|$

NAÏVE SET THEORY CONTINUED
INTRO TO PROOFS

1. Let **A**, **B** and **C** be three sets such that:

Set **A** = {2, 4, 6, 8, 10, 12}, set **B** = {3, 6, 9, 12, 15} and set **C** = {1, 4, 7, 10, 13, 16}.

Find:

(i) A ∪ B

(ii) A ∩ B

(iii) B ∩ A

(iv) B ∪ A

(v) B ∪ C

(vi) A – B

(vii) A – (B ∪ C)

(viii) A – (B ∩ C)

(ix) Is A ∪ B = B ∪ A?

(x) Is B ∩ C = B ∪ C?

2. Complete each of the following:

(i) Associativity of set union and intersection:

$A\cup(B\cup C)=$ $\qquad$ $A\cap(B\cap C)=$

(ii) Commutativity: $A\cup B=$ $\qquad$ $A\cap B=$

(iii) Distributivity: $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$, $\qquad$ $A\cap(B\cup C)=$

(iv) De Morgan Laws: $\overline{(A\cup B)}=$ $\qquad$ $\overline{(A\cap B)}=$

(v) Complementation: $A\cup\bar{A}=$ $\qquad$ $A\cap\bar{A}=$

(vi) Double complement: $\overline{(\bar{A})}=$

3. True or False? Give proof or counterexample.

(a) $A \cup B \subseteq A \cap B$

(b) $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

(c) $A \cup (B \cap C) \supseteq (A \cup B) \cap (A \cup C)$

(d) $A - (B \cap C) = (A - B) \cup (A - C)$

(e) $A - B = \bar{B} - \bar{A}$

(f) $(A \cup B) \cap C \supseteq (A \cup B) \cap (A \cup C)$

4. [Halmos, **Naïve Set Theory**] A necessary and sufficient condition the $(A \cap B) \cup C = A \cap (B \cup C)$ *is that* $C \subseteq A$. Observe that the condition has nothing to do with the set B. Explain.

5. [Halmos, **Naïve Set Theory**]

(a) Prove that $\mathcal{P}(E) \cap \mathcal{P}(F) = \mathcal{P}(E \cap F)$

(b) Prove that $\mathcal{P}(E) \cup \mathcal{P}(F) \subseteq \mathcal{P}(E \cup F)$