CLASS DISCUSSION: 28 JANUARY 2019

EXISTENTIAL AND UNIVERSAL QUANTIFIERS

REVIEW:

R1. Which of the following are well-formed propositional formulas (abbreviated wff)?

1. $\vee pq$
2. $(\neg(p \to (q \wedge p)))$
3. $(\neg(p \to (q = p)))$
4. $(\neg(\Diamond(q \vee p)))$
5. $(p \wedge \neg q) \vee (q \to r)$
6. $p\neg r$

R2 Let's consider a propositional language where *p* means "Paola is happy", *q* means "Paola paints a picture", *r* means "Renzo is happy". Formalize the following sentences:

1. "if Paola is happy and paints a picture then Renzo isn't happy"
2. "if Paola is happy, then she paints a picture"
3. "Paola is happy only if she paints a picture"

R3 Let's consider a propositional language where A ="Angelo comes to the party", B ="Bruno comes to the party", C ="Carlo comes to the party", D ="Davide comes to the party". Formalize the following sentences:

1. "Angelo comes to the party while Bruno doesn't"
2. "Either Carlo comes to the party, or Bruno and Davide don't come" 14 2.3 Propositional Formalization
3. "If Angelo and Bruno come to the party, then Carlo comes provided that Davide doesn't come"
4. "Carlo comes to the party if Bruno and Angelo don't come, or if Davide comes"
5. "If Angelo comes to the party then Bruno or Carlo come too, but if Angelo doesn't come to the party, then Carlo and Davide come"

QUANTIFIERS:

I Employing existential and/or universal quantifiers ($\exists$ or $\forall$), convert each statement into one that uses quantifiers. Assume that *X, A, B, C* are sets.

(a) For all *x* in *A* and all *y* in *B* there exists a *z* in *C* satisfying the condition $x < z < y$.

(b) For each *a* in *A* there is a *b* in *B* such that, for every *c* in C, $c > a+b$.

(c) There exists an x in X such that for all y in A there exists a z in B such that $x < z < y$.

(d) For every p in A there exists a q in B such that for all r in X either $r < 3p$ or $r > 5q$.

II Translate each of the following into an English sentence.

(a) $\forall x \in A \ \forall y \in B \ \exists z \in X, \ z \geq xy$

(b) $\exists p \in X \ \exists q \in B \ \forall r \in C, \ r + p < q$

(c) $\exists c \in C \ \forall x \in X \ \forall y \in B, \ c > x - y$

(d) $\forall x \in R \ \forall \varepsilon \in B \ \exists r \in Q, |x - r| < \varepsilon$

III Let variables x, y, z denote people at Loyola University Chicago. Let L be the predicate L(x, y) = "x loves y."

Translate each of the following statements into a logical statement.

(a) Swann loves himself.

(b) Everybody loves Albertine.

(c) There is someone whom Albertine doesn't love.

(d) Albertine loves no one.

(e) Everybody loves someone.

(f) There is someone whom everybody loves.

(g) There is someone whom no one loves.

(h) There is someone who loves everybody.

(i) There is no one who loves everyone.

(j) There is no one who loves no one.

IV Translate from English into 1^{st} order predicate calculus. Define an appropriate universe of discourse.

(a) There is a math major who is not a biology major.

(b) There is a student who likes to swim but dislikes skiing.

(c) There is a student who loves Game of Thrones but dislikes The Night Of.

(d) There is a student who likes either to kayak or compete in half marathons but not both.

(e) There is a student who likes Joyce Carol Oates and Clive Barker but not Sir Arthur Conan Doyle.

(f) Every Actuarial Science minor plans to become an Actuary.

(g) Not every Actuarial Science minor plans to become an Actuary.

V Translate the following from natural language into an appropriate logical statement.

Let the universe of discourse, S, be the set of all positive integers.

(a) If *n* is even then n^3 is even.

(b) If *n* is an odd number then *n* + 2 is also odd.

(c) If *n* is a perfect square, then 5n is a perfect square.

(d) If *n* is a multiple of 8 then *n* is a multiple of 16.

(e) If *n* is a multiple of 9 then *n* is a multiple of 3.

VI Explain why the following are equivalent:

$$\sim \forall x \;\; P(x) \quad \text{and} \quad \sim \exists x \sim P(x)$$

Analogously, $\sim \exists x \; P(x)$ and $\forall x \sim P(x)$ are equivalent.

Give examples from natural language.

VII Distribute the negation through each of the following statements.

(a) $\sim \exists x \in A, \;\; x > 0$

(b) $\sim \forall x \in A, \;\; x > 0$

(c) $\sim \forall x \in A \; \forall y \in B \;\; \exists z \in X, \; z > xy$

(d) $\sim \exists p \in X \;\; \exists q \in B \;\; \forall r \in C, \;\; r + p < q$

(e) $\sim \exists c \in C \;\;\; \forall x \in X \;\;\; \forall y \in B, \;\; \ln(1 + c^2) > x^2 + y^4 - 5$

(f) $\sim \exists a, b, c \in Z^+ \;\;\; a^4 + b^4 = c^4$