CLASS DISCUSSION: BI-CONDITIONAL (IFF) STATEMENTS

11 MARCH 2019

❖ Bi-conditional statements

Exercises for Chapter 7

Prove the following statements. These exercises are cumulative, covering all techniques addressed in Chapters 4–7.

1. Suppose $x \in \mathbb{Z}$. Then x is even if and only if $3x+5$ is odd.
2. Suppose $x \in \mathbb{Z}$. Then x is odd if and only if $3x+6$ is odd.
3. Given an integer a, then a^3+a^2+a is even if and only if a is even.
4. Given an integer a, then a^2+4a+5 is odd if and only if a is even.
5. An integer a is odd if and only if a^3 is odd.
6. Suppose $x, y \in \mathbb{R}$. Then $x^3+x^2y = y^2+xy$ if and only if $y = x^2$ or $y = -x$.
7. Suppose $x, y \in \mathbb{R}$. Then $(x+y)^2 = x^2+y^2$ if and only if $x = 0$ or $y = 0$.
8. Suppose $a, b \in \mathbb{Z}$. Prove that $a \equiv b \pmod{10}$ if and only if $a \equiv b \pmod 2$ and $a \equiv b \pmod 5$.
9. Suppose $a \in \mathbb{Z}$. Prove that $14 \mid a$ if and only if $7 \mid a$ and $2 \mid a$.
10. If $a \in \mathbb{Z}$, then $a^3 \equiv a \pmod 3$.
11. Suppose $a, b \in \mathbb{Z}$. Prove that $(a-3)b^2$ is even if and only if a is odd or b is even.

❖ Review: conditional statements

Exercises for Chapter 6

A. Use the method of proof by contradiction to prove the following statements. (In each case, you should also think about how a direct or contrapositive proof would work. You will find in most cases that proof by contradiction is easier.)

1. Suppose $n \in \mathbb{Z}$. If n is odd, then n^2 is odd.
2. Suppose $n \in \mathbb{Z}$. If n^2 is odd, then n is odd.
3. Prove that $\sqrt[3]{2}$ is irrational.
4. Prove that $\sqrt{6}$ is irrational.
5. Prove that $\sqrt{3}$ is irrational.
6. If $a, b \in \mathbb{Z}$, then $a^2-4b-2 \neq 0$.
7. If $a, b \in \mathbb{Z}$, then $a^2-4b-3 \neq 0$.

8. Suppose $a, b, c \in \mathbb{Z}$. If $a^2 + b^2 = c^2$, then a or b is even.

9. Suppose $a, b \in \mathbb{R}$. If a is rational and ab is irrational, then b is irrational.

10. There exist no integers a and b for which $21a + 30b = 1$.

11. There exist no integers a and b for which $18a + 6b = 1$.

12. For every positive $x \in \mathbb{Q}$, there is a positive $y \in \mathbb{Q}$ for which $y < x$.

13. For every $x \in [\pi/2, \pi]$, $\sin x - \cos x \geq 1$.

14. If A and B are sets, then $A \cap (B - A) = \emptyset$.

15. If $b \in \mathbb{Z}$ and $b \nmid k$ for every $k \in \mathbb{N}$, then $b = 0$.

16. If a and b are positive real numbers, then $a + b \geq 2\sqrt{ab}$.

17. For every $n \in \mathbb{Z}$, $4 \nmid (n^2 + 2)$.

18. Suppose $a, b \in \mathbb{Z}$. If $4 \mid (a^2 + b^2)$, then a and b are not both odd.