MATH 201: CLASS DISCUSSION 1 FEBRUARY 2019

REVIEW: PREP FOR TEST I (REVISED)

1. Let A, B, C be sets. Prove that $(A \cup B) - C \subseteq (A - C) \cup (B - C)$

2. Let A, B, C be sets. Assume that $B \subseteq C$. Prove that $A \times B \subseteq A \times C$

3. Let P(x, y) be a predicate. Are the two statements below logically equivalent? Justify your answer.

$$\forall x\, \exists y\, P(x, y)$$
$$\exists y\, \forall x\, P(x, y)$$

4. Consider the statement, "If it is Groundhog Day or there is snow on the ground then there is snow on the ground."

a. Translate the above statement into a proposition. Clearly state which statement is P and which is Q.
b. Make a truth table for the statement.
c. Assuming the statement is true, what (if anything) can you conclude if there is snow on the ground?
d. Assuming the statement is true, what (if anything) can you conclude if there is no snow on the ground?
e. Suppose you found out that the statement is a lie. What can you conclude?

5. Create a truth table for the statement

$$\sim P \wedge (Q \rightarrow P)$$

What can you conclude about P and Q if you know the statement is true?

6. Are the statements

$$P \rightarrow (Q \vee R)$$

and

$$(P \rightarrow Q) \vee (P \rightarrow R)$$

logically equivalent?

7. Simplify each of the following statements (so that negation only appears just before variables).

(a) $\sim (P \rightarrow \sim Q)$
(b) $\sim (P \vee \sim Q) \rightarrow (\sim Q \wedge R)$
(c) $\sim ((P \rightarrow \sim Q) \vee \sim (R \wedge \sim R))$
(d) It is false that if Albertine does not like horror films then Boris likes documentary films, and that Boris does not like horror films.

8. We can also simplify statements in predicate logic using our rules for passing negations over quantifiers, and then applying propositional logical equivalence to the "inside" propositional part. Simplify the statements below (so negation appears only directly next to predicates). Assume that P(x) and Q(x) are predicates.

(a) $\sim \exists x\, \forall y\, (\sim P(x) \vee Q(y))$
(b) $\sim \forall x \sim \forall y \sim (x < y \wedge \exists z (x < z \vee y < z))$
(c) There is a number ***n*** for which no other number is either less than or equal to ***n***.
(d) It is false that for every number ***n*** there are two other numbers, one larger than ***n***, the other smaller than ***n***.

Problems from recent Test 1:

1. (a) Is the following argument correct? Explain.

Lucky will buy a house only if Pozzo buys a car. Pozzo will buy a car only if Estragon buys a motorcycle. Estragon will not buy a motorcycle. So, Lucky will not buy a house.

(b) Write the *negation* of the following sentence. Simplify as much as possible.

$$\exists p \in X\ \exists q \in B\ \forall r \in C\quad p \le r \le q$$

2. For each of the five logical formulas, indicate whether or not it is true when the domain of discourse is the set of non-negative integers, the integers, the rationals, or the reals.
For example, in the set of all non-negative integers, there does not exist a number whose square is two, but in the set of all real numbers there does exist a number whose square is 3.

Each section should have five T or F responses. No penalty for guessing.

❖ Domain of discourse (our "universe") is the set of *all non-negative integers*.

(a) $\exists x\ (x^2 = 2)$
(b) $\forall x\ \ \exists y\ (x^2 = y)$
(c) $\forall y\ \ \exists x\ (x^2 = y)$
(d) $\forall x \neq 0\ \ \exists y\ (xy = 1)$
(e) $\exists x\, \exists y\ (x + 2y = 2) \wedge (x + 4y = 5)$

❖ Domain of discourse is the set of *all integers*.

(f) $\exists x\ (x^2 = 2)$
(g) $\forall x\ \ \exists y\ (x^2 = y)$
(h) $\forall y\ \ \exists x\ (x^2 = y)$
(i) $\forall x \neq 0\ \ \exists y\ (xy = 1)$
(j) $\exists x\, \exists y\ (x + 2y = 2) \wedge (x + 4y = 5)$

❖ Domain of discourse is the set of *all rationals*.

(k) $\exists x\ (x^2 = 2)$
(l) $\forall x\ \ \exists y\ (x^2 = y)$
(m) $\forall y\ \ \exists x\ (x^2 = y)$
(n) $\forall x \neq 0\ \ \exists y\ (xy = 1)$
(o) $\exists x\, \exists y\ (x + 2y = 2) \wedge (x + 4y = 5)$

❖ Domain of discourse is the set of *all reals.*

(p) $\exists x\ (x^2 = 2)$
(q) $\forall x\ \ \exists y\ (x^2 = y)$
(r) $\forall y\ \ \exists x\ (x^2 = y)$
(s) $\forall x \neq 0\ \ \exists y\ (xy = 1)$
(t) $\exists x\, \exists y\ (x + 2y = 2) \wedge (x + 4y = 5)$

3. Find the flaw in the following bogus proof. Explain!

- Step 1: Let $a = b$.
- Step 2: Then $a^2 = ab$

- Step 3: Then $a^2 + a^2 = a^2 + ab$
- Step 4: Then $2a^2 = a^2 + ab$
- Step 5: Then $2a^2 – 2ab = a^2 + ab – 2ab$
- Step 6: Then $2a^2 – 2ab = a^2 – ab$
- Step 7: This can be written as $2(a^2 – ab) = 1(a^2 – ab)$
- Step 8: Dividing each side by $(a^2 – ab)$ yields $1 = 2$

4. Introducing appropriate predicates, express each of the following statements in first-order predicate logic. Assume that the universe of discourse is the set of all people who live in Illinois.

(a) Someone walks and talks.

(b) Someone walks and someone talks.

(c) Everyone who walks is unable to talk.

(d) No one who talks can walk.

(e) No one can both walk and talk.

5. Use a truth table to determine if the following statement is always true:

$$\sim p \Rightarrow \big((p \wedge \sim q) \Rightarrow (\sim p \vee q)\big)$$