MATH 201 SOLUTIONS: QUIZ III 22 FEBRUARY 2019

1. Fix the following proof so that it will earn full-credit from your grader.

Proposition: For all n ≥ 1, $\frac{(3n)!}{(3!)^n}$ is an integer.

"Bad Proof": We will use the method of mathematical induction.

Let us imagine that there exists an integer *n* for which $\frac{(3n)!}{(3!)^n}$ is an integer.

Now $\frac{(3n+1)!}{(3!)^{n+1}} = \frac{(3n)!(1!)}{((3!)^n)^1} = \frac{(3n+1)!}{(3!)^{n+1}} = \frac{(3n)!}{(3!)^n}$

which must be an integer from our assumption.

Thus we have proven the case n + 1, and the induction is complete.

Solution: *This "proof" contains many errors, but the result is correct. (We now write a corrected proof.)*

Proposition: For all n ≥ 1, $\frac{(3n)!}{(3!)^n}$ is an integer.

Proof: For each n ≥ 1, let P(n) represent the statement $\frac{(3n)!}{(3!)^n}$ is an integer.

We will use the method of mathematical induction.

Base case: *For n = 1:* $\frac{(3(1))!}{(3!)^1} = 1 \in \mathbb{Z}$.

Inductive step: Assume that n ≥ 1 is fixed and that P(n) is true.

Now $\frac{(3(n+1))!}{(3!)^{n+1}} = \frac{(3n+3)!}{(3!)^{n+1}} = \frac{(3n)!}{(3!)^n}\left(\frac{(3n+1)(3n+2)(3n+3)}{3!}\right) =$

$$\frac{(3n)!}{(3!)^n}\left(\frac{3n+3}{3}\right)\frac{(3n+1)(3n+2)}{2} = \frac{(3n)!}{(3!)^n}(n+1)\frac{(3n+1)(3n+2)}{2}$$

Now, by the inductive hypothesis, P(n), $\frac{(3n)!}{(3!)^n}$ is an integer. Furthermore, the product of two consecutive integers is even. Hence 2 is a divisor of $(3n+1)(3n+2)$.

Thus we have proven P(n+1), and the induction proof is complete.

2. Proposition: For all $n \in N$, $4|(3^{2n} + 7)$. (That is, $3^{2n} + 7$ is a multiple of 4.)

Proof: For $n \in N$ let S_n be the statement $4|(3^{2n} + 7)$.

Base Case: $n = 1$ is true, since $3^2 + 7 = 16$ is divisible by 4.

Inductive Step:

For a fixed $k \geq 1$, assume that the proposition is true for $n = k$, so that $4|(3^{2k} + 7)$.

Then $3^{2k} + 7 = 4L$ for some $L \in Z$.

Now, $3^{2(k+1)} + 7 = 9(3^{2k}) + 7 = 8(3^{2k}) + 3^{2k} + 7 = 8(3^{2k}) + 4L = 4(2(3^{2k}) + L)$.

So $4|(32^{(k+1)} + 7)$, and we see that the proposition is true for $n = k+1$.

- **This is the only "error": $32^{(k+1)}$ is actually $2(3^{2k})$.**

Therefore, by the principle of mathematical induction, the proposition is true for all $n \in N$.

→ ***Which of the following statements is correct?***

No: The proposition is false, but the proof is correct.

- The proof contains arithmetic mistakes that make it incorrect.

No: The proof incorrectly assumes what it is trying to prove.

- The proof is a correct proof of the stated result.

No: None of the above is true.

Both answers (2 and 4) are acceptable since the proof is "virtually" correct but for one minor mistake.