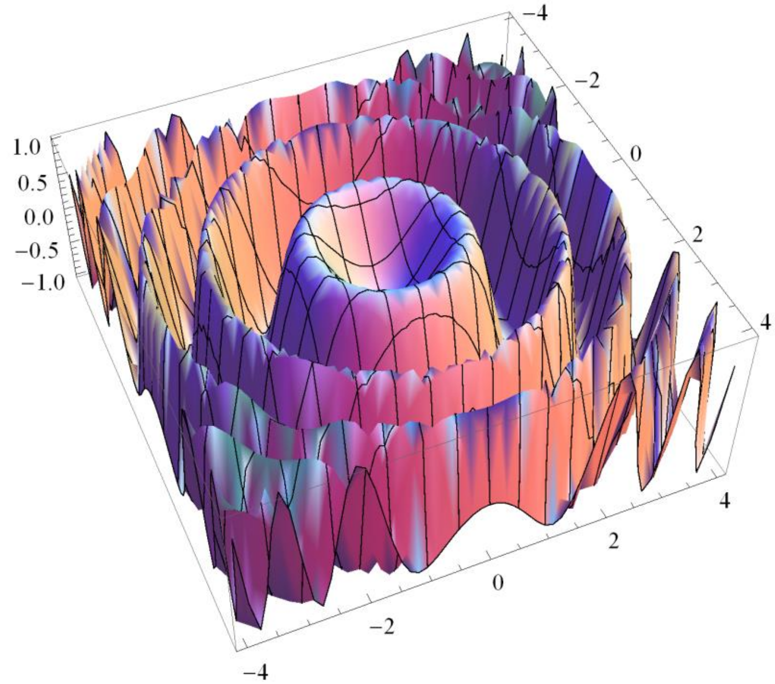
MATHEMATICA LAB I

Visualizing 3-dimensional curves and surfaces: Contour diagrams



(Lab report due: 15 February 2019)

Before beginning this lab, study the examples given in graphing. Submit a *printed version* of your Mathematica notebook. *Copy the statement of each problem into your notebook.* You may work with other students and compare results, but ultimately you must submit *your own* lab results --- not a shared copy.

I. Using **ParametricPlot3D** or **ParametricPlot**, graph each of the following parameterized curves:

$\gamma(t) = (\cos t)\,\mathbf{i} + (\sin t)\,\mathbf{j} + t\,\mathbf{k}$

$\sigma(t) = (\cos t)\,\mathbf{i} + (\sin t)\,\mathbf{j} + (1 - \cos t)\,\mathbf{k}$

$\alpha(t) = (\cos t + t \sin t)\,\mathbf{i} + (\sin t - t \cos t)\,\mathbf{j}$

$\beta(t) = (t - \sin t)\,\mathbf{i} + (1 - \cos t)\,\mathbf{j}$

$\Gamma(t) = (2+\cos(24t))\cos(t)\,\mathbf{i} + (2+\cos(24t))\sin(t)\,\mathbf{j} + \sin(24t)\,\mathbf{k}$

II. The "witch of Agnesi" is a curve studied by Maria Agnesi in 1748 in her book *Instituzioni analitiche ad uso della gioventù Italiana* (the first surviving mathematical work written by a woman). The curve is also known as cubique d'Agnesi or agnésienne and had been studied earlier by Fermat and Guido Grandi in 1703. The name "witch" derives from a mistranslation of the term *averisera* ("versed sine curve," from the Latin *vertere*, "to turn") in the original work as *avversiera* ("witch" or "wife of the devil") in an 1801 translation of the work by Cambridge Lucasian Professor of Mathematics John Colson.

Consider the following parameterization of the witch:

$$x(t) = 2 \tan t, \quad y(t) = 2 \cos^2(t) \quad \text{for } -\pi/2 < t < \pi/2$$

Find an equation (without using Mathematica) of the tangent line to the witch of Agnesi at $t = 1.2$. Then plot the witch and the tangent line on the same pair of axes. Use the domain $-1.4 < t < 1.4$ and the instruction *AspectRatio* → 1 which requires a unit on the y-axis to be of the same length as a unit on the x-axis.

III. For each of the following surfaces, use Plot3D to plot the surface and ContourPlot to plot a contour diagram in the xy-plane. (Notice that by moving the mouse pointer over a level curve, you can see the value of the function on that particular level curve. Alternatively, you may use the ContourLabels command.)

Note: In statistics and mathematics, exp(f) means e^f. In Mathematica, use the built-in function Exp[f].

$z = \sin(4x^2 + y^2)$

$z = x^2 - y^2$

$z = (x^2 + 3y^2) \exp(1 - x^2 - y^2)$

$z = \exp(-x^2 - y^2) \sin(x^2) \cos(y^2)$

$z = \exp(-x^2 - y^2)$

IV. For each of the following implicitly-defined functions $f(x, y, z) = 0$, use ContourPlot3D to plot a particular level set that corresponds to the given surface. Of course, you will need to select an xyz-region wisely.

1. $(x - 1)^2 + y^2 + (z - 3)^2 = 4.$

2. $9 \ln(x^2 + y^2 + z^2) = 1$

3. $x^2 + z^2 = 1$

4. $x + y^2 - 4z^2 = 1$

5. $\sin x - (\cos y)(x^2 + z^2)^{1/2} = 3$

V. For each of the following functions w = f(x, y, z), use ContourPlot3D to plot a contour diagram (that consists of several level sets). You may need to adjust **Opacity** to make the surfaces visible. $w = x^2 + y^2 + z^2$

1. $w = x^2 + y^2$
2. $w = xyz$
3. $w = z^2 - x^2 - y^2$

VI. Using **PolarPlot**, graph each of the following curves:

1. $$r = \frac{2 - \sin(7\theta) - \dfrac{\cos(30\theta)}{2}}{100 + \left(\theta - \dfrac{\pi}{2}\right)^8}$$

Let θ vary from $-\pi/2$ to 2π. What shape do you see?

Now use: PlotSyle → {Red, Thickness[0.005]}

2. $r = 1 + \cos(100\,\theta)$. Let θ vary from 0 to 2π. Use PlotSyle → {Green}

"It's very good jam," said the Queen.
"Well, I don't want any today, at any rate."
"You couldn't have it if you did want it," the Queen said. "The rule is jam tomorrow and jam yesterday but never jam today."
"It must come sometimes to 'jam to-day,'" Alice objected.
"No it can't," said the Queen. "It's jam every other day; today isn't any other day, you know."
"I don't understand you," said Alice. "It's dreadfully confusing."

- Lewis Carroll, **Through the Looking Glass**