**Mathematica  Lab II**

*Taylor Polynomials, Unconstrained Extrema, LaGrange multipliers*

***due: March 25th***

1. For each of the following functions and associated points, *P*, compute the equation of the Taylor quadratic approximation to the function at *P*.  Graph the original surface and its Taylor approximation on the same set of axes, and identify each surface.

(a)   z = sin(xy),  P = (1, /2)

(b)    z = exp(–x2 – y2),  P = (0.3, 0.4)

*2*. When it rains on the surface z = 1/x + 1/y + xy, at what point (if any) will a puddle form? Explain! Use contour diagrams in your explanation.

*3*. Find and classify the critical points of each of the following functions.  (If the second derivative test fails, then you are *not required* to investigate further.  But successful investigation will merit extra credit.) Draw contour diagrams.

(a)  z = x2 + xy + y2   
(b)  z = x3 + y3 + 3xy + 8   
(c)  z = x3 – 3x + y2 – 6y   
(d)  z = 120x3 – 30x4 + 18x 5 + 5x6 + 30xy2

*4*. Let z = F(x, y) be defined by:

F(x, y) = (2x3 – 3x2  + 1) exp(–y2) + (2x3 – 3x2) exp(–y)

Assume that the domain of *F* is the entire xy-plane.

(a)  Graph this surface over the region:   |x| ≤ 0.1, |y|  ≤ 0.1

(b)   Find and classify all critical points (on the xy-plane) of *F*. *(NSolve[ ] may need some help from you. For example, factor when possible.)*

(c)    Does *F* possess a global maximum or global minimum?   Explain.  How does this example stand in stark contrast to our optimization experience for functions of a single variable?

*5*. Consider the *monkey saddle* f(x, y) = x3 – 3xy2. Find the critical points of this function and then draw a contour diagram for *f* near the unique critical point.  Identify which level curves are positive and which are negative. (Try using the commands: *ContourLabels → All*, *ContourShading → None*. Also, experiment with graphing particular contour values using *Contours → {-2, -1, 0, 1, 2, 3.5}* where we have chosen some particular contour values. You should choose your own values to improve the clarity of your plot.)

***6.***   Find and classify all local extrema of the function

z = (3x4 – 4x3 – 12x2 + 18) / (1 + 4y2).

Also, display the graph of this surface as well as contour diagrams.

***7***. Using LaGrange multipliers find all local and global extrema of f(x, y) = x2 – 4xy + 8y2 + 5x + 9y – 79 subject to the constraint g(x,y) = 8 – x2 – y3 + x. Be certain to draw contour diagrams to see all points of tangency.

  

[Brook Taylor](http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Taylor.html) (1685 – 1731)