MATHEMATICA LAB II

Taylor Polynomials, Unconstrained Extrema, LaGrange multipliers

due: March 25th

1. For each of the following functions and associated points, *P*, compute the equation of the Taylor quadratic approximation to the function at *P*. Graph the original surface and its Taylor approximation on the same set of axes, and identify each surface.

(a) $z = \sin(xy)$, $P = (1, \pi/2)$

(b) $z = \exp(-x^2 - y^2)$, $P = (0.3, 0.4)$

2. When it rains on the surface $z = 1/x + 1/y + xy$, at what point (if any) will a puddle form? Explain! Use contour diagrams in your explanation.

3. Find and classify the critical points of each of the following functions. (If the second derivative test fails, then you are *not required* to investigate further. But successful investigation will merit extra credit.) Draw contour diagrams.

(a) $z = x^2 + xy + y^2$

(b) $z = x^3 + y^3 + 3xy + 8$

(c) $z = x^3 - 3x + y^2 - 6y$

(d) $z = 120x^3 - 30x^4 + 18x^5 + 5x^6 + 30xy^2$

4. Let $z = F(x, y)$ be defined by:

$$F(x, y) = (2x^3 - 3x^2 + 1)\exp(-y^2) + (2x^3 - 3x^2)\exp(-y)$$

Assume that the domain of *F* is the entire xy-plane.

(a) Graph this surface over the region: $|x| \le 0.1$, $|y| \le 0.1$

(b) Find and classify all critical points (on the xy-plane) of *F*. *(NSolve[ ] may need some help from you. For example, factor when possible.)*

(c) Does *F* possess a global maximum or global minimum? Explain. How does this example stand in stark contrast to our optimization experience for functions of a single variable?

5. Consider the *monkey saddle* $f(x, y) = x^3 - 3xy^2$. Find the critical points of this function and then draw a contour diagram for *f* near the unique critical point. Identify which level curves are positive and which are negative. (Try using the commands: *ContourLabels → All*, *ContourShading → None*. Also, experiment with graphing particular contour values using *Contours → {-2, -1, 0, 1, 2, 3.5}* where we have chosen some particular contour values. You should choose your own values to improve the clarity of your plot.)

6. Find and classify all local extrema of the function

$$z = (3x^4 - 4x^3 - 12x^2 + 18) / (1 + 4y^2).$$

Also, display the graph of this surface as well as contour diagrams.

7. Using LaGrange multipliers find all local and global extrema of $f(x, y) = x^2 - 4xy + 8y^2 + 5x + 9y - 79$ subject to the constraint $g(x,y) = 8 - x^2 - y^3 + x$. Be certain to draw contour diagrams to see all points of tangency.

Brook Taylor (1685 – 1731)