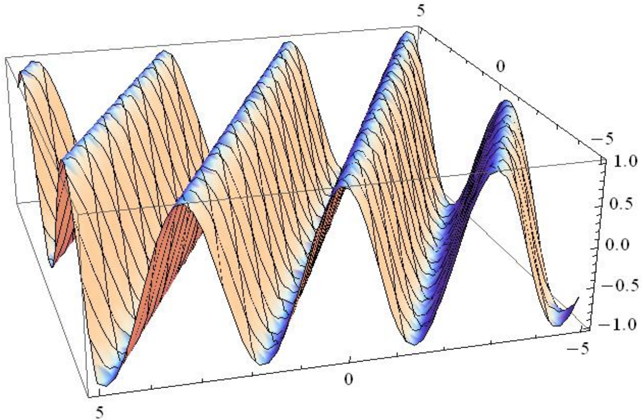
MATH 263 CLASS DISCUSSION 24 FEB 2019

SEARCHING FOR LOCAL EXTREMA ON OPEN SETS

PART I (Taylor polynomials)

(a) Find the second-order Taylor approximation for the function $f(x, y) = \sin(x + 2y)$ at the origin.



(b) Find the second-order Taylor approximation, $Q(x, y)$, of the function $F(x, y) = \sin(xy)$ at the point $P = (1, \pi/2)$.

(c) Find the quadratic Taylor approximation, $Q(x, y)$, of the function $g(x,y) = (x - 1)/(y - 1)$ at the point $P = (2,3)$.

(d) Derive the quadratic Taylor approximation of $h(x, y) = \exp(-x-y)$ about the origin in two different ways: (1) using the two-variable approximation, and (2) substituting $t = -x-y$ into the one-variable approximation for e^t.

(e) Find the second-order Taylor approximation for the function $f(x, y) = \sin(xy) + \cos(xy)$ at the origin.

PART II (local extrema)

1) Locate local/global extrema of $x^2 + 2xy + 2y^2$ by locating critical points and then factoring.

2) Locate local/global extrema of $F(x, y) = x^2 - 4xy - 4y + 2x + 5y^2 - 5$. (*Hint:* find critical point(s) and then factor *F*.)

3) Again, through factorization, determine the behavior of $G(x, y) = ax^2 + bxy + cy^2$ near the origin.

In the following, you may employ the Second-Derivative Test for Max/Min.

4. Explore the critical point(s) of the function $z = x + y + 1/(xy)$. Can you say anything about global extrema?

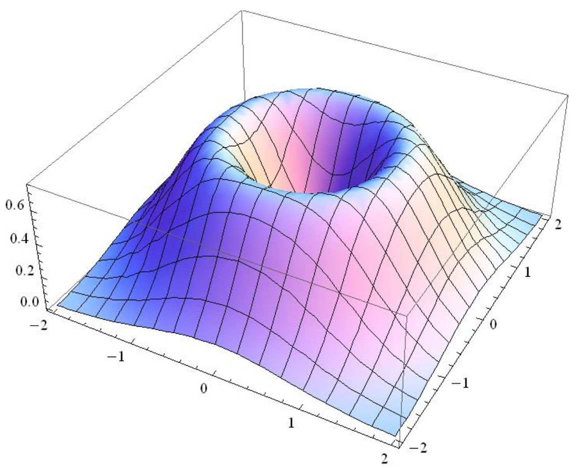
5. Find all local extrema of the function $z = x^3 + y^2 - 2xy + 7x - 8y + 3$

6. Determine all critical points of $f(x, y) = e^x + 3y + x + e^y$.

7. Determine all local maxima and minima of $g(x, y) = xy + 2x - \ln(x^2 y)$ in the first quadrant or show why there are none.

8. When it rains on the surface $z = 1/x + 1/y + xy$, at what point will a puddle form?

9. Show that the critical points of the "volcano" $z = 2(x^2 + y^2) \exp(-x^2-y^2)$ occur at (0, 0) and on the circle $x^2 + y^2 = 1$.



10. Consider $z = x^2 - y^2$. Show that P = (0, 0) is a critical point. Is it a local extremum?

11. Find the *minimum distance* from Q = (1, 2, 0) to the cone $z^2 = x^2 + y^2$.

12. A rectangular box, open at the top, is to hold 256 cubic cm. of dog food. Find the dimensions for which the surface area (the bottom and four sides) is minimized.

13. Find all local extrema of $z = x^2 + 3xy + y^2 + 16$.

14. Classify all critical points of the surface

$$g(x, y) = (x^2 - y^2) \exp((-1/2)(x^2 + y^2)).$$

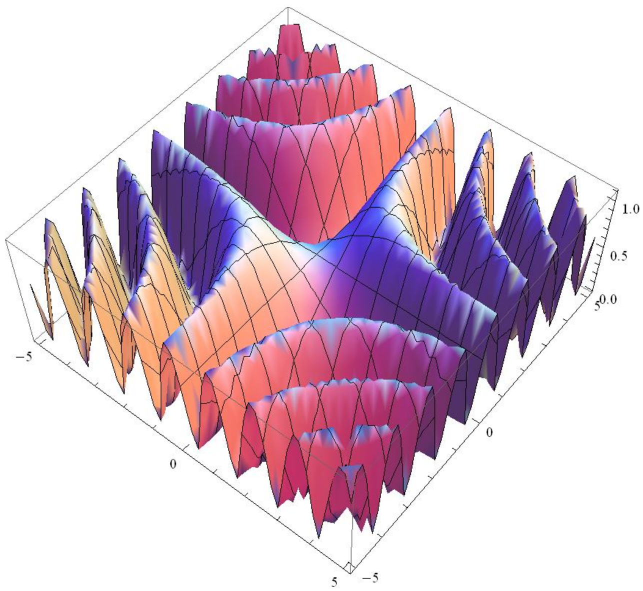
15. Determine the behavior of the function

$z = (x^2 + y^2) \cos(x + 2y)$ at the origin.

16. Let $f(x, y) = x^2 - 2xy + y^2$. Show that the second-derivative test fails for the critical point (0, 0). Can you determine whether the critical point(s) are local minima, local maxima, or saddle points.

(B) Locate and classify all critical points of each of the following surfaces:

1. $z = x^2 + y^2 + 16x - 4y + 13$
2. $z = x^2 - xy + y^2 + 5$
3. $z = x^2 - y^2 - xy$
4. $z = x^2 + xy^2 + y^4$
5. $z = \exp(1 + x^2 - y^2)$
6. $z = \ln(2 + \sin(xy))$. Consider only (0, 0).



7. $z = \sin(x^2 + y^2)$. Consider only (0, 0).
8. $z = x^5y + xy^5 + xy$
9. $z = \ln(x^2 + y^2 + 1)$
10. $z = x^2 + y^3$. Consider only (0, 0).
11. $z = \ln(ax^2 + by^2 + 1)$ where *a* and *b* are positive constants.