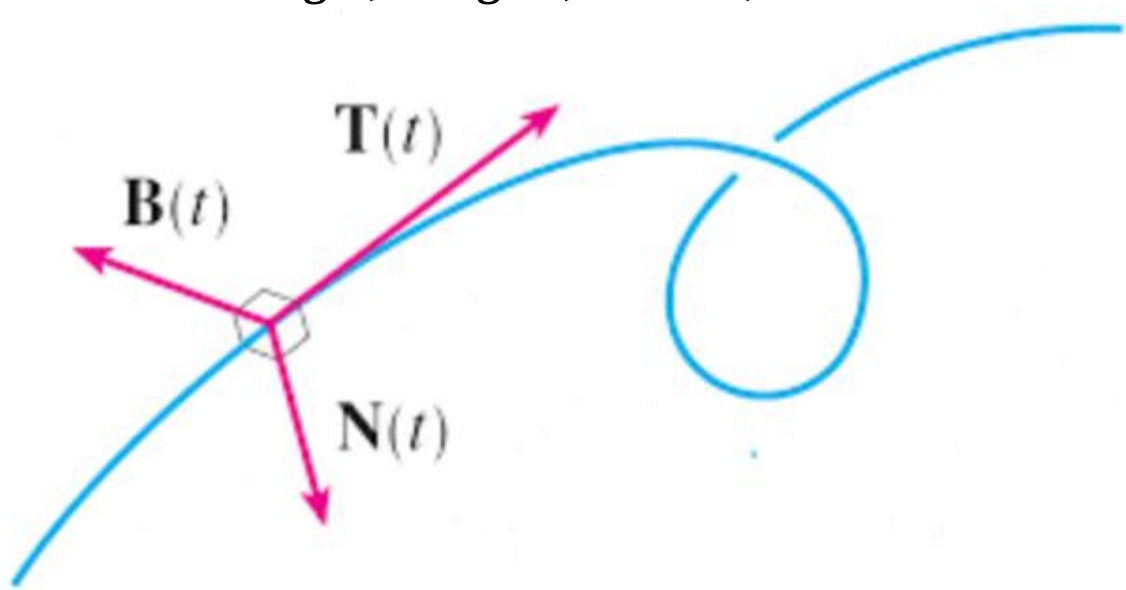
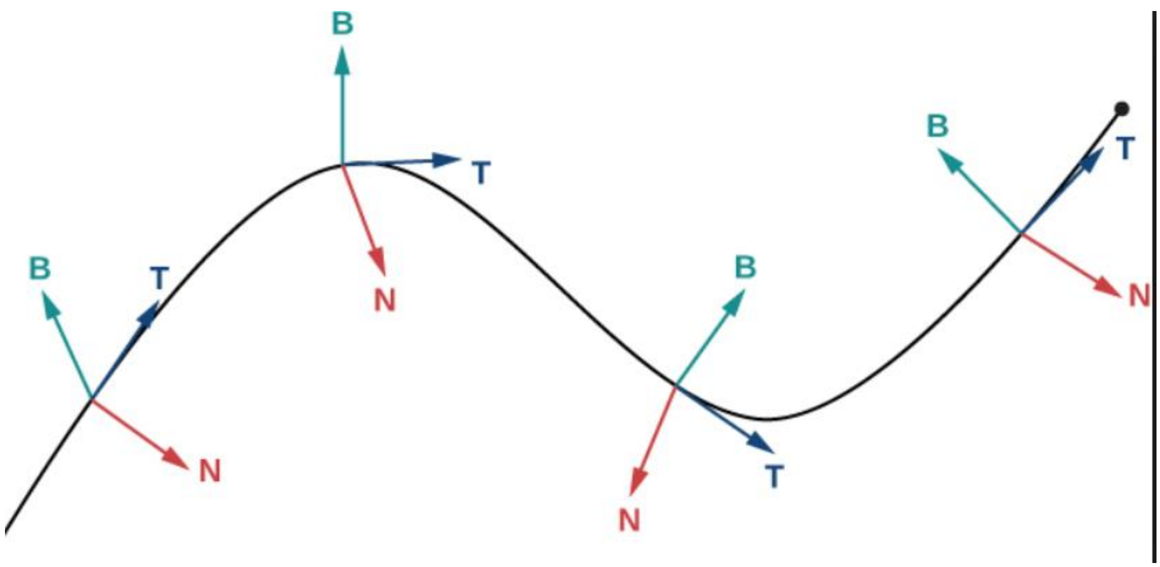
CLASS DISCUSSION: 4 FEBRUARY 2019

Arc length, Tangent, Normal, Binormal



1. Let v(t) and w(t) be curves in R^3. Prove that

$$\frac{d}{dt}\, v(t) \cdot w(t) = v'(t) \cdot w(t) + v(t) \cdot w'(t)$$



2. Prove that if r(t) is a curve on the unit sphere centered at the origin, then $r'(t)$ *is orthogonal to* $r(t)$. Can this result be extended to any sphere?

3. Define the unit tangent to r(t) as follows: $T(t) = \frac{r(t)}{\|r'(t)\|}$ What condition is required for this definition to be meaningful?

Define N(t) = $N(t) = \frac{T'(t)}{\|T'(t)\|}$. Why is N(t) orthogonal to T(t) ? Define the binormal B(t) = $B(t) = T'(t) \times N(t)$. What is the norm of B(t)? The orthogonal coordinate system T(t), N(t), B(t) is called the Frenet frame.

4. Frenet frame animations:
https://www.google.com/search?q=frenet+frame+animation&rlz=1C1GCEV_en&biw=960&bih=469&tbm=isch&source=iu&ictx=1&fir=ShCUsF0TXULf8M%253A%252CY_OlCYEgvi749M%252C_&usg=AI4_-kSjQYKnhSRp3x0WtCsTSCXDcR0V2w&sa=X&ved=2ahUKEwjEtMeXqKLgAhUjyoMKHdKBDiQQ9QEwDnoECAUQBA&cshid=1549291630101705#imgrc=Nd7JEZRE_0uWwM:

https://www.google.com/search?q=frenet+frame+animation&rlz=1C1GCEV_en&biw=960&bih=469&tbm=isch&source=iu&ictx=1&fir=ShCUsF0TXULf8M%253A%252CY_OlCYEgvi749M%252C_&usg=AI4_-kSjQYKnhSRp3x0WtCsTSCXDcR0V2w&sa=X&ved=2ahUKEwjEtMeXqKLgAhUjyoMKHdKBDiQQ9QEwDnoECAUQBA&cshid=1549291630101705#imgdii=ShCUsF0TXULf8M:&imgrc=raToSWH-WOt9-M:

5. Find an equation of the tangent line to
$r(t) = \cos(4t)\, i + 3\sin(4t)\, j + t^2\, k$ at $t = \pi$.

6. Find the Frenet frame for each of the following curves:

(a) $r(t) = t^2\, \mathbf{i} + 2 \sin t\, \mathbf{j} + 2 \cos t\, \mathbf{k}$ at an arbitrary time, t

(b) $r(t) = t^2\, \mathbf{i} + 2 \sin t\, \mathbf{j} + 2 \cos t\, \mathbf{k}$ at $t = \frac{\pi}{3}$.

(c) $r(t) = t\, \mathbf{i} + 3 \sin t\, \mathbf{j} + 3 \cos t\, \mathbf{k}$

7. Find the Frenet frame for each of the following curves:

(a) $r(t) = (e^{4t} \sin t, e^{4t} \cos t, 2)$.

(b) $r(t) = (2t, ½\, t^2, \ln(t^2))$

8. Define arc length: $L(r) = \int_{t_0}^{t_1} \|r'(t)\|\ dt$.

9. Find the arc length of each of the following curves:

(a) $r(t) = (a \cos t, a \sin t)$ where a is a constant and $0 \le t \le 2\pi$

(b) $r(t) = (\cos t, \sin t, t^2)$

(c) Consider the cycloid $(t - \sin t, 1 - \cos t)$. Find the velocity, the speed, and the length of one arch.
https://www.mathcurve.com/courbes2d.gb/cycloid/cycloid.shtml

http://mathworld.wolfram.com/Cycloid.html

10. (Marsden) A billiard ball on a pool table follows the path $r = (|t|, |t - ½|, 0)$, for $-½ \le t \le ½$ Find the distance traveled by the ball. (Caution: This curve is not smooth!)