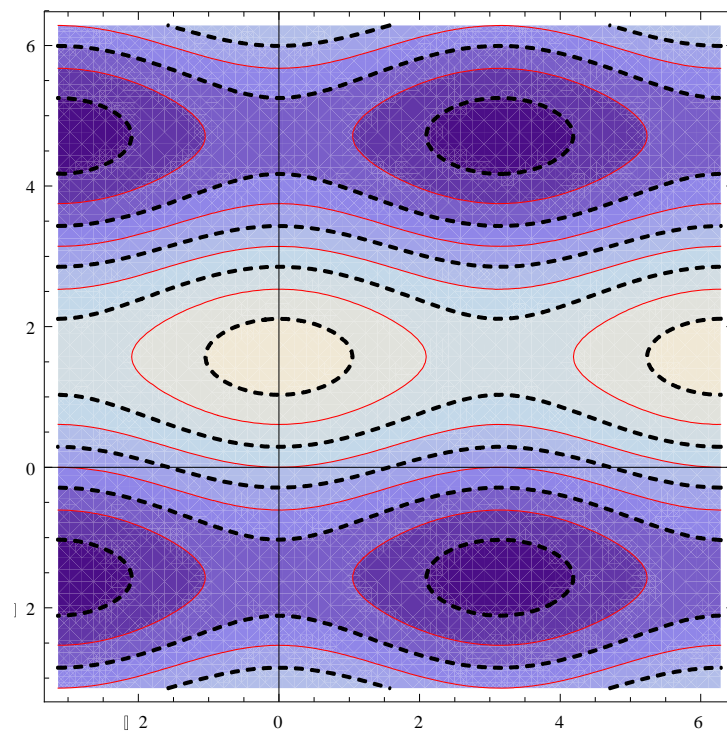


## CLASS DISCUSSION: 6 FEBRUARY 2019

### FUNCTIONS OF TWO VARIABLES: LEVEL SETS, PARTIAL DERIVATIVES



```
ContourPlot[2 Cos[x] + 7 Sin[x], {x, -Pi, 2 Pi}, {y, -Pi, 2 Pi},  
            Axes -> True, ContourStyle -> {Red, Dashed}]
```

1. Draw *level curves* for each of the following functions:

- (a)  $z = x + y + 3$
- (b)  $z = x^2 + y^2$
- (c)  $z = x^2 - y^2$
- (d)  $z = x^2 + y$
- (e)  $z = -xy$
- (f)  $z = x^3 - 1$
- (g)  $z = 4x^2 + 9y^2 + 1$
- (h)  $z = x/y$

$$(i) z = \sqrt{100 - x^2 - y^2}$$

$$(j) z = \sqrt{x^2 + y^2}$$

2. Find the *domain* and *range* of each of the following functions:

$$(a) f(x, y) = y - x + 1$$

$$(b) f(x, y) = \sqrt{y - x}$$

$$(c) g(x, y) = 4x^2 + 25y^2 + 3$$

$$(d) H(x, y) = x^2 - y^2 + 3$$

$$(e) F(x, y) = xy$$

$$(f) G(x, y) = \ln(x^2 + y^2)$$

$$(g) f(x, y) = \sqrt{25 - x^2 - y^2}$$

$$(h) f(x, y) = \frac{1}{\sqrt{25 - x^2 - y^2}}$$

3. Compute each of the following limits or explain why the limit fails to exist:

$$(a) \lim_{(x,y) \rightarrow (0,0)} \frac{e^{-xy}}{y + 3}$$

$$(b) \lim_{(x,y) \rightarrow (0,0)} \frac{(x - y)^2}{x^2 + y^2}$$

$$(c) \lim_{(x,y) \rightarrow (0,0)} \frac{1 - \frac{x^2}{2} - \cos x}{x^2 + y^2}$$

$$(d) \lim_{(x,y) \rightarrow (0,0)} \frac{x^2}{x^2 + y^2}$$

$$(e) \lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y}{x^2 + y^2}$$

$$(f) \lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y}{x^4 + y^2}$$

$$(g) \lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + y^2}{x^3 + y^2}$$

$$(h) \lim_{(x,y) \rightarrow (0,0)} \frac{x^2}{\sqrt{x^2 + y^2}}$$

$$(i) \lim_{(x,y) \rightarrow (0,0)} \frac{\sin(x^2 + y^2)}{x^2 + y^2}$$

4. For each function below, compute the *partial derivatives*

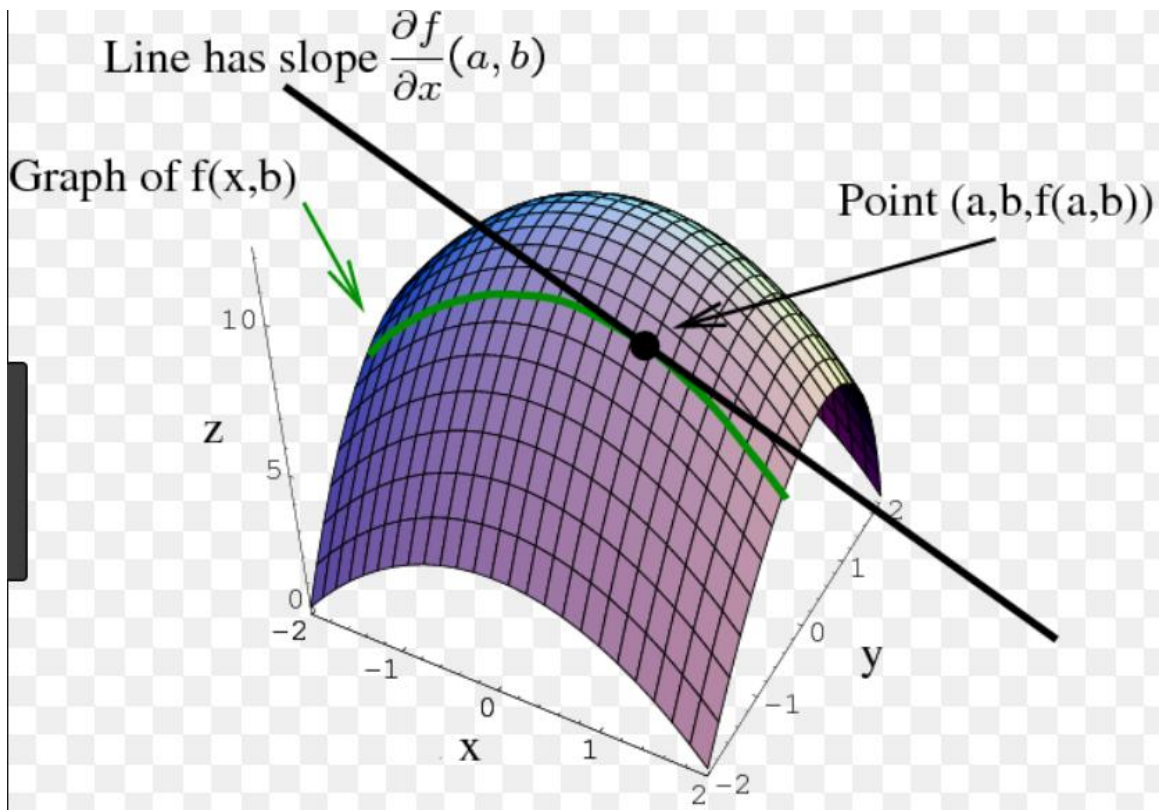
$$\frac{\partial z}{\partial x} \text{ and } \frac{\partial z}{\partial y}$$

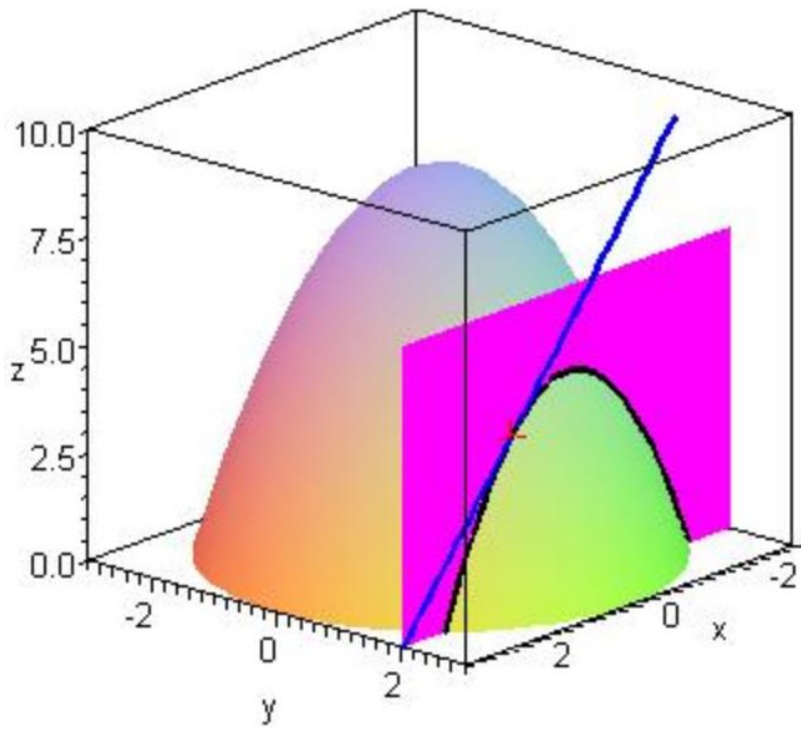
(a)  $z = ax + by + c$

(b)  $z = \ln(x^2 y^3)$

(c)  $z = x \sin(x + y)$

(d)  $z = (y + x^5)e^{x+5y}$





## Tangent planes

