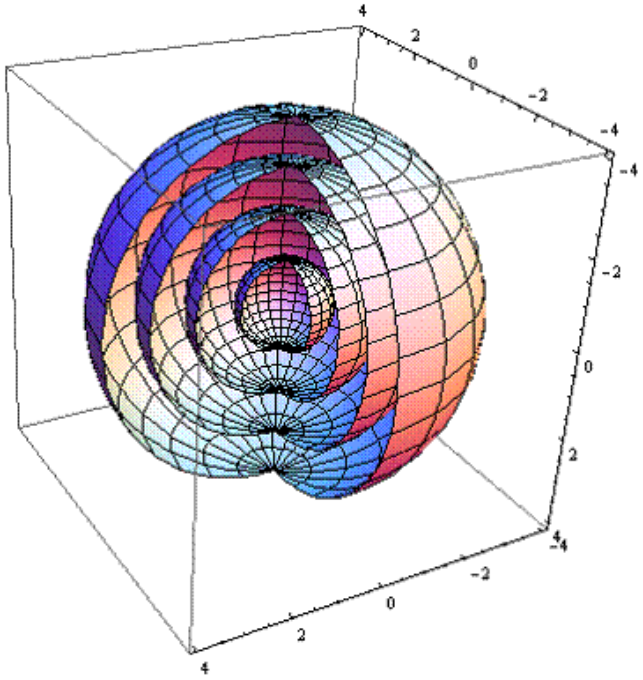
QUESTIONS FOR CLASS DISCUSSION (14 JANUARY)

VECTORS IN 2-SPACE AND 3-SPACE



Concentric spheres drawn with Mathematica

1. Find the *distance* between the two points (3, -1, 4) and (5, 5, -3).

2. Find the *equation of a sphere* with center (1, 2, 3) and radius 4.

3. Consider the sphere defined by the equation

$$(x - 2)^2 + (y - 5)^2 + (z - 8)^2 = 8.$$

Find the *equation of the sphere* centered at (7, 17, 21) that is *tangent* to the given sphere. *Hint:* Begin with a picture.

4. Do the two spheres $(x - 1)^2 + (y - 2)^2 + (z - 2)^2 = 4$ and

$$(x + 1)^2 + (y + 2)^2 + (z - 3)^2 = 4$$ intersect? Why?

5. Let $\mathbf{v} = \mathbf{i} + 2\,\mathbf{j}$ and $\mathbf{w} = 4\,\mathbf{i} + 3\,\mathbf{j}$. Compute $\mathbf{v} + \mathbf{w}$ and display its geometrical meaning.

6. Referring to problem (5), compute the *norm* of **v**, the *norm* of **w**, and the *norm* of **v**+**w**.

7. Find a *unit vector* in the direction of $\mathbf{v} = \mathbf{i} + 3\,\mathbf{j} + \mathbf{k}$.

8. Express the vector from the point A = (2, 3, 4) to the point B = (7, 1, 0) as a vector in *standard position*.

9. Explain the geometric meaning of *scalar multiplication* of a vector.

10. Find all vectors in 2-space that have norm of 13 and **i**-component of 5.

11. Find the equation of the line L passing through (1, 0, 0) in the direction of **j**.

12. Find the equation of the line L passing through (3, -1, 2) in the direction of $2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$.

13. Do the two lines $L_1(t) = (t, -6t + 1, 2t - 8)$ and $L_2(t) = (3t + 1, 2t, 0)$ intersect? Justify your answer!

15. Does the line $L(t) = (2, -1, 2) + t(2, 3, 1)$ intersect the plane $5x - 3y - z = 6$? If so, where?

16. *Normalize* the vector $5\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}$

Exercises from Stuart:

1. Suppose you start at the origin, move along the x-axis a distance of 4 units in the positive direction, and then move downward a distance of 3 units. What are the coordinates of your position?
2. Sketch the points $(1, 5, 3)$, $(0, 2, -3)$, $(-3, 0, 2)$, and $(2, -2, -1)$ on a single set of coordinate axes.
3. Which of the points $A(-4, 0, -1)$, $B(3, 1, -5)$, and $C(2, 4, 6)$ is closest to the yz-plane? Which point lies in the xz-plane?
4. What are the projections of the point $(2, 3, 5)$ on the xy-, yz-, and xz-planes? Draw a rectangular box with the origin and $(2, 3, 5)$ as opposite vertices and with its faces parallel to the coordinate planes. Label all vertices of the box. Find the length of the diagonal of the box.
5. What does the equation $x = 4$ represent in $\mathbb{R}^2$? What does it represent in $\mathbb{R}^3$? Illustrate with sketches.
6. What does the equation $y = 3$ represent in $\mathbb{R}^3$? What does $z = 5$ represent? What does the pair of equations $y = 3$, $z = 5$ represent? In other words, describe the set of points (x, y, z) such that $y = 3$ and $z = 5$. Illustrate with a sketch.
7. Describe and sketch the surface in $\mathbb{R}^3$ represented by the equation $x + y = 2$.

"...treat Nature by the sphere, the cylinder, and the cone..."

- Paul Cézanne
(1839-1906)