CLASS DISCUSSION: JAN 18TH

DOT AND CROSS PRODUCTS

Review (Stuart exercises)

I. Find an equation of a sphere if one of its diameters has endpoints (5, 4, 3) and (1, 6, -9) .

II. Describe in words the region of represented by the equation(s) or inequality.

(a) $x = 5$

(b) $z \geq -1$

(c) $x^2 + y^2 = 4$; $z = -1$

(d) $x^2 + y^2 + z^2 \leq 4$

(e) $x^2 + z^2 \leq 9$

III. Consider the points P such that the distance from P to A(-1, 5, 3) is twice the distance from P to B(6, 2, -2). Show that the set of all such points is a sphere, and find its center and radius.

IV. Find the equation of the set of all points equidistant from the points A(-1, 5, 3) and B(6, 2, -2). Describe the set.

Dot products

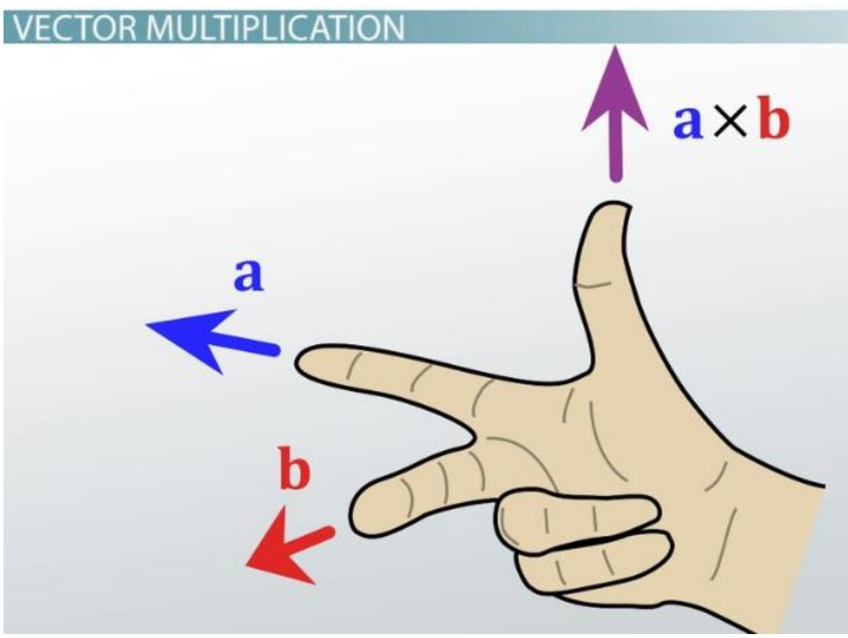
Properties of the Dot Product

If **a**, **b**, and **c** are vectors in V_3 and c is a scalar, then

1. $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2$
2. $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$
3. $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$
4. $(c\mathbf{a}) \cdot \mathbf{b} = c(\mathbf{a} \cdot \mathbf{b}) = \mathbf{a} \cdot (c\mathbf{b})$
5. $\mathbf{0} \cdot \mathbf{a} = 0$

1. Find the *angle* between the two vectors 2**i** – 3**j** + 4**k** and 3**i** – 4**j** + **k**.
2. Find the *projection* of **v** = 2**i** + **j** – 3**k** onto **w** = – **i** + **j** + **k**.
3. Find two non-parallel vectors, each *orthogonal* to (1, 1, 1).
4. If the vectors **a** and **b** have lengths 4 and 6, respectively, and the angle between them is p/3, find ***a*** · ***b***.
5. Find the angle between the vectors **a** = (2, 2, -1) and **b** = (5, -3, 2).

Cross products



1, 2, 3, 4, 5, 6 and 7 Find the cross product $\mathbf{a} \times \mathbf{b}$ and verify that it is orthogonal to both $\mathbf{a}$ and $\mathbf{b}$.

1. $\mathbf{a} = \langle 2, 3, 0 \rangle, \quad \mathbf{b} = \langle 1, 0, 5 \rangle$

   Answer

2. $\mathbf{a} = \langle 4, 3, -2 \rangle, \quad \mathbf{b} = \langle 2, -1, 1 \rangle$

3. $\mathbf{a} = 2\mathbf{j} - 4\mathbf{k}, \quad \mathbf{b} = -\mathbf{i} + 3\mathbf{j} + \mathbf{k}$

   Answer

4. $\mathbf{a} = 3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}, \quad \mathbf{b} = 3\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$

5. $\mathbf{a} = \frac{1}{2}\mathbf{i} + \frac{1}{3}\mathbf{j} + \frac{1}{4}\mathbf{k}, \quad \mathbf{b} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$

   Answer

6. $\mathbf{a} = t\,\mathbf{i} + \cos t\,\mathbf{j} + \sin t\,\mathbf{k}, \quad \mathbf{b} = \mathbf{i} - \sin t\,\mathbf{j} + \cos t\,\mathbf{k}$

7. $\mathbf{a} = \langle t, 1, 1/t \rangle, \quad \mathbf{b} = \langle t^2, t^2, 1 \rangle$

8. Show that 2 **i** + 2 **j** – **k** is perpendicular to 5 **i** – 4 **j** + 2 **k**.
9. Consider the line L(t) = (1, 0, 3) + t (1, 1, 2) and the point P = (1, 2, 3). Calculate the *distance* from *P* to the line (a) using a dot product, and (b) using a cross-product.
10. Find the *area* of the parallelogram spanned by the vectors **a** = **i** + 2**j** + 3**k** and **b** = – **i** – **k**.
11. Find a unit vector *orthogonal* to the vectors **i** + **j** and **j** + **k**.
12. Find an equation for the plane that is perpendicular to **i** + **j** + **k** and contains the point (1, 0, 0).
13. Find an equation of the plane containing the points (1, 1, 1), (2, 0, 0) and (1, 1, 0).
14. Find the *distance* from the point A = (2, 0, –1) to the plane 3x – 2y + 8z + 1 = 0.
15. Find an equation for the plane containing the two (parallel) lines

    $L_1(t) = (0, 1, -2) + t(2, 3, -1)$ and $L_2(t) = (2, -1, 0) + t(2, 3, -1)$.

16. Find the *distance* between the (parallel) planes x – 2y + 8z + 1 = 0 and x – 2y + 8z + 5 = 0.
17. Find the *angle* between the two planes 3x – 2y + 8z = 7 and 5x – 3y + z = 1.
18. Find the *distance* between the two parallel lines $L_1(t) = (1, 1, 1) + t(2, 1, -1)$ and $L_2(t) = (2, 5, 0) + t(2, 1, -1)$ in two different ways (one using *dot products*, the other using *cross products*).

19. Two lines in 3-space are said to be *skew lines* if they do not intersect and are not parallel. Show that the two lines $L_1(t) = (1, 1, 1) + t(3, 1, 0)$ and $L_2(t) = (2, 5, 0) + t(2, 1, -1)$ are skew lines. Find the distance between these two lines. (*Hint:* Find a pair of parallel planes, each containing one of the two lines.)

Explore the properties of the cross-product

If $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are vectors and c is a scalar, then

1. $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$
2. $(c\mathbf{a}) \times \mathbf{b} = c(\mathbf{a} \times \mathbf{b}) = \mathbf{a} \times (c\mathbf{b})$
3. $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$
4. $(\mathbf{a} + \mathbf{b}) \times \mathbf{c} = \mathbf{a} \times \mathbf{c} + \mathbf{b} \times \mathbf{c}$
5. $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$
6. $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$