MATH 263 CLASS DISCUSSION: 23 JAN 2019

VECTOR ANALYSIS IN R^3, CONTINUED

1. Find the *angle* between the two vectors 2**i** – 3**j** + 4**k** and 3**i** – 4**j** + **k**.
2. Find the *projection* of **v** = 2**i** + **j** – 3**k** onto **w** = – **i** + **j** + **k**.
3. Find two non-parallel vectors, each *orthogonal* to (1, 1, 1).
4. Consider the line L(t) = (1, 0, 3) + t (1, 1, 2) and the point P = (1, 2, 3). Calculate the *distance* from *P* to the line (a) using a dot product, and (b) using a cross-product.
5. Find the *area* of the parallelogram spanned by the vectors

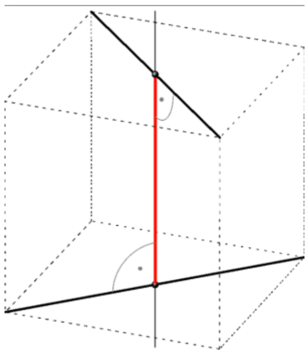
   a = **i** + 2**j** + 3**k** and **b** = – **i** – **k**.
6. Find a unit vector *orthogonal* to the vectors **i** + **j** and **j** + **k**.
7. Find an equation for the plane that is perpendicular to **i** + **j** + **k** and contains the point (1, 0, 0).
8. Find an equation of the plane containing the points (1, 1, 1), (2, 0, 0) and (1, 1, 0).
9. Find the *distance* from the point A = (2, 0, –1) to the plane 3x – 2y + 8z + 1 = 0.
10. Find an equation for the plane containing the two (parallel) lines

    $L_1(t) = (0, 1, -2) + t\,(2, 3, -1)$ and $L_2(t) = (2, -1, 0) + t(2, 3, -1)$.
11. Find the *distance* between the (parallel) planes

    x – 2y + 8z + 1 = 0 and x – 2y + 8z + 5 = 0.

12. Find the *angle* between the two planes 3x – 2y + 8z = 7 and 5x – 3y + z = 1.
13. Find the *distance* between the two parallel lines $L_1(t) = (1, 1, 1) + t(2, 1, -1)$ and $L_2(t) = (2, 5, 0) + t(2, 1, -1)$ in two different ways (one using *dot products*, the other using *cross products*).

14. Two lines in 3-space are said to be *skew lines* if they do not intersect and are not parallel. Show that the two lines $L_1(t) = (1, 1, 1) + t(3, 1, 0)$ and $L_2(t) = (2, 5, 0) + t(2, 1, -1)$ are skew lines. Find the distance between these two lines. (*Hint:* Find a pair of parallel planes, each containing one of the two lines.)



15. (a) Suppose that $\mathbf{v}\cdot\mathbf{w} = \mathbf{u}\cdot\mathbf{w}$ for all vectors $\mathbf{w}$. Must it follow that $\mathbf{v} = \mathbf{u}$?

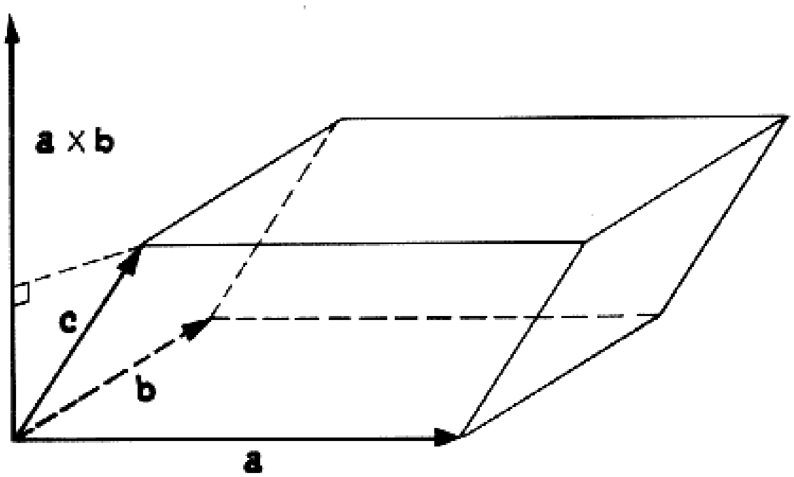
(b) Suppose that $\mathbf{v}\times\mathbf{w} = \mathbf{u}\times\mathbf{w}$ for all vectors $\mathbf{w}$. Must it follow that $\mathbf{v} = \mathbf{u}$?

16. Find an equation for the plane that passes through the points (5, 0, 3), (1, 1, 1) and (2, -1, 0).

17. Find an equation for the plane that passes through the point (2, -1, 3) and is perpendicular to the line $L(t) = (1, -2, 1) + t(1, -2, 3)$.

18. Find an equation for the plane that passes through the points (3, 2, -1) and (1, -1, 2) and that is parallel to the line $L(t) = (1, -1, 0) + t(3, 2, -2)$.

19. Find the volume of the *parallelepiped* determined by the vertices (0, 1, 0), (1, 1, 1), (0, 2, 0), (3, 1, 2).

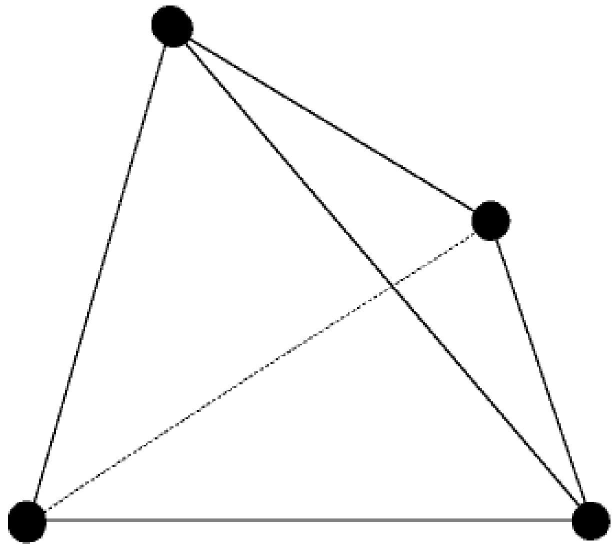


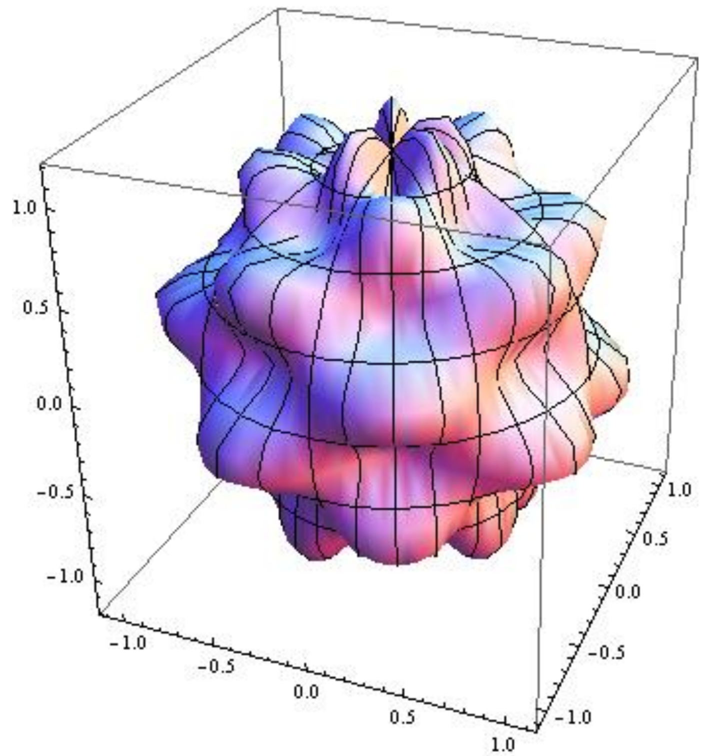
20. Find a vector *parallel* to the line of intersection of the planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$.

21. The volume of a *tetrahedron* with concurrent edges **a**, **b**, **c** is given by

$$V = \frac{1}{6} a \cdot (b \times c)$$

Find the volume when **a** = **i** + **j** + **k**, **b** = **i** – **j** + **k**, **c** = **i** + **j**.





SphericalPlot3D[1 + 0.2Sin[8θ]Sin[4ϕ], {θ, 0,2Pi}, {ϕ, 0, Pi}]

This "Bumpy Sphere" has been used to model tumors.
Cf. *Heat therapy for tumors: Applications of calculus to medicine*
(UMAP modules in undergraduate mathematics and its applications), Leah Edelstein-Keshet