

CLASS DISCUSSION: 28 JANUARY 2019

curves in \mathbb{R}^2 and \mathbb{R}^3

1(a) (review)

Are the following quantities vectors or scalars? Explain.

a. The cost of a theater ticket

Answer ▾

b. The current in a river

Answer ▾

c. The initial flight path from Houston to Dallas

Answer ▾

d. The population of the world

1(b) (review)

If \mathbf{v} lies in the first quadrant and makes an angle $\pi/3$ with the positive x -axis and $|\mathbf{v}| = 4$, find \mathbf{v} in component form.

2. Find the domain of each of the following vector valued functions.

$$\mathbf{r}(t) = \left\langle \ln(t+1), \frac{t}{\sqrt{9-t^2}}, 2^t \right\rangle$$

Answer ▾

$$\mathbf{r}(t) = \cos t \mathbf{i} + \ln t \mathbf{j} + \frac{1}{t-2} \mathbf{k}$$

3. Find the following limits

$$\lim_{t \rightarrow 0} \left(e^{-3t} \mathbf{i} + \frac{t^2}{\sin^2 t} \mathbf{j} + \cos 2t \mathbf{k} \right)$$

Answer ▾

$$\lim_{t \rightarrow 1} \left(\frac{t^2 - t}{t - 1} \mathbf{i} + \sqrt{t + 8} \mathbf{j} + \frac{\sin \pi t}{\ln t} \mathbf{k} \right)$$

$$\lim_{t \rightarrow \infty} \left\langle \frac{1+t^2}{1-t^2}, \tan^{-1} t, \frac{1-e^{-2t}}{t} \right\rangle$$

Answer ▾

$$\lim_{t \rightarrow \infty} \left\langle te^{-t}, \frac{t^3 + t}{2t^3 - 1}, t \sin \frac{1}{t} \right\rangle$$

4. Sketch the curve with the given vector equation. Indicate with an arrow the direction in which increases.

$$\mathbf{r}(t) = \langle \sin t, t \rangle$$

Answer ▾

$$\mathbf{r}(t) = \langle t^2 - 1, t \rangle$$

$$\mathbf{r}(t) = \langle t, 2 - t, 2t \rangle$$

Answer ▾

$$\mathbf{r}(t) = \langle \sin \pi t, t, \cos \pi t \rangle$$

$$\mathbf{r}(t) = \langle 3, t, 2 - t^2 \rangle$$

Answer ▾

$$\mathbf{r}(t) = 2 \cos t \mathbf{i} + 2 \sin t \mathbf{j} + \mathbf{k}$$

$$\mathbf{r}(t) = t^2 \mathbf{i} + t^4 \mathbf{j} + t^6 \mathbf{k}$$

Answer ▾

$$\mathbf{r}(t) = \cos t \mathbf{i} - \cos t \mathbf{j} + \sin t \mathbf{k}$$

5. Find a vector equation and parametric equations for the line segment that joins P to Q.

$$P(2, 0, 0), Q(6, 2, -2)$$

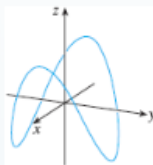
Answer ▾

$$P(-1, 2, -2), Q(-3, 5, 1)$$

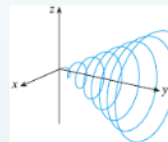
$$P(0, -1, 1), Q\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right)$$

6. Match the parametric equations with the graphs (labeled I–VI). Give reasons for your choices.

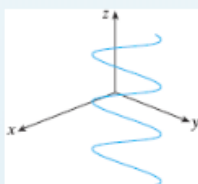
I.



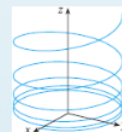
II.



III.

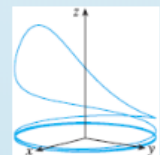
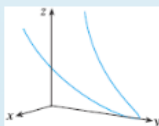


IV.



VI.

V.



$$x = t \cos t, \quad y = t, \quad z = t \sin t, \quad t \geq 0$$

Answer ▾

$$x = \cos t, \quad y = \sin t, \quad z = 1/(1 + t^2)$$

$$x = t, \quad y = 1/(1 + t^2), \quad z = t^2$$

Answer ▾

$$x = \cos t, \quad y = \sin t, \quad z = \cos 2t$$

$$x = \cos 8t, \quad y = \sin 8t, \quad z = e^{0.8t}, \quad t \geq 0$$

Answer ▾

$$x = \cos^2 t, \quad y = \sin^2 t, \quad z = t$$

7.

Show that the curve with parametric equations $x = t \cos t$, $y = t \sin t$, $z = t$ lies on the cone $z^2 = x^2 + y^2$, and use this fact to help sketch the curve.

8.

At what points does the curve $\mathbf{r}(t) = t \mathbf{i} + (2t - t^2) \mathbf{k}$ intersect the paraboloid $z = x^2 + y^2$?

9.

Show that the curve with parametric equations $x = t^2$, $y = 1 - 3t$, $z = 1 + t^3$ passes through the points $(1, 4, 0)$ and $(9, -8, 28)$ but not through the point $(4, 7, -6)$.

10.

For the curve $\mathbf{r}(t) = \sqrt{t} \mathbf{i} + (2 - t) \mathbf{j}$, find $\mathbf{r}'(t)$ and sketch the position vector $\mathbf{r}(1)$ and the tangent vector $\mathbf{r}'(1)$.

11. Let $\mathbf{r}(t)$ be the position of a particle in the xy -plane at time t . Find an equation in x and y whose graph is the path of the particle. Then find the particle's velocity and acceleration vectors at the given time t . Also find the speed of the particle at time t . Write the particle's velocity at that time as the product of its speed and direction.

(a) $\mathbf{r}(t) = (\cos 2t) \mathbf{i} + 3\sin(2t) \mathbf{j}$, $t = 0$

(b) $\mathbf{r}(t) = e^t \mathbf{i} + 3e^{2t} \mathbf{j}$ at time $t = 0$.

(c) $\mathbf{r}(t) = (t - \sin t) \mathbf{i} + (1 - \cos t) \mathbf{j}$, $t = \pi/6$. (This is a *cycloid*.)

12. Let $\mathbf{r}(t)$ be the position of a particle in \mathbf{R}^3 at time t . Find the particle's velocity and acceleration vectors at the given time t . Also find the speed of the particle at time t . Write the particle's velocity at that time as the product of its speed and direction.

(a) $\mathbf{r}(t) = (t + 1) \mathbf{i} + (t^2 - 1) \mathbf{j} + 2t \mathbf{k}$, $t = 1$

(b) $\mathbf{r}(t) = (\sec t) \mathbf{i} + (\tan t) \mathbf{j} + (4/3)t \mathbf{k}$, $t = \pi/6$

13. For each of the following:

a. Sketch the plane curve with the given vector equation.

b. Find $\mathbf{r}'(t)$.

c. Sketch the position vector $\mathbf{r}(t)$ and the tangent vector $\mathbf{r}'(t)$ for the given value of t .

$$\mathbf{r}(t) = \langle t - 2, t^2 + 1 \rangle, \quad t = -1$$

Answer ▾

$$\mathbf{r}(t) = \langle t^2, t^3 \rangle, \quad t = 1$$

$$\mathbf{r}(t) = e^{2t} \mathbf{i} + e^t \mathbf{j}, \quad t = 0$$

Answer ▾

$$\mathbf{r}(t) = e^t \mathbf{i} + 2t \mathbf{j}, \quad t = 0$$

$$\mathbf{r}(t) = 4 \sin t \mathbf{i} - 2 \cos t \mathbf{j}, \quad t = 3\pi/4$$

Answer ▾

$$\mathbf{r}(t) = (\cos t + 1) \mathbf{i} + (\sin t - 1) \mathbf{j}, \quad t = -\pi/3$$

14. Find the unit tangent vector $\mathbf{T}(t)$ at the point with the given value of the parameter t .

$$\mathbf{r}(t) = \left\langle t^2 - 2t, 1 + 3t, \frac{1}{3}t^3 + \frac{1}{2}t^2 \right\rangle, t = 2$$

Answer ▾

$$\mathbf{r}(t) = \langle \tan^{-1} t, 2e^{2t}, 8te^t \rangle, t = 0$$

$$\mathbf{r}(t) = \cos t \mathbf{i} + 3t \mathbf{j} + 2 \sin 2t \mathbf{k}, t = 0$$

Answer ▾

$$\mathbf{r}(t) = \sin^2 t \mathbf{i} + \cos^2 t \mathbf{j} + \tan^2 t \mathbf{k}, t = \pi/4$$

- 15.

The curves $\mathbf{r}_1(t) = \langle t, t^2, t^3 \rangle$ and $\mathbf{r}_2(t) = \langle \sin t, \sin 2t, t \rangle$ intersect at the origin. Find their angle of intersection correct to the nearest degree.

- 16.

At what point do the curves $\mathbf{r}_1(t) = \langle t, 1 - t, 3 + t^2 \rangle$ and $\mathbf{r}_2(s) = \langle 3 - s, s - 2, s^2 \rangle$ intersect? Find their angle of intersection correct to the nearest degree.