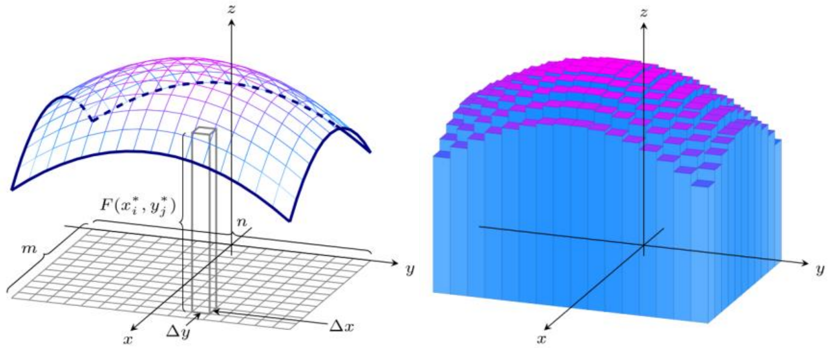
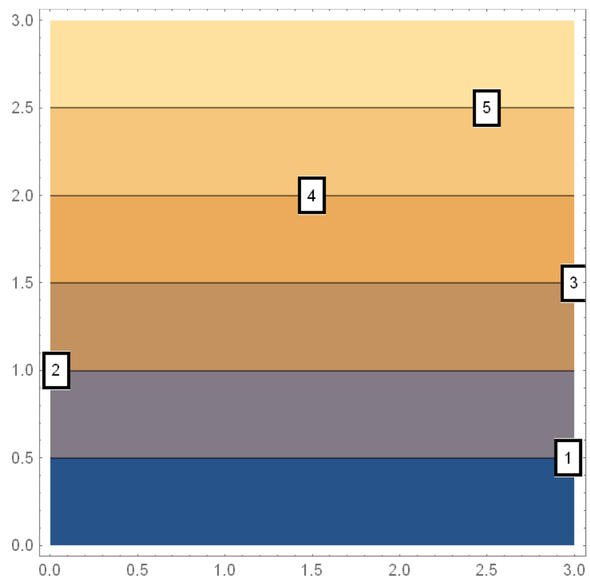
DOUBLE INTEGRALS OVER RECTANGLES
FUBINI'S THEOREM

13 March 2019

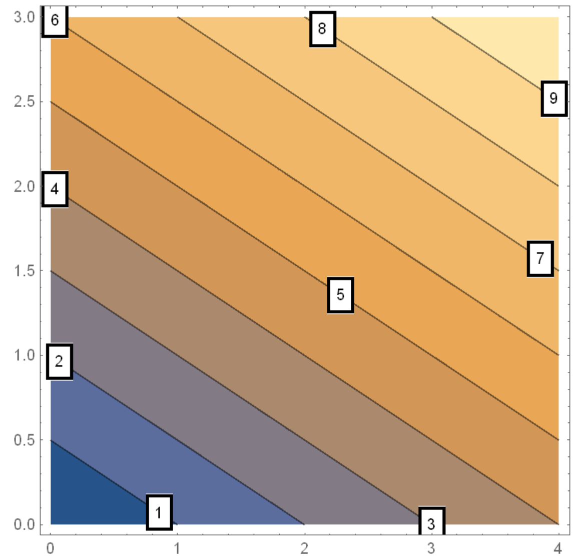


1. (a) Using the following contour diagram, *estimate* the volume beneath the surface $z = 2y$ and above the rectangle R = [0, 3]× [0, 3] in the xy-plane.



(b) Using a double integral, compute the *exact* value of this volume.

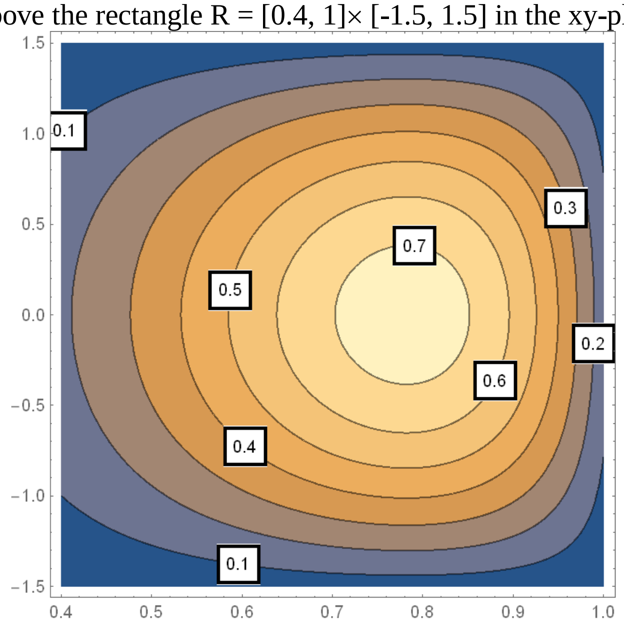
2. (a) Using the following contour diagram, estimate the volume beneath the surface $z = x + 2y$ and above the rectangle $R = [0, 4]\times[0, 3]$ in the xy-plane.



(b) Using a double integral, compute the exact value of this volume.

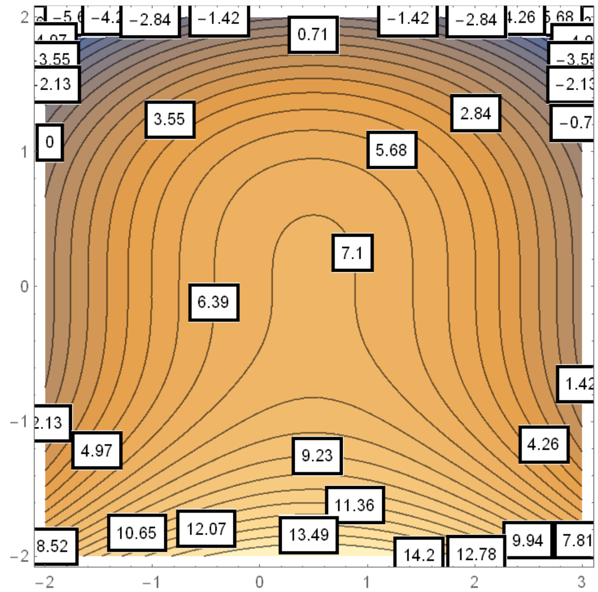
3.

(a) Using the following contour diagram, estimate the volume beneath the surface $z = x \sin(3x^2) \cos y$ and above the rectangle $R = [0.4, 1] \times [-1.5, 1.5]$ in the xy-plane.



(b) Using a double integral, compute the *exact* value of this volume.

4. (a) Using the following contour diagram, estimate the volume beneath the surface $z = 7 - x^2 - y^3 + x$ and above the rectangle $R = [-2, 3] \times [-2, 2]$ in the xy-plane.



(b) Using a double integral, compute the exact value of this volume.

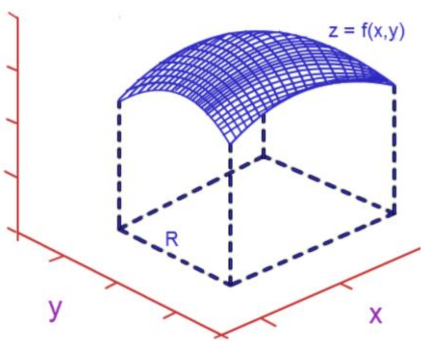
5. Calculate *under* and *over*-estimates of the double integral

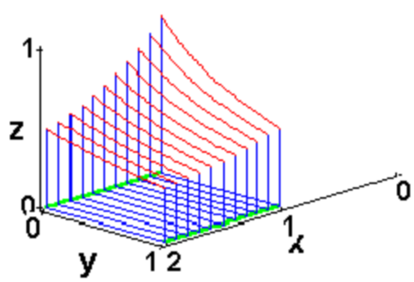
$$\iint_R f(x,y)\,dA$$

6.

y\x	0	3	6
0	2	3	4
2	6	4	3
4	18	15	12

Using the table of values to the right, where ***R*** is the rectangle [0, 6] × [0, 4].





6. For the following iterated integral, sketch the region of integration and evaluate the integral.

$$\int_0^{\frac{\pi}{2}} \left(\int_0^{\pi} (\sin x + \cos y)\,dx \right) dy$$

7. Evaluate each of the following iterated integrals. What is their relationship to one another?

$$(a)\ \int_1^2 \left(\int_0^1 xye^x\,dy \right) dx \qquad (b)\ \int_0^1 \left(\int_1^2 xye^x\,dx \right) dy$$

8. If D is a rectangular plate defined by $1 \le x \le 2$, $0 \le y \le 1$ (measured in cm.), and its mass density is given by $\delta(x,y) = ye^{xy}$ grams per square cm, integrate δ over D to find the *mass* of the plate.

9. Find the *volume* of the region bounded above by the paraboloid $z = x^2 + y^2$ and below by the square ***S***: $-1 \le x \le 1$, $-1 \le y \le 1$.

10. Find the *volume* of the region bounded above by the plane $z = 2 - x - y$ and below by the square **S**: $0 \le x \le 1$, $0 \le y \le 1$.

11. (Hughes-Hallett)

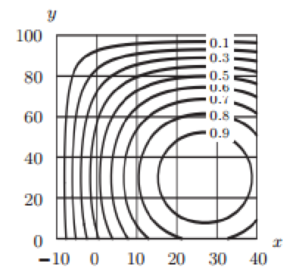


Figure 16.1

Let R be the region $0 \le x \le 20$, $0 \le y \le 40$. The function $f(x,y)$ represents the performance index of a machine; its values range from 0 to 1. Use the grid in Figure 16.1, which is a contour map of f, to make upper and lower estimates of

$$I = \int_R f(x,y)\,dA.$$

12. The integral $\int_0^1 \int_0^1 x^2\,dx\,dy$ represents the

(a) Area under the curve $y = x^2$ between $x = 0$ and $x = 1$.
(b) Volume under the surface $z = x^2$ above the square $0 \le x, y \le 1$ on the xy-plane.
(c) Area under the curve $y = x^2$ above the square $0 \le x, y \le 1$ on the xy-plane.

Guido Fubini (1879 – 1943)

After years of finding mathematics easy, I finally reached integral calculus and came up against a barrier. I realized that this was as far as I could go, and to this day I have never successfully gone beyond it in any but the most superficial way.

- Isaac Asimov