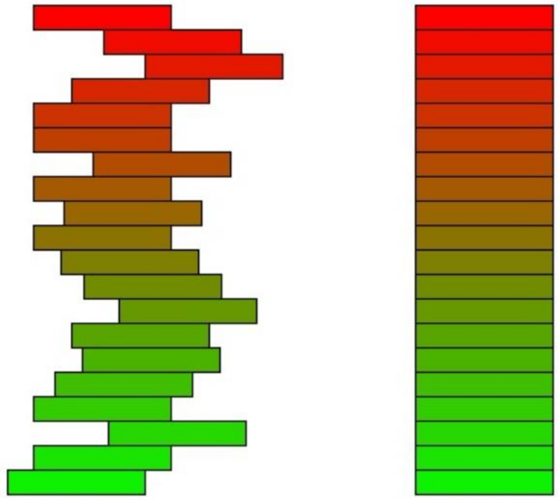
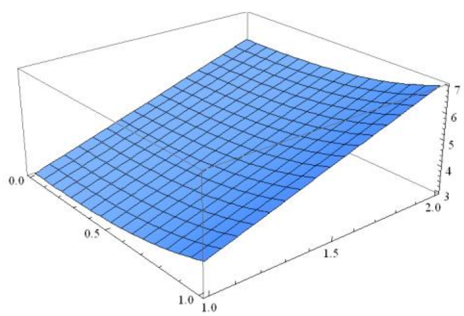
CLASS DISCUSSION: 18 MARCH 2019

DOUBLE & ITERATED INTEGRALS: FUBINI'S THEOREM

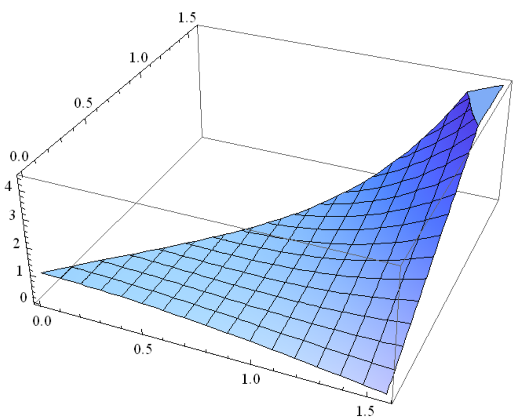


Cavalieri's Principle

1. (a) Compute the volume of the solid bounded by the graph $z = x^3 + 3y$, the rectangle $R = [0, 1] \times [1, 2]$ and the "vertical sides" of R.



(b) *Find* $\iint_T (x^3 y + \cos x)\, dA$, *where T is the triangle bounded by* $y = x,\ y = 0,\ x = \dfrac{\pi}{2}$



(c) Evaluate the following integral and sketch the corresponding region of integration.

$$\int_0^1 \int_0^{x^2} 1 \, dy \, dx$$

What does this integral represent?

2.

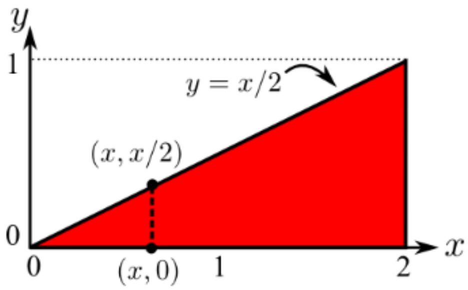
Using a double integral, find the area of an ellipse with semi-axes of length *a* and *b*.

3. Evaluate $\iint_R xy^2\,dA$ where $R = [0,2] \times [0,1]$ by writing it as an iterated integral where:

(a) We use horizontal slices

(b) We use vertical slices

4. Evaluate $\iint_T xy^2\,dA$ where T is the triangle with vertices (0, 0), (2, 0), and (2, 1).



5. *[Stewart]* Evaluate each of the following double integrals (using geometry or symmetry without actually integrating):

(a) $\iint_R 13\,dA$ where $R = [-2, 5]\times[-8, 3]$

(b) $\iint_D \sqrt{R^2 - x^2 - y^2}\,dA$ where D is the disk with center at the origin and radius R.

6. Evaluate each of the following double integrals by converting it to an iterated integral:

(a) $\iint_R (x^2 + y^2)\,dA$ where $R = [0, 1]\times[0, 1]$

(b) $\iint_R xe^{x+3y}\,dA$ where $R = [0, 1]\times[0, 1]$

(c) $\iint_R xe^{xy}\,dA$ where $R = [0, 1]\times[0, 1]]$

(d) $\iint_R (ax + by + c)\,dA$ where $R = [0, 3]\times[0, 1]$

7. Compute the *volume* of each of the following regions:

(a) Beneath the saddle $z = xy$ and above the unit square $[0, 1]\times[0, 1]$

(b) Beneath the paraboloid $z = x^2 + y^2$ and above the rectangle $[-1, 1]\times[0, 1]$

(c) Bounded by the surface $z = \sin y$, the planes $x = 1$, $x = 0$, $y = 0$, $y = \pi/2$ and the xy-plane.

(d) Beneath the surface $z = x^2 + y$ and above the rectangle $[0, 1]\times[1, 2]$

8. Compute the *average temperature* over the given plate:

(a) $T(x, y) = x + 11$; $R = [0, 3]\times[-4, 1]$

(b) $T(x, y) = xy$; R = the triangle with vertices (0, 0), (1, 0) and (1, 3)

(c) $T(x, y) = x \sin y$; R = region enclosed by the curves $y = 0$, $y = x^2$, and $x = 1$

(d) $T(x, y) = y/(1 + x^2)$; $R = [0, \pi]\times[0, 2\pi]$

(e) $T(x, y) = y \ln x - 5$; $R = [1, e]\times[1/e, 1]$

9. Find the *mass* of each of the following metallic sheets given the density function:

(a) $\delta(x, y) = x^2y^3$ kg/cm^2; $R = [0, 2]\times[0, 4]$

(b) $\delta(x, y) = x\, e^{x+y}$ g/cm^2; $R = [0, 2]\times[0, 9]$

(c) $\delta(x, y) = x \sin^2 y$ lb/ft^2 ; $R = [1, 3]\times[1, 9]$

10. Evaluate each of the following double integrals over the given region. Begin by sketching the region of integration. Identify whether the region is x-simple, y-simple, or neither.

$$(a)\quad \iint_R \sqrt{1-x^2}\,dA,\ \ R=\{(x,y): 0\le x\le 1,\ 0\le y\le x\}$$

$$(b)\quad \iint_R (x^3+2y)\,dA,\ \ R=\{(x,y): 0\le x\le 2,\ x^2\le y\le 2x\}$$

$$(c)\quad \iint_R xy\,dA,\ \ R=\{(x,y): -1\le x\le 2,\ -x^2\le y\le 1+x^2\}$$

$$(d)\quad \iint_R (1+2x+2y)\,dA,\ \ R=\{(x,y): 0\le y\le 1,\ y\le x\le 2y\}$$

$$(e)\quad \iint_R \frac{1}{xy}\,dA,\ \ R=\{(x,y): 1\le y\le e,\ y\le x\le y^2\}$$

$$(f)\quad \iint_R (\sin x - y)\,dA,\ \ R \text{ bounded by } y=\cos x,\ y=0,\ x=0,\ x=\frac{\pi}{2}$$

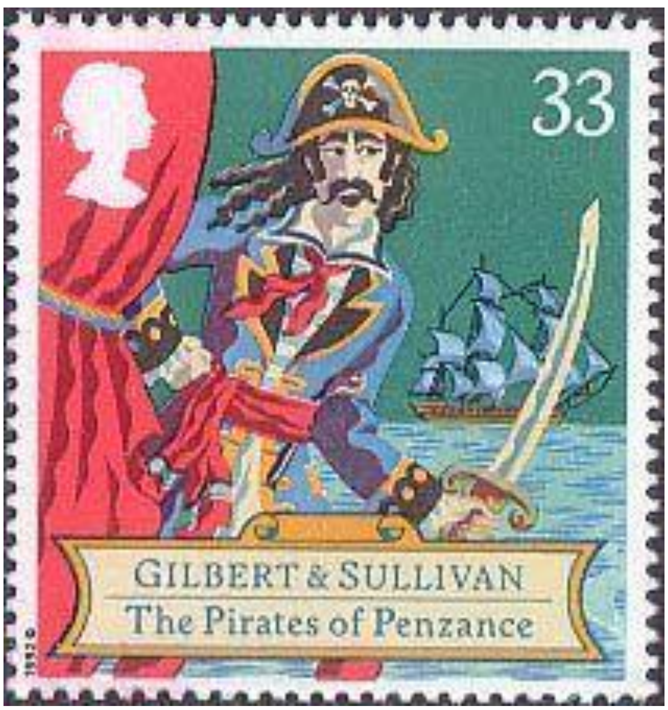
11. Evaluate each of the following iterated integrals by *reversing the order* of integration. (The first step is to identify the region of integration!)

$$(a)\quad \int_0^1\int_y^1 \frac{\sin x}{x}\,dx\,dy$$

$$(b)\quad \int_0^1\int_{2y}^2 e^{-x^2}\,dx\,dy$$

$$(c)\quad \int_0^4 \int_{\sqrt{x}}^2 \sin(y^3)\, dy\, dx$$

$$(d)\quad \int_0^4 \int_{\sqrt{y}}^2 \frac{1}{\sqrt{x^3+1}}\, dx\, dy$$



I'm very good at integral and differential calculus, I know the scientific names of beings animalculous; In short, in matters vegetable, animal, and mineral, I am the very model of a modern Major-General.

- Gilbert, W. S. (1836 - 1911) **The Pirates of Penzance**