PRACTICE FINAL-A

PART I (5 PTS EACH)

1. Find the distance between the two points P = (1, -1, 3) and Q = (2, 2, 4).

2. $(\mathbf{i} + \mathbf{k}) \times (\mathbf{j} + 3\,\mathbf{k}) =$

3. Let $\mathbf{a} = (\mathbf{i} + 2\,\mathbf{j} + 3\,\mathbf{k})$ and $\mathbf{b} = (-3\,\mathbf{i} + 5\,\mathbf{j} - 13\,\mathbf{k})$. Then $\mathbf{a} \cdot (\mathbf{a} \times \mathbf{b}) =$

4. Let *R* denote the annulus (or ring-shaped region) in the xy-plane, defined by $1 \le r \le 3$. Compute

$$\iint_R 5\,dA$$

5. Let $h(x, y, z) = 3xyz - (xy)^2 - (yz)^3 + 4(xz)^2$. Let *C* denote that portion of a helix given by $\boldsymbol{\sigma}(t) = (\cos \pi t, \sin \pi t, t)$ for $0 \le t \le 1$. Evaluate:

$$\int_C \nabla h \cdot d\vec{s}$$

6. Let *S* denote the sphere of radius 3 centered at the origin. Let ***F*** denote the 3-dimensional vector field given by $\mathbf{F}(x, y, z) = x\,\mathbf{i} + y\,\mathbf{j} + 3z\,\mathbf{k}$. Compute the flux integral

$$\iint_S F \cdot n\,dS$$

7. Let $\mathbf{G}(x,y) = x^3y\,\mathbf{i} + xy^3\mathbf{j}$. Is ***G*** a *conservative* vector field? Why?

8. Let *B* denote the unit ball centered at the origin. Suppose that the density of the ball is given by $\delta(x, y, z) = 3(x^2 + y^2 + z^2)$. Find the total mass of the ball. (*Hint:* Use spherical coordinates.)

9. Find a *potential function* for the vector field $\mathbf{F}(x, y) = (\sin y + 1)\,\mathbf{i} + (x \cos y - 1)\,\mathbf{j}$.

10. Consider the surface defined implicitly by the equation $xz^2 + yz + xyz = -3$. Find a unit vector that is normal to this surface at the point P = (1, 2, -1).

11. Evaluate

$$\int_{-1}^{1}\int_{0}^{1}\left(x^4y + y^2\right)dy\,dx$$

12. Compute

$$\frac{\partial}{\partial x}\left(x \ln y + yz \sin x - y \ln x\right)$$

13. Let $f(x, y) = \ln(x + 3y)$, and let $\mathbf{v} = 3\,\mathbf{i} + 4\,\mathbf{j}$. Find the *directional derivative* of *f* in the direction of ***v*** at the point Q = (1, 1).

14. Captain Odette finds herself on the sunny side of Mercury and notices that her spacesuit is melting. The temperature in a rectangular coordinate system in her vicinity is given by $T(x, y, z) = e^{-x} + e^{-2y} + e^{-3z}$.

If Odette is at the point P = (1, 1, 1), in which direction should she start to move in order to cool down most rapidly?

15. Let ***F*** be the vector field given by $\mathbf{F}(x, y, z) = x^2\mathbf{i} + xy^2\,\mathbf{j} + xyz^2\,\mathbf{k}$.

Compute div **F** at the point (1, 2, 3).

16. Suppose that Albertine tries to compute the volume of a cone, *V*, by measuring the radius, *r*, and height, *h*, of the cone. (Recall that $V = (\pi/3)r^2h$.) If Albertine can measure *r* with an accuracy of 1% and *h* with an accuracy of only 3%, what is the maximum percentage error in her computed value of *V*?

17. Let *D* be the unit disk centered at the origin. Evaluate

$$\iint_D e^{x^2+y^2}\, dA$$

(*Hint:* Use polar coordinates.)

18. Let $\mathbf{G}(x, y, z) = (x + y + z)\,\mathbf{i} + (x + 3y - 4z)\,\mathbf{j} + (2x - y + 3z)\,\mathbf{k}$. Compute curl **G**.

19. Let $f(x, y) = xy$, $x = u^2 - 3v^2$, and $y = u^2 + 4v^2$. Compute f_u when u = 3 and v = 2.

20. Suppose that a duck is swimming in a straight line $x(t) = 4 + 3t$, $y(t) = 3 - t$, while the water temperature is given by $T(x, y) = x^3 \sin y - y^2 \cos x$. Find dT/dt when t = 1.

21. Find and classify *all critical points* of the function $f(x, y) = x^3 - 3x + y^2 - 6y$.

PART II (10 PTS EACH)

1. Reverse the order of integration of the following integral. Do not evaluate!

$$\int_{1}^{2}\int_{0}^{\ln x}(x-3)\sqrt{1+e^{2y}}\,dy\,dx$$

(Be sure to make a sketch of the region of integration.)

2. Find the equation of the *tangent plane* to the surface $3z + \cos(\pi xyz) = 4x - y - 1$ at the point P = (1, 1, 1).

3. Let S be the rectangle in the xy-plane with vertices (0, 0), (2, 0), (0, 1), and (2, 1). Let C denote the boundary of S endowed with the positive orientation.

Let $\mathbf{F}(x,y) = xy\,\mathbf{i} + xy\,\mathbf{j}$. Using Green's Theorem, evaluate

$$\oint_{C} F \cdot ds$$

4. Compute the *volume* of the region lying below the saddle $z = 4xy$ and above the triangle in the xy-plane having vertices (0, 0), (0, 3), and (1, 1).

5. Verify Stokes' Theorem for $\mathbf{F}(x, y, z) = y\,\mathbf{i} + z\,\mathbf{j} + x\,\mathbf{k}$ and *S*, the paraboloid $z = 1 - (x^2 + y^2)$, $z \geq 0$, oriented upward.


MONACO
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$F = G \frac{mm'}{r^2}$
1687-1987
ENONCE DES LOIS SUR L'ATTRACTION UNIVERSELLE
Sir Isaac Newton