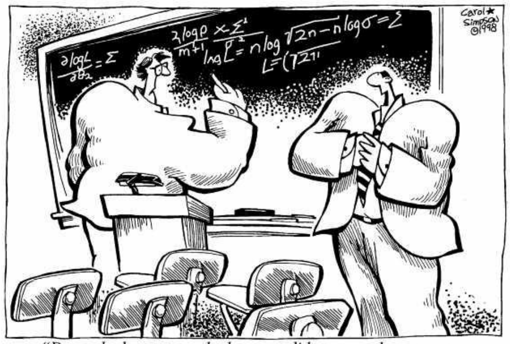
PRACTICE FINAL-B



"Due to budget cuts we had to consolidate some departments. You'll be teaching your advanced calculus class in French."

PART I (7 PTS EACH)

1. Find the radius of the sphere whose equation is given by:

   $x^2 - 2x + y^2 + 8y + z^2 = 19$.

2. $(\mathbf{i} + \mathbf{j} + k) \times (\mathbf{j} + 2\,\mathbf{k}) =$

3. Find the angle (to the nearest degree) between the two vectors $\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\mathbf{b} = \mathbf{i} + \mathbf{k}$.

4. Determine the distance between the origin and the plane $x + 2y + 3z = 6$.

5. Let *R* denote the annulus (or ring-shaped region) in the xy-plane, defined by $1 \le r \le 4$. Compute

$$\iint_R \left(x^2 + y^2\right) dA$$

6. Write a double integral that equals the volume of the tetrahedron bounded by the three coordinate planes and the plane $x + 4y + z = 8$. Do not evaluate.

7. Let $h(x,y,z) = \ln(x^2 + y + 3z + 1) + xyz$. Let *C* denote the curve given by $\boldsymbol{\sigma}(t) = (t, t(5t - 3), 5t^4 - 2t^3)$ for $0 \le t \le 1$. Evaluate the line integral:

$$\int_C \nabla h \cdot d\vec{s}$$

8. Find the length of the curve given by $\mathbf{r}(t) = (3 \cos t)\,\mathbf{i} + (3 \sin t)\,\mathbf{j} + 2t^{3/2}\,\mathbf{k}$, $0 \le t \le 3$.

9. Let *B* denote the ball of radius 2 centered at the origin and *S* the boundary of *B*. Let ***F*** denote the 3-dimensional vector field given by $\mathbf{F}(x, y, z) = x\,\mathbf{i} - y\,\mathbf{j} + z\,\mathbf{k}$. Using the Divergence Theorem, compute the flux integral:

$$\oiint_S F \cdot n \, dS$$

10. Find a potential function for the vector field

$$\mathbf{F}(x, y) = (3x^2 \sin y + 1 - y \sin x)\,\mathbf{i} + (x^3 \cos y + \cos x + 2y)\,\mathbf{j}$$

11. Consider the surface defined implicitly by the equation $xyz + 3z = x + 4$. Find a unit vector that is normal to this surface at the point P = (1, 2, 1).

12. Let $\mathbf{F} = yz\,\mathbf{i} + xz\,\mathbf{j} + xy\,\mathbf{k}$ and let *C* be the curve parameterized by: $\mathbf{r}(t) = t\,\mathbf{i} + t^2\,\mathbf{j} + t^3\,\mathbf{k}$, defined for $0 \le t \le 1$. Evaluate:

$$\int_C F \cdot d\vec{s}$$

13. Let $f(x, y) = 3y + \ln(x + y)$, and let $\mathbf{v} = 3\,\mathbf{i} + 4\,\mathbf{j}$. Find the directional derivative of *f* in the direction of ***v*** at the point Q = (1, 1).

14. In *which direction* is the directional derivative of $z = (x^2 - y^2)/(x^2 + y^2)$ at (1, 1) equal to zero?

15. Let *T* be given by the formula $T(x, y) = x(e^y + e^{-y})$. In conducting an experiment, Albertine finds that $x = 2$ with a maximum possible error of 0.1 and that $y = \ln 2$ with a maximum possible error of 0.02. Estimate the maximum possible error in the computed value of *T*.

16. Let $\mathbf{F} = (x+z)\,\mathbf{i} + x^2\,\mathbf{j} + y^2\,\mathbf{k}$ represent the velocity field of a fluid (velocity measured in meters/second). Compute how many cubic meters of fluid per second are crossing the xy-plane through the square $0 \le x \le 1$, $0 \le y \le 1$.

17. Suppose that a fish in the ocean is swimming according to the rule:

$$x(t) = 1 + 3t,\ y(t) = 3 + t,\ z(t) = \sin(2\pi t).$$

If the water temperature in degrees Fahrenheit at (x, y, z) is given by $T(x, y, z) = 76 + (0.01)(x \ln(y + z))$, find dT/dt when $t = 1$.

PART II (10 PTS EACH)

Answer any 6 of the following 8 problems. You may answer more than six for extra credit.

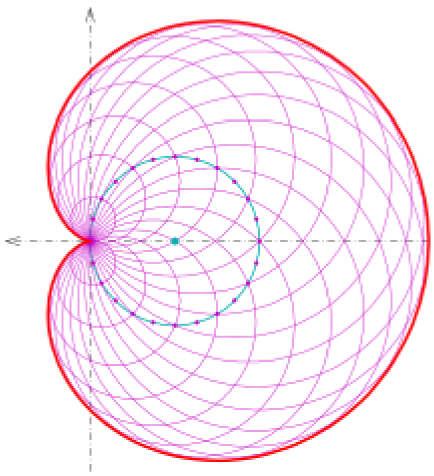
1. Use the surface (curl) integral in Stokes' Theorem to calculate the circulation of the field

$$\mathbf{F}(x, y, z) = (y^3 + z^4)\,\mathbf{i} + (x^2 + y^2)\,\mathbf{j} + (9x^2y^2 + z)\,\mathbf{k}$$

around the curve *C*, where *C* is the square bounded by $x = \pm 1$ and $y = \pm 1$ in the xy-plane, counterclockwise when viewed from above.

Hint: Begin by computing curl **F**.

2. Consider the cardioid pictured below. Click here for an animation.



This curve is parameterized by the equation $\boldsymbol{\sigma}(t) = 3(1 + \cos t)(\cos t)\,\mathbf{i} + 3(1 + \cos t)(\sin t)\,\mathbf{j}$, $0 \le t \le 2\pi$. Using *Green's area formula*, express the area of the cardioid as a Riemann integral.

Note: Do not evaluate the integral that you obtain.

3. *Reverse* the order of integration of the following integral. *Do not evaluate*. (Be sure sketch the region of integration.)

$$\int_0^{\frac{1}{16}} \int_{y^{\frac{1}{4}}}^{\frac{1}{2}} \cos(16x^5)\,dx\,dy$$

4. Find a vector of length 2 that is parallel to the line of intersection of the planes

$$x + 2y + z - 1 = 0 \text{ and } x + y + 2z + 7 = 0.$$

5. Using the method of Lagrange multipliers, find all points on the ellipse $x^2 + 2y^2 = 1$ where the function $f(x, y) = xy$ achieves a local maximum or minimum.

6. Find and classify all critical points of $z = x^3 + y^3 + 3x^2 - 3y^2$.

7. Let T be the triangle in the xy-plane with vertices (0, 0), (2, 0), and (0, 4). Let C denote the boundary of T endowed with the positive orientation. Let $\mathbf{F}(x, y) = xy^2\,\mathbf{i} + (x^2y + x^2)\,\mathbf{j}$. Using Green's Theorem, evaluate the circulation of $\boldsymbol{F}$ around C.

8. Find the *average value* of the function $g(x, y, z) = xyz$ over the cube bounded by the coordinate planes and the planes $x = 2$, $y = 2$, $z = 2$ in the first octant.