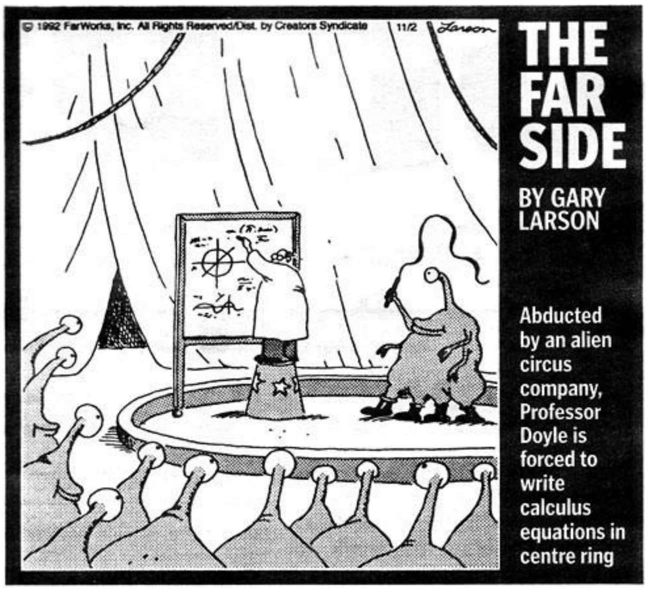
PRACTICE FINAL EXAM - C



PART I

1. (a) On an exam in the Abarat, Candy Quackenbush wrote the following answer:

   "The level curve of f(x, y) at the point (1, 2) is $x^2 - 2xy = 3$."

   This answer is wrong. How can you tell without knowing the formula for *f* ?

(b) Suppose that we know that f(1, 2) = 5, and the level curve of f(x, y) at the point (1, 2) is given by $x^2 - 2xy = -3$. Write down a possible formula for f(x, y).

2. The points A= (2, 2, 5), B = (6, -1, 2), and C = (1, 3, -4) are the vertices of a triangle in 3-space.
   (a) Which of the vertices is *closest to the xz-plane*?
   (b) Which of the vertices is *closest to the origin*?
   (c) Which is the *longest side* of the triangle, and what is its length?

3. Find (to the nearest degree) the *angle* between the planes x + y + z = 4 and x – 2y – z = 11.

4. Find (to the nearest hundredth) the *distance* between the lines

L(t) = (1, 2, 3) + t(4, 5, 6) and S(t) = (3, 1, 4) + t(4, 5, 6).

5. Find the equation of the plane that passes through the three points (1, 1, 1), (1, 0, 2) and (3, 1, 3).

6. Consider the paraboloid $z = x^2 + y^2$ and the sphere $x^2 + y^2 + z^2 = 6$. Do these surfaces intersect? If so, find the curve of intersection. If not, explain.

7. An object is pulled by a force $\boldsymbol{F_1}$ of magnitude 15 lb at an angle of 20 degrees north of east. In what direction must a force $\boldsymbol{F_2}$ of magnitude 10 lb pull to ensure that the object moves due east? (Express your answer in degrees to the nearest hundredth.)

8. Given the points P = (1, 2, 3), Q = (3, 5, 7), and R = (2, 5, 3), find the *area* of triangle PQR.

9. Consider a fly whose path is described by the equation:

$$\boldsymbol{\sigma}(t) = (2\cos 3t,\ t^2 + 1,\ 2\sin 3t).$$

Does this fly ever land upon or pass through the sphere of radius 2 centered at the origin? Justify your answer.

10. The tangent plane to the surface $x^2 - 2xy + az = 2$ at the point (1, 0, 1/a) is *parallel* to the plane $3x + by + 3z = 5$. What are the values of *a* and *b*?

11. Let $f(x, y, z) = x^3 + 3xz + 2yz + z^2$. Find the directional derivative of *f* at the point Q = (1, 1, 1) in the direction of the vector $v = -\mathbf{i} + \mathbf{j} - \mathbf{k}$.

12. Find and classify all the critical points of the function:

$$g(x,y) = xy + \frac{8}{x^2} + \frac{8}{y^2}$$

13. Find all of the critical points of $f(x, y) = (x^2 + y^2)e^x$ and classify each.

14. Find the maximum value achieved by $f(x, y) = x^2 + 3xy + y^2$ subject to the constraint $x^2 + y^2 = 5$.

15. Let *R* denote the portion of an annulus in the first quadrant of the xy-plane, defined by $1 \le r \le 4$, $x \ge 0$, $y \ge 0$. Compute

$$\iint_R x\, dA.$$

16. Let S denote the sphere of radius 2 centered at the point Q = (3, 2, 5). Let $\boldsymbol{F}$ denote the 3-dimensional vector field given by

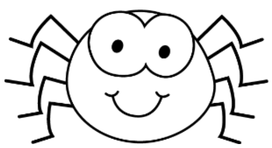
$\mathbf{F}(x, y, z) = (x + yz)\,\mathbf{i} + (x - 2y + z^2)\,\mathbf{j} + (xy + 3z)\,\mathbf{k}$.

Using the Divergence Theorem, compute the flux integral

$$\iint_S F \cdot d\vec{S}.$$

17. Find a *potential function* for the vector field

$$\vec{F}(x, y, z) = (y^2 - \frac{1}{x+z})\vec{i} + (2xy + 5)\vec{j} + (3z^2 - \frac{1}{x+z})\vec{k}$$

18.  A spider, finding herself in a toxic environment caused by chemical XBC, decides to move in a direction that will *decrease the concentration* of XBC the most rapidly. If the concentration of XBC is given by

$\mathbf{c}(x, y, z) = e^{-3x} + \sin(yz) + \exp(-z^2)$,

with the spider at (0, 0, 0), in which direction should she move?

19. The position of a particle is given by $\boldsymbol{\sigma}(t) = (t - \sin t)\,\mathbf{i} + (1 - \cos t)\,\mathbf{j}$, $0 \le t \le 2\pi$.

Find the time or times in the given time interval when the velocity and acceleration vectors are perpendicular to each other.

PART II

Answer any 7 of the following 9 problems. You may answer more than seven for extra credit.

1. *Sketch the region of integration* of each of the following double integrals. Do *not evaluate* the integrals.

$$(a)\ \int_0^1 \int_0^{(1-x^2)^{1/2}} \ln(x+y)\, dy\, dx \qquad (b)\ \int_0^1 \int_{y^2}^{y} \exp(xy)\, dx\, dy$$

2. Let $\boldsymbol{F}$ be the vector field $\mathbf{F}(x, y, z) = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$. Let C be the closed curve parameterized by:

$\boldsymbol{\sigma}(t) = (\cos t + \sin t)\,\mathbf{i} + \cos^3 t\,\mathbf{j} + \sin^4 t\,\mathbf{k}$, for $0 \le t \le 2\pi$.

Using Stokes' Theorem, compute the circulation of $\boldsymbol{F}$ around C.

3. Let $\boldsymbol{F}(x, y, z)$ be a vector field satisfying: $\text{curl}\,\mathbf{F} = axyz\,\mathbf{i} + (by^2z + byz)\,\mathbf{j} + 3z^2\,\mathbf{k}$, where a and b are constants.

Determine the values of *a* and *b*.

4. *Let* $F(x,y,z) = 2xy^2 z\exp(x^2y^2z)\vec{i} + 2x^2yz\exp(x^2y^2z)\,\vec{j} + x^2y^2\exp(x^2y^2z)\,\vec{k}$

Compute $\int_C F$ where C is the curve parameterized by $\sigma(t) = (t, t^2, t^3)$, for $0 \le t \le 1$.

5. Using Green's Theorem, compute

$$\oint_C (5 - xy - y^2)\,dx + (x^2 - 2xy)dy$$

where C is the boundary of the square with vertices (0, 0), (1, 0), (1, 1), and (0, 1).

6. Express as a double integral the volume under the surface $z = xy^2$ that lies above the triangle in the xy-plane with vertices (0, 0), (0, 5), (3, 1).

You need not evaluate the integral.

7. Evaluate the following integral by changing to polar coordinates:

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2\,dy\,dx}{(1+x^2+y^2)^2}$$

8. In a neighborhood clinic, the number of patient visits can be described as a function of the number of doctors, *x*, and the number of nurses, *y*, by $f(x, y) = 1000x^{0.6}y^{0.3}$.

With upcoming budget cuts, the clinic must reduce the number of doctors at the rate of 2 per month. Estimate the rate at which the number of nurses has to be increased in order to maintain the current service.

Currently there are 30 doctors and 50 nurses. (*Hint:* Use the chain rule.)

9. For each of the following regions, $\mathbb{R}$, in the xy-plane, give the pre-image of the region under the given transformation. ***Sketch both the region in the xy-plane and its pre-image in the uv-plane.***

(a) The region $\mathbb{R}$ is defined by the inequality $x^2/4 + y^2/25 \le 1$. The transformation is given by $u = x$, $v = 2y/5$.

(b) The region $\mathbb{R}$ is bounded by the line $x + y = 2$ and the two coordinate axes. The transformation is given by $u = y - x$, $v = y + x$.

(c) The region R is a parallelogram with vertices (1, 1), (4, 1), (3, 5) and (6, 5). The transformation is given by $u = 2x + y$ and $v = x - 3y$.