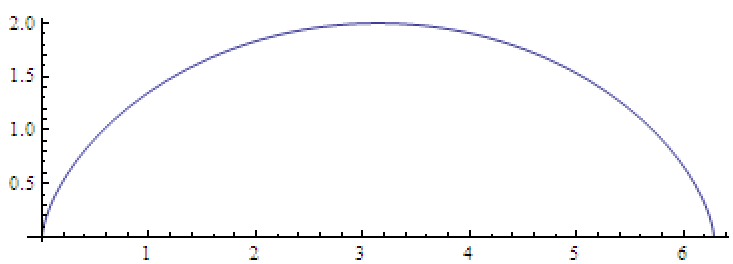
PRACTICE FINAL EXAM - D

1) Use a line integral to evaluate the area of the region bounded by the x-axis and one arch of the cycloid with parametric equation:

$$x = a\,(t - \sin t)\ , \quad y = a\,(1 - \cos t) \quad where \ \ 0 \le t \le 2\pi$$



2) Let C be the curve given by $\boldsymbol{\sigma}(t) = 4t\ \mathbf{i} + 3t\sin(t^3\pi/2)\ \mathbf{j} - (1 - 4t^{5/2})\ \mathbf{k}$, $0 \le t \le 1$.

3) Evaluate the line integral $\int_C 2xyz\ dx + x^2 z\ dy + x^2 y\ dz$

4) Evaluate the surface integral $\iint_R z\ dS$ where R is given by

$\Phi(u,v) = (u+v)i + u\ j + (u-v)k$ and $0 \le u \le 2,\ 0 \le v \le \pi$.

5) A cone-shaped lamina R is given by $z = a\left(a - \sqrt{x^2+y^2}\right),\ 0 \le z \le a^2$. At each point on S, the density is proportional to the distance between the point and the z-axis.

Sketch the cone-shaped lamina and find its mass.

6) Find the flux over the sphere S centered at the origin with radius a, given that $\vec{F}(x,\ y,\ z) = \dfrac{kq\vec{r}}{\|r\|^3}$

where $\mathbf{r} = x\ \mathbf{i} + y\ \mathbf{j} + z\ \mathbf{k}$.

Assume that S is directed outward. (This results in *Gauss's Law*, a basic principle of electrostatics.)

(*Note:* This vector field is not defined at the origin.)

7) Let R be the surface defined by $z = 9 - x^2 - y^2$, $z \ge 0$. Let $\mathbf{F}(x, y, z) = (z - y)\ \mathbf{i} + (x - z)\ \mathbf{j} + (x - y)\ \mathbf{k}$. Verify Stokes' Theorem.

8) Let V be the volume of a solid bounded by a closed surface R. Let $\mathbf{F}(x, y, z) = x\ \mathbf{i} + y\ \mathbf{j} + z\ \mathbf{k}$.

Why does $\iint_R \vec{F}\cdot\vec{n}\ dS = 3V$?

9) Use the Divergence Theorem to evaluate the flux integral $\iint_R \vec{F}\cdot\vec{n}\ dS$ given that $\mathbf{F}(x, y, z) = xyz\ \mathbf{j}$ and R is the surface bounded by $x^2 + y^2 = 4$, $z = 0$, $z = 5$.

10) Find the global max and min of $f(x, y) = x^2 + 2y^2 - 2x + 3$.

11) Maximize $f(x, y) = (6 - x^2 - y^2)^{1/2}$ subject to the constraint $x + y - 2 = 0$.

12) Evaluate the double integral $\iint_R 5(x^2+y^2)\ dA$ where R is the region bounded by the square with vertices (0, 0), (1, 1), (5, 0), (4, -1). Use the change of variables theorem:

$x = ½ (u + v),\ y = ½ (u - v)$

13) Using an appropriate change of variables, evaluate $\iint_R \sqrt{(x-y)(x+4y)}\, dS$ where R is the region bounded by the parallelogram with vertices (0, 1), (0, -1), (1, 0), (-1, 0).

14) Evaluate the following double integral by first switching the order of integration:

$$\int_0^1 \int_{\frac{y}{2}}^{\frac{1}{2}} e^{-x^2}\, dx\, dy$$

15) Let *W* be the region in 3-space satisfying $x + y + z \leq 1,\ x \geq 0,\ y \geq 0,\ z \geq 0$. Find the *average value* of $g(x, y, z) = y$ on the region *W*.

16) Let $\mathbf{G}(x, y, z) = (x^2, -yz, z - xz)$.

(a) Compute the divergence and curl of **G**.

(b) Show that **G** is neither the gradient of a function nor the curl of a vector field.

17) Let R be the region bounded by the square with vertices (0, 1), (1, 2), (2, 1) and (1, 0). We wish to evaluate the following integral over the region R.

$$\iint_R (x+y)^2 \sin^2(x-y)\, dA_{xy}$$

(a) Consider the change of variables $u = x + y$ and $v = x - y$. Find the pre-image of R under this transformation. (Sketch R in the xy-plane and the pre-image of R in the uv-plane.)

(b) Find the *Jacobian* of this transformation.

(c) Using the *Change of Variables Theorem*, evaluate the given integral.

M. C. Escher, **Möbius Strip**