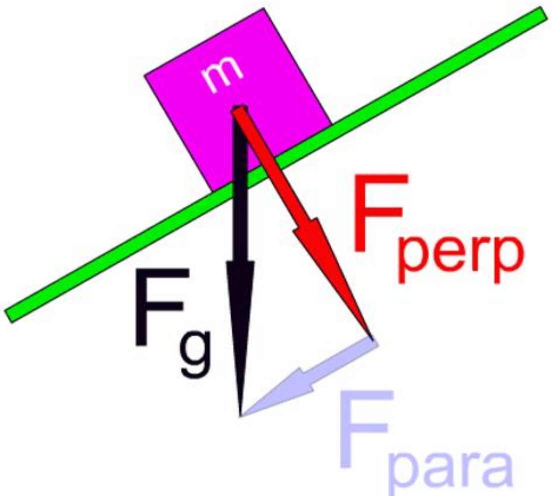
MATH 263 PRACTICE PROBLEMS FOR TEST I

(REVISED 7 FEB)

1. Compute each of the following:
   (a) A unit vector normal to the plane x – 3 y + 5 z = 0.
   (b) A vector perpendicular to each of the vectors **i** – **j** + **k** and **i** + **j** + **k**.
   (c) The angle between the vectors 2 **i** + **j** + **k** and – **i** + **k**.
   (d) The area of the triangle whose vertices are (-1, -1,- 1), (-1, 0 ,1) and (1, 0, -1).
2. Let **a** and **b** be any vectors in 3-space. What is the value of **a** · (**a** × **b**)? Explain!
3. Suppose that an object moving in the xy-plane in the direction **i** + **j** is acted upon by a force given by the vector 2 **i** + **j**. Express this force as a sum of a force in the direction of motion and a force perpendicular to the direction of motion.



4. Determine an equation of the plane containing the points A = (1, 2, 3), B = (3, 5, 7), and C = (2, 5, 3).
5. Determine the (shortest) distance from the point P = (1,3,-1) to the plane 2x + y + z =1.
6. Find the length of the curve defined by the vector-valued function

$$r(t) = (2\sqrt{t}, e^{t}, e^{-t}) \text{ for } 0 \le t \le 1$$

7. Write an equation of a plane that passes through the point (4, 9, 8) and has normal vector **i** +3 **j** + 5**k**.

8. Does the following function, defined for all (x, y) other than (0, 0), possess a limit as (x,y) approaches (0, 0) ? Justify your answer!

$$f(x,y) = \frac{x^4 y}{x^2 + y^2}$$

9. Verify that the circumference of the unit circle is 2π.

10. Consider the pair of planes $z = x + y$ and $z = 2x + y$.

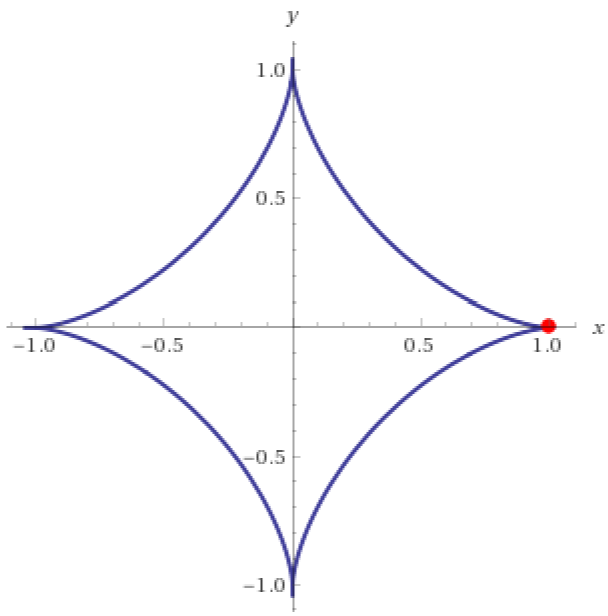
(a) Find a vector *parallel* to the line of intersection of the two planes.
(b) Find the *angle* between the two planes.

11. A bird is flying with velocity $v = 12\mathbf{i} + 3\mathbf{j}$ (measured in meters/sec) relative to the air. The wind is blowing at a speed of 4 meters/sec parallel to the x-axis but opposing the bird's motion.

(a) Find the velocity vector of the bird relative to the ground.

(b) Find the speed of the bird relative to the ground.

12. The curve below is parametrized by $x(t) = \cos^3(t)$, $y(t) = \sin^3(t)$ on the interval $0 \le t \le 2\pi$. Find its arc length.



13. Find the *distance* between the pair of lines:

$$L_1(t) = (1,0,1) + t\,(3,0,4) \quad \text{and} \quad L_2(t) = (2,5,0) + t\,(1,1,2).$$

14. (a) Find the center and radius of the sphere whose equation is:

$$x^2 + 8x + y^2 + 6y + z^2 - 2z = 11$$

(b) Find the *domain* of the function

$$f(x,y) = \ln\frac{x-2y}{x^2+y^2} + \sqrt{1-x^2-y^2}$$

Is this domain bounded? open? closed?

15. Find the distance between the two skew lines:

$L_1(t) = (1,0,1) + t(2,1,0)$ and
$L_2(t) = (0,2,3) + t(5,1,3)$.

16. Find the tangent, normal and binormal vectors for $r(t) = \langle t, 3\sin t, 3\cos t\rangle$

17. Find the speed of the particle whose position is $r(t) = (2t, t^3, 1 + 5t)$ at t = 1.
At t = 1, write the velocity vector at t as the speed times the unit tangent vector.
Find an equation of the tangent line to r(t) at t = 1.

18. Find the distance between the parallel lines L(t) = (1, 1, 1) + t (2, 3, 4) and

L(t) = (9, 1, 5) + t (2, 3, 4).

19. A force $\mathbf{F} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ newtons is applied to a spacecraft with velocity vector $\mathbf{v} = 3\mathbf{i} - \mathbf{j}$. Express **F** as a sum of a vector parallel to **v** and a vector perpendicular to **v**.

20. Consider the points P = (1, 2, 1), Q = (3, 4, -2) and R = (-1, 2, 2).

(a) Find a vector of magnitude 5 perpendicular to each of **PQ** and **PR**.
(b) Find the equation of the plane which contains the three points P, Q, and R.
(c) Find the angle between the vectors **PQ** and **PR**.

21. Find the distance between the two planes x + 2y + 6z = 1 and x + 2y + 6z = 10.

22. Let $g(x,y) = 1 + \cos\left(\dfrac{2\pi}{x^2 + y^2}\right)$.

(a) Find the domain of *g*. Explain.
(b) Find the range of *g*. Explain.
(c) Describe or draw the level curves of *g*.
(d) Does the limit of *g* exist as (x, y) approaches (0, 0)? Explain.

23. (a) Let G(x,y) be given by:

$$G(x,y) = \frac{\sqrt{4 - 3x - y}}{5 + \sqrt{x} + \sqrt{y}}$$

What is the *domain* of G?

(b) Let H(x, y) be given by:

$$H(x,y) = \frac{1}{\ln(x^2 + y^2 + e)}$$

What is the *range* of H?

24. Where does the line L(t) = (1, 2, 4) + t (3, 1,t) intersect the plane x – 2y + z = 11?

How can it be that mathematics, being after all a product of human thought independent of experience, is so admirably adapted to the objects of reality?

\- Albert Einstein