MATH 263 PRACTICE PROBLEMS FOR TEST II

11 MARCH 2019

1. (a) Find any and all *critical points* of $f(x, y) = -3x^2 - 4xy - y^2 - 12y + 16x$.

(b) Classify each (and every) critical point.

(c) Is there a *global maximum* of $z = f(x, y)$ in the first quadrant,

$x \geq 0, y \geq 0$? If so, find it; if not explain why.

2. Suppose that a function is given in terms of rectangular coordinates,

$w = f(x, y, z)$. $\quad$ *If* $x = \rho\cos\theta\sin\emptyset$, $y = \rho\sin\theta\sin\varphi$, *and* $z = \rho\cos\varphi$, *use the chain rule to express*

$$\frac{\partial w}{\partial \rho}, \quad \frac{\partial w}{\partial \theta}, \quad \text{and} \quad \frac{\partial w}{\partial \emptyset} \text{ in terms of } \frac{\partial w}{\partial x}, \quad \frac{\partial w}{\partial y} \text{ and } \frac{\partial w}{\partial z}.$$

3. Odette wishes to find the *global maximum and minimum* values of the function

$f(x, y) = x^3 - y^2$ on the closed disk $x^2 + y^2 \leq 4$.

(a) State the theorem that guarantees the existence of the global max and global min.

(b) Find the points at which the global maximum and minimum occur and compute *their values. Sketch.*

4. Albertine's utility function for x units of item A and y units of item B is given by

$$f(x, y) = 6x^{\frac{1}{3}}y^{\frac{1}{2}}.$$

Each unit of item *A* costs \$80, and each unit of item *B* costs \$30. What choice of *x* and *y* will *maximize* her utility function if she spends \$200?

5. In a neighborhood clinic, the number of patient visits, *N*, per month can be modeled by a function of the number of doctors, *x*, and the number of nurses, *y*, according to the formula:

$$N(x, y) = 1000x^{0.6}y^{0.3}.$$

With upcoming federal budget cuts, the clinic must reduce the number of doctors at the *rate* of 2 per month. Estimate the *rate* at which the number of nurses has to be increased to maintain the current service (i.e., maintain the same number of patient visits). Currently, there are 30 doctors and 50 nurses. (*Hint: Use the chain rule.)*

6. Find the Lagrange multiplier equations for the point of the surface below at which x is largest. (Do not solve.)

$$x^4 + y^4 + z^4 + xy + yz + zx = 6$$

7. Let $T = f(x, y)$ be the temperature in degrees Celsius at a point (x, y) in the xy-plane (where x and y are measured in cm). Suppose that f has the following properties:

$$f(5,3) = 38,\ f_x(5,3) = 3, \text{ and } f_y(5,3) = -2.$$

Charlotte, the spider, finds herself at the point R = (5, 3).

(A) In which direction should she move to cool off as quickly as possible? (Express your answer in vector form.)

(B) What is her rate of cooling in the direction that you found in part (A)? (Use appropriate units.)

(C) In which direction should she move to remain at the same temperature? (Express your answer in vector form.)

(D) If Charlotte were to move in the direction **i + j**, would she be warming up or cooling off? At what rate? (Use appropriate units.)

8. Consider the surface:

$$z = f(x, y) = x \cos y - ye^{x}.$$

(A) Find the equation of the tangent plane to this surface at the point (0, 0).

(B) Using your result from part (A), estimate the value of f(0.02, -0.03).

9. Consider the surface $\cos(x + y) = e^{xz+2.}$ Check that the point Q = (-1, 1, 2) lies on the surface and, viewing the surface as a level surface of a function of three variables, find the equation of the tangent plane to the surface at Q.

10. Given z = f(x, y), x = g(u, v), y = h(u, v), and g(1, 2) = 5, h(1, 2) = 3, calculate $z_u(1, 2)$ in terms of some of the numbers *a, b, c, d, e, k, p, q* where

$f_x(1, 2) = a$, $f_y(1, 2) = c$, $g_u(1, 2) = e$, $h_u(1, 2) = p$, $f_x(5, 3) = b$, $f_y(5, 3) = d$, $g_v(1, 2) = k$, $h_v(1, 2) = q$.

11. Let $g(x,y) = x(x+y)^3$. Show that $g_{xy} = g_{yx}$.

12. The lengths *x*, *y*, *z* of the edges of a closed rectangular box are changing with time.

(A) What is the equation of the volume *V* of the box? Of the surface area *S* of the box?

(B) At time = 5 minutes, x = 1 m, y = 2 m, z = 3 m, dx/dt = 1 m/sec, dy/dt = 1 m/sec, and dz/dt = -3 m/sec. At what rates are *V* and *S* changing at t = 5 minutes? (Show your work.)

(C) Is the major diagonal of the box increasing or decreasing at t = 5 minutes? (Justify your answer.)

13. Let $g(x,y) = 1 + \cos\left(\frac{2\pi}{x^2+y^2}\right)$.

(A) Find the domain of *g*. Explain.

(B) Find the range of *g*. Explain.

(C) Describe or draw the level curves of *g*.

(D) Does the limit of *g* exist as (x,y) approaches (0,0)? Explain.

14. An experiment to measure the toxicity of formaldehyde yielded the data shown in the table below. The values show the percent, P = f(t, c), of rats surviving exposure to formaldehyde at a concentration of *c* (in parts per million, ppm) after t months. Estimate $f_t(18, 6)$ and $f_c(18, 6)$. *Using complete sentences, interpret each answer in terms of formaldehyde toxicity.*

Time t (months)

c (ppm) \ t (months)	**14**	**16**	**18**	**20**	**22**	**24**
0	100	100	100	99	97	95
2	100	99	98	97	95	92
6	96	95	93	90	86	80
15	96	93	82	70	58	36

(A) $f_t(18, 6)$ = ________

Meaning: ____

(B) $f_c(18, 6)$ = ________

Meaning: ____

15. The energy, *E*, of a body of mass m moving with speed *v* is given by the formula

$$E = mc^2\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right)$$

The speed, *v*, is non-negative and less than the speed of light, *c*, which is a constant.

(A) Find E_m. What would you expect the sign of E_m to be? Explain!

(B) Find E_v. What would you expect the sign of E_v to be? Explain!

16. To make different people comparable in studies of cardiac output, researchers divide the measured cardiac output by the body surface area to find the *cardiac index* C;

$$C = (\text{cardiac output}) / (\text{body surface area}).$$

The body surface area *B* of a person with weight *w* and height *h* is approximated by the formula

$$B = 71.84 w^{0.425} h^{0.725},$$

which gives *B* in square centimeters when *w* is measured in kilograms and *h* in centimeters. You are about to calculate the cardiac index of a person with the following measurements:

Cardiac output:	7 liters / min
Weight:	70 kg
Height:	180 cm.

Which will have a greater effect on the calculation: a 1 kg error in measuring the weight or a 1 cm error in measuring the height? Explain your reasoning! (Use calculus in your solution.)

17.

Calculate the directional derivative of $f(x, y) = x^2 + y^2$ at $(1, 0)$ in the direction of the vector $\vec{i} + \vec{j}$.

18.

In Exercises 32–37, use the contour diagram of f in Figure 14.34 to decide if the specified directional derivative is positive, negative, or approximately zero.

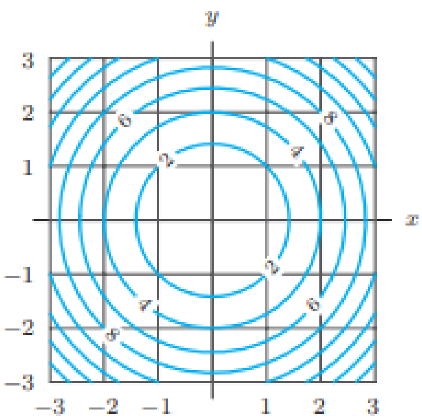


Figure 14.34

32. At point $(-2, 2)$, in direction $\vec{i}$.

33. At point $(0, -2)$, in direction $\vec{j}$.

34. At point $(0, -2)$, in direction $\vec{i} + 2\vec{j}$.

35. At point $(0, -2)$, in direction $\vec{i} - 2\vec{j}$.

36. At point $(-1, 1)$, in direction $\vec{i} + \vec{j}$.

37. At point $(-1, 1)$, in direction $-\vec{i} + \vec{j}$.

19.

45. Match the functions $f(x, y, z)$ in (a)–(d) with the descriptions of their gradients in (I)–(IV). No reasons needed.

(a) $x^2 + y^2 + z^2$ **(b)** $x^2 + y^2$

(c) $\dfrac{1}{x^2 + y^2 + z^2}$ **(d)** $\dfrac{1}{x^2 + y^2}$

I Points radially outward from the z-axis.
II Points radially inward toward the z-axis.
III Points radially outward from the origin.
IV Points radially inward toward the origin.

46. Find the equation of the tangent plane at $(2, 3, 1)$ to the surface $x^2 + y^2 - xyz = 7$. Do this in two ways:

(a) Viewing the surface as the level set of a function of three variables, $F(x, y, z)$.
(b) Viewing the surface as the graph of a function of two variables $z = f(x, y)$.

20.

Consider the functions $f(x, y) = 4 - x^2 - 2y^2$ and $g(x, y) = 4 - x^2$. Calculate a vector perpendicular to each of the following:

(a) The level curve of f at the point $(1, 1)$ (b) The surface $z = f(x, y)$ at the point $(1, 1, 1)$
(c) The level curve of g at the point $(1, 1)$ (d) The surface $z = g(x, y)$ at the point $(1, 1, 3)$

21.

Let $f(x, y) = \sqrt{x + 2y + 1}$.

(a) Compute the local linearization of f at $(0, 0)$.
(b) Compute the quadratic Taylor polynomial for f at $(0, 0)$.
(c) Compare the values of the linear and quadratic approximations in part (a) and part (b) with the true values for $f(x, y)$ at the points $(0.1, 0.1)$, $(-0.1, 0.1)$, $(0.1, -0.1)$, $(-0.1, -0.1)$. Which approximation gives the closest values?

Figure 14.71 gives a contour diagram for the number n of foxes per square kilometer in southwestern England. Estimate $\partial n/\partial x$ and $\partial n/\partial y$ at the points A, B, and C, where x is kilometers east and y is kilometers north.

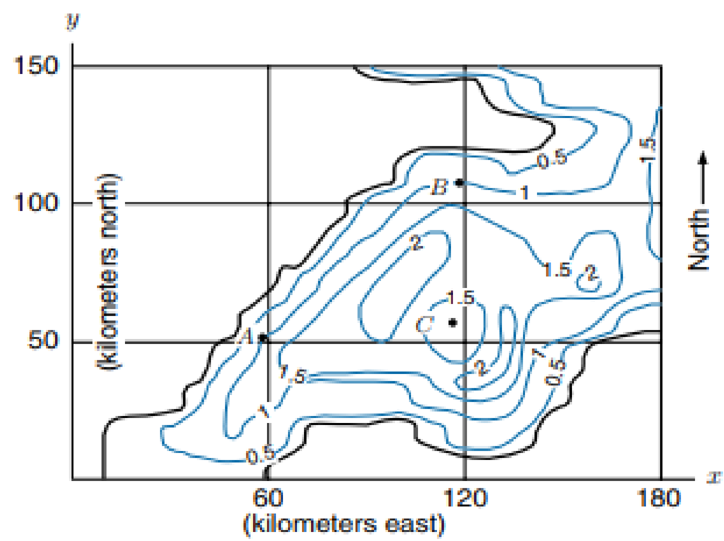


Figure 14.71

22.

Design a rectangular milk carton box of width w, length l, and height h which holds 512 cm^3 of milk. The sides of the box cost 1 cent/cm^2 and the top and bottom cost 2 cent/cm^2. Find the dimensions of the box that minimize the total cost of materials used.

23.

Decide whether each point appears to be a maximum, minimum, or neither for the function f constrained by the loop in Figure 15.30.

(a) P **(b)** Q **(c)** R **(d)** S

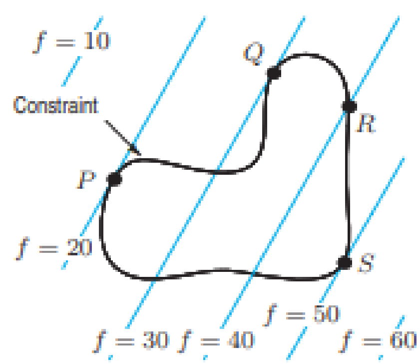


Figure 15.30

24.

For Exercises 7–10, find the local maxima, local minima, and saddle points of the function. Decide if the local maxima or minima are global maxima or minima. Explain.

7. $f(x, y) = 10 + 12x + 6y - 3x^2 - y^2$

8. $f(x, y) = x^2 + y^3 - 3xy$

9. $f(x, y) = x + y + \dfrac{1}{x} + \dfrac{4}{y}$

10. $f(x, y) = xy + \ln x + y^2 - 10, \qquad x > 0$

25.

For Exercises 11–22, use Lagrange multipliers to find the maximum and minimum values of f subject to the constraint.

11. $f(x, y) = 3x - 4y, \quad x^2 + y^2 = 5$

12. $f(x, y) = x^2 + y^2, \quad x^4 + y^4 = 2$

13. $f(x, y) = x^2 + y^2, \quad 4x - 2y = 15$

26.

Figure 15.40 shows contours labeled with values of $f(x, y)$ and a constraint $g(x, y) = c$. Mark the approximate points at which:

(a) $\operatorname{grad} f = \lambda \operatorname{grad} g$
(b) f has a maximum
(c) f has a maximum on the constraint $g = c$.

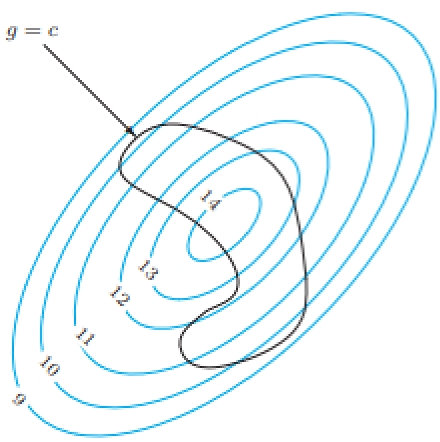
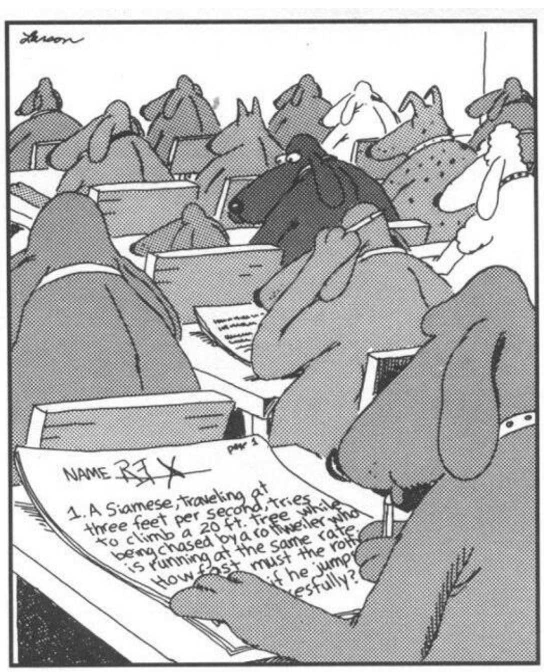


Figure 15.40



Before their admission to any canine university, dogs must first do well on the CATs.