MATH 263 SOLUTIONS: QUIZ I 25 JANUARY 2019

1. *[8 pts]* Let $v = 2i + j - k$ and $w = i + j - k$.

Compute $v \times w$.

Solution: *Using the definition of cross product:*

$$\boldsymbol{v} \times \boldsymbol{w} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 2 & 1 & -1 \\ 1 & 1 & -1 \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 1 & -1 \end{vmatrix} \mathrm{i} - \begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix} \mathrm{j} + \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} \mathrm{k} =$$

$$0\mathrm{i} - \big(-2 - (-1)\big)\mathrm{j} + (2 - 1)k = j + k$$

2. *[8 pts]* Find the *center* and *radius* of the sphere $x^2 + y^2 + z^2 + 6x - 8y + 4z = 0$.

Solution: *Rearranging terms, we obtain*

$$x^2 + 6x + y^2 - 8y + z^2 + 4z = 0.$$

Completing the squares:

$$(x + 3)^2 + (y - 4)^2 + (z + 2)^2 = 9 + 16 + 4$$

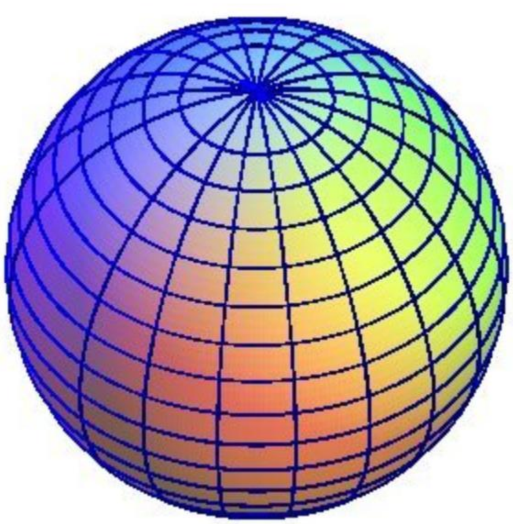
which simplifies to

$$(x + 3)^2 + (y - 4)^2 + (z + 2)^2 = 29$$

Hence:

center = (-3, 4, -2)

radius = $\sqrt{29}$



3. *[6 pts]* Find *one point* on the plane $5x - 3y + 4z = 1$.

Solution: *Here we use the method of "judicious guessing".*

Letting $x = y = 0$, we obtain $z = \frac{1}{4}$. Thus $P = (0, 0, \frac{1}{4})$ lies on the plane.

4. *[8 pts]* Find the angle between the two vectors $v = i - 2j + 2k$ and $w = 3j + 4k$. Express your answer to the nearest tenth of a degree.

Solution: *We use the Dot Product Theorem:*

$$v \cdot w = \|v\| \|w\| \cos\theta$$

Hence

$$\cos\theta = \frac{v \cdot w}{\|v\| \|w\|} = \frac{0 - 6 + 8}{\sqrt{9}\sqrt{25}} = \frac{2}{15}$$

And so $\theta = \arccos\frac{2}{15} \approx 82.3°$

The real voyage of discovery consists not in seeking new landscapes but in having new eyes.

- Marcel Proust