MATH 263 SOLUTIONS: QUIZ II 15 FEBRUARY 2019

1. Let $z = y^5 \cos x + x^{413} \sin x + e^{x+y} + 2019$. Find $z_{xxxyyyyx}$. Show your work!

Solution:

$z_{xxxyyyyx} = z_{yyyyxxxx} = (z_y)_{yyyxxxx} = (5y^4 \cos x + e^{x+y})_{yyyxxxx} =$

$(5 \cdot 4y^3 \cos x + e^{x+y})_{yyxxxx} = (5 \cdot 4 \cdot 3\, y^2 \cos x + e^{x+y})_{yxxxx} =$

$(5 \cdot 4 \cdot 3 \cdot 2\, y \cos x + e^{x+y})_{xxxx} = (-5 \cdot 4 \cdot 3 \cdot 2\, y \sin x + e^{x+y})_{xxx} =$

$(-5 \cdot 4 \cdot 3 \cdot 2 \cos x + e^{x+y})_{xx} = (5 \cdot 4 \cdot 3 \cdot 2 \sin x + e^{x+y})_{x} =$

$5! \cos x + e^{x+y}$

2. *Let* $z = \cos(xy^3) + \ln(2019x + y) + x^{2019}$. *Compute*:

(a) z_x

Solution: $z_x = -y^3 \sin(xy^3) + \frac{2019}{2019x+y} + 2019\, x^{2018}$.

(b) z_y

Solution: $z_y = -3xy^2 \sin(xy^3) + \frac{1}{2019x+y}$

3. Let $z = g(x, y) = (xy - 2x^3 + 2)^3$.

Find an equation of a *level curve* of *g* that passes through the point Q = (1, 2).

Solution: *Every level curve of this function must be of the form g(x, y) = constant.*

So $(xy - 2x^3 + 2)^3 = c$ is the most general level curve of g.

If this curve passes through Q, then $(2 - 2 + 2)^3 = c$, or c = 8.

So the curve that we seek is $(xy - 2x^3 + 2)^3 = 8$, or $xy - 2x^3 + 2 = 2$. (Here we must choose the *positive* square root else Q does not lie on the curve.)

4. Let $z = f(x, y) = x^8y^3$.

Find an equation of the *tangent plane* to this surface at the point P = (1, 2, 8).

Solution: Differentiating yields $z_x = 8x^7y^3$ and $z_y = 3x^8y^2$.

Thus $z_x(P) = 64$ and $z_y(P) = 12$. So an equation of the tangent plane to the surface at P is:

$$\mathbf{z - 8 = 64(x - 1) + 12(y - 2)}$$

or

$$\mathbf{z = 64x + 12y - 80}$$

5. *Matching:* For each of the six functions below, write the letter of the graph that best represents it. (*You will not be penalized for guessing.)*

(1) $z = x^2 - y^2$ saddle: E

(2) $z = x^2 + y^2 + 1$ paraboloid: C

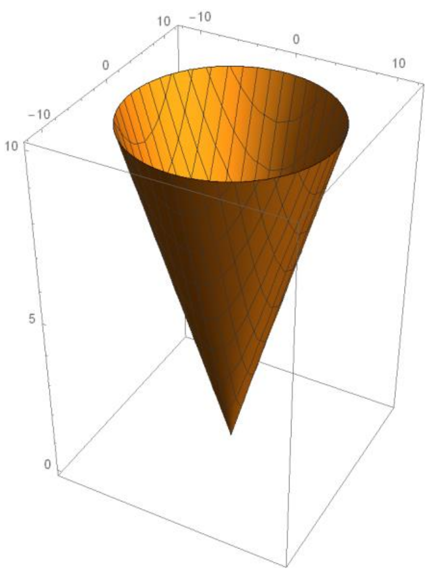
(3) $z = e^{-\frac{1}{4}(x^2+y^2)}$ "cousin" of normal distribution: F

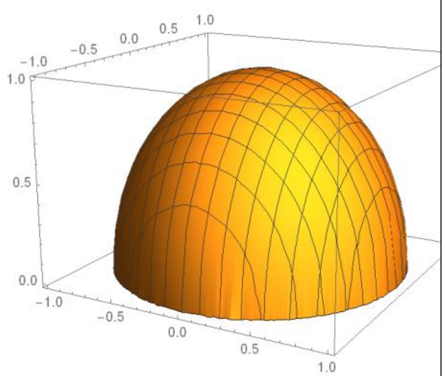
(4) $z = 3x + y - 1$ plane: D

(5) $z = \sqrt{1 - x^2 - y^2}$ hemisphere: B

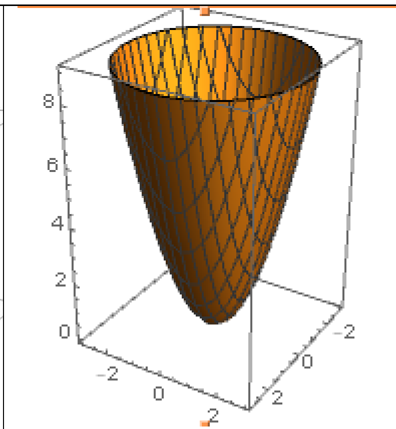
(6) $z = \sqrt{x^2 + y^2}$ cone: A

A

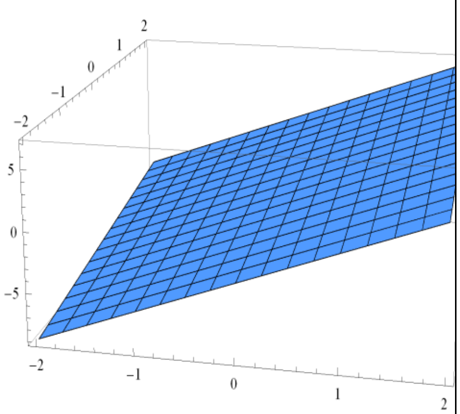




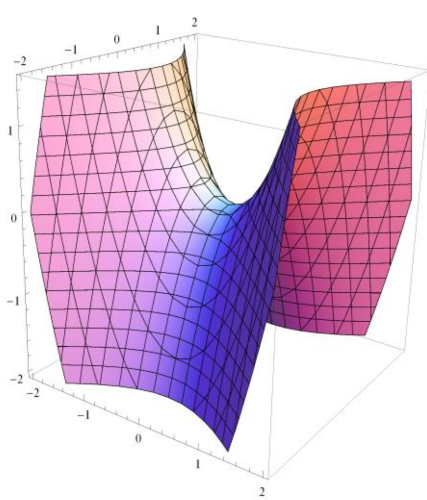
B



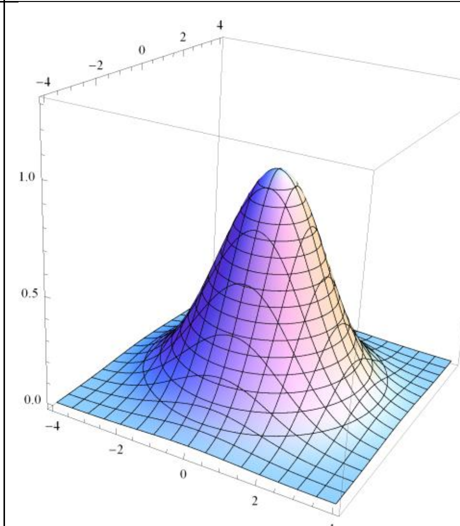
C



D



E



F

Extra Credit: The surface area S of a human body (in square meters) in terms of its weight W (in kg) and height H (in cm) can be modeled by the formula

$$S = 0.007184W^{0.425}H^{0.725}$$

(a) Compute $\partial S/\partial W$ when W = 70 and H = 180.

Solution:

$$S_W(W,H) = 0.007184(0.4250)W^{-0.575}H^{0.725}$$

$$S_W(70,180) = 0.007184(0.4250)(70)^{-0.575}(180)^{0.725} = 0.01145$$

meters^2 / kg

(b) Compute $\partial S/\partial H$ when W = 70 and H = 180.

Solution:

$$S_H(W,H) = 0.007184(0.725)W^{0.425}H^{-0.275}$$

$$S_H(70,180) = 0.007184(0.725)(70)^{0.425}(180)^{-0.275} = 0.00759$$

meters^2 /cm.

(c) Assume that Albertine weighs 70 kg and is 180 cm in height. Find the approximate *change* in Albertine's surface area if she were to lose 2.3 kg and gain 1.1 cm in height? *Use only the results* of (a) and (b) in this estimation.

Solution:

$$\Delta S = S_W(70,\ 180)\Delta W + S_H(70,\ 180)\Delta H =$$

$$0.01145\,\Delta W + 0.00759\,\Delta H =$$

$$0.01145(\text{-}2.3) + 0.00759(1.1) = -0.0179\ m^2$$

This is a ***loss of 0.0179 meters^2***.

I don't know what it is I don't know,
and yet am supposed to know,
and I feel I look stupid
if I seem both not to know it
and not know what it is I don't know.
Therefore I pretend I know it.

- R. D. Laing, **Knots**