MATH 263 SOLUTIONS: QUIZ III 22 FEBRUARY 2019

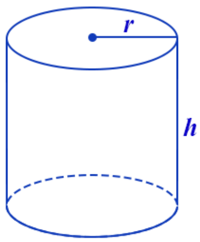
1. The radius of a right circular cylinder is increasing at the rate of 0.3 cm/sec while its height is decreasing at the rate of 0.4 cm/sec. Find the rate at which the volume is changing when the radius of the cylinder is 60 cm and its height is 130 cm. Is the volume increasing or decreasing at that moment?

Solution:

$V = \pi r^2 h$ may be regarded as a function of t, since each of *r* and *h* is a function of *t*.

Using the Chain Rule:

$$dV/dt = (\partial V/\partial r)(dr/dt) + (\partial V/\partial h)(dh/dt) = 3240\,\pi\,\frac{\text{cm}^3}{sec} \approx 10{,}178.8\,\frac{\text{cm}^3}{sec}$$



2. While climbing a mountain near Chamonix, Albertine notices that her oxygen mask has begun to leak. If the surface of the mountain is given by $z = 5 - x^2 - 2y^2$ km. and she is currently located at P = (1/2, -1/2, 17/4), in which *direction* should Albertine turn to *descend* most rapidly? (Express your answer as a *two-dimensional unit* vector.)

Solution:

*To **descend** most rapidly, Albertine should move in the direction of* $-\nabla z$. *Expressed as a unit vector this equals:*

$$-\frac{\nabla z}{\|\nabla z\|}\Big|_P = -\left(\frac{-2x\vec{i} - 4y\,\vec{j}}{\|\nabla z\|}\right)\Big|_P = \frac{i - 2j}{\sqrt{5}}$$

3. Find an equation of the *tangent plane* to the surface $xyz + x^3 + y^3 + z^3 = 14$ at P = (3,-2, 1).

Solution: Let $G(x, y, z) = xyz + x^3 + y^3 + z^3$.

Then $\nabla G = (yz + 3x^2)i + (xz + 3y^2)j + (xy + 3z^2)k$.

At P = (3, -2, 1), $\nabla G = (-2 + 3(9))i + (3 + 3(4))j + (-6 + 3(1))k = 25i + 15j - 3k$.

Hence, the tangent plane to the given surface at P is given by:

$$\mathbf{25(x-3) + 15(y+2) - 3(z-1) = 0}$$

4. Suppose that $T(x, y) = x^2 + 2y^2 - x$ is the temperature (in degrees Celsius) at the point (x, y). Assume that distance is measured in meters. If you are standing at the point Q = (-2, 1) and decide to proceed in the direction of the point P = (1, -3), will the temperature be increasing or decreasing at the moment you begin? At what rate?

Solution: QP =(1, -3) – (-2, 1) = (3, -4).

Now $\nabla T = (2x - 1)i + 4yj$. At Q = (-2, 1), $\nabla T = -5i + 4j$.

Now, the unit vector representing PQ is: $u = \frac{3i-4j}{5}$.

Hence, at P, $D_u T = \nabla T \cdot u = \frac{-15-16}{5} = -\frac{31}{5}\frac{℃}{meter}$. And so, the temperature is *decreasing* at Q in the direction of u at the rate of $-\mathbf{\frac{31}{5}\frac{℃}{meter}}$

There are many paths to the top of a mountain, but the view is always the same.

- Chinese proverb