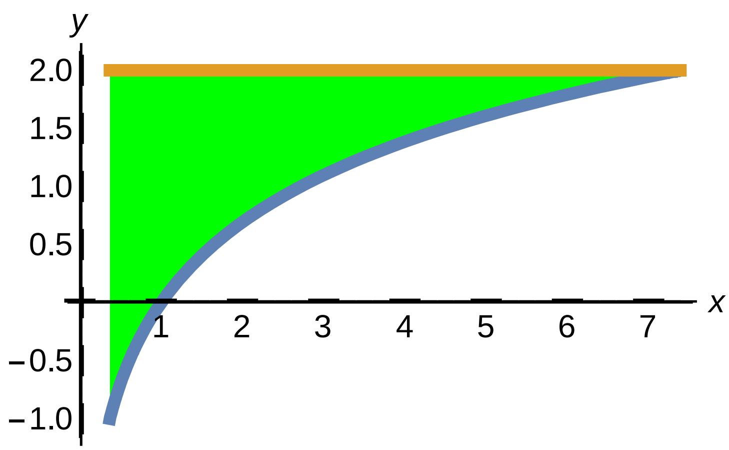
MATH 263 SOLUTIONS: QUIZ IV 22 MARCH 2019

1. Sketch the *region* of integration of the following iterated integral. Be certain that your sketch captures all the essentials (i.e., label curves and all relevant points of intersection).

$$\int_{\frac{1}{e}}^{e^2} \int_{\ln x}^{2} f(x, y)\, dy\, dx$$

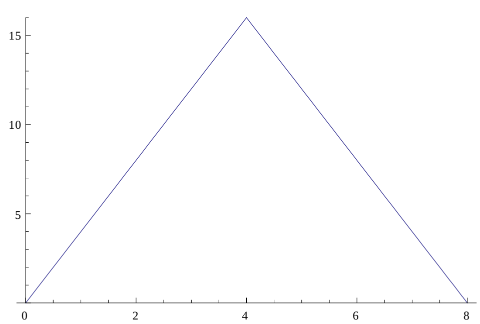
Solution:



2. Express as an *iterated double integral* the *volume* of the solid that is bounded above by the plane x + 3y – z + 5 = 0 and below by the triangle in the xy-plane having vertices (0, 0), (4, 16) and (8, 0). *Sketch the region of integration. Do not evaluate.*

Solution:

The function that defines the plane is given by: z = f(x, y) = x + 3y + 5.



The equations of the non-horizontal sides of the triangles are: y = 4x and *y = -4x + 32. Since the is a x-simple region, we should fix y and integrate with respect to x. Hence the volume beneath the plane that lies above the given triangle is given by:*

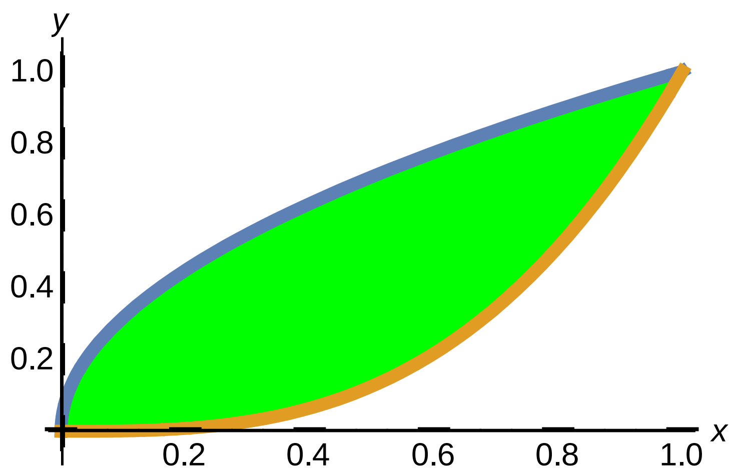
$$\int_0^{16} \int_{\frac{y}{4}}^{8-\frac{y}{4}} (x+3y+5)\ dx\,dy$$

3. Let g(x, y) be a continuous function defined on the xy-plane. Reverse the order of integration of the following iterated integral:

$$\int_0^1 \int_{x^3}^{\sqrt{x}} g(x,y)\ dy\,dx\ .$$

Be certain to sketch the region of integration!

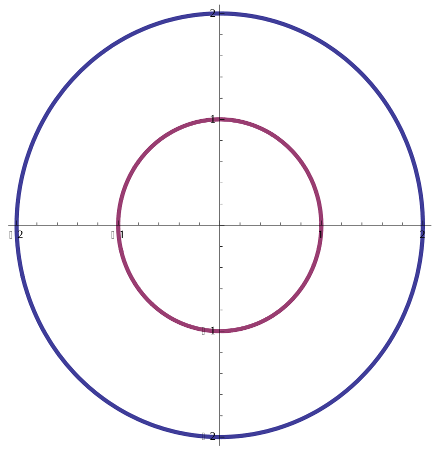
Solution:



$$\int_0^1 \int_{y^2}^{y^{\frac{1}{3}}} g(x,y)\ dx\,dy$$

4. The function $G(x, y) = c(x^2 + y^2 + 1)$ has an *average value* of 7 on the annulus $1 \le x^2 + y^2 \le 4$. Determine the value of the constant c. Hint: Convert to polar coordinates.

Solution:



The area of the annulus is $\pi(2)^2 - \pi(1)^2 = 3\pi$. *Switching to polar coordinates, recall that* $x^2 + y^2 = r^2$ *and that* $dx\,dy = r\,dr\,d\theta$. *Using the definition of average value:*

$$Average\,value = 7 = \frac{1}{area\ of\ annulus}\iint_R G(x,y)\,dA =$$

$$\frac{1}{3\pi}\int_{\theta=0}^{2\pi}\int_{r=1}^{r=2} c(r^2+1)\,r\,dr\,d\theta = \frac{c}{3\pi}\int_{\theta=0}^{2\pi}\int_{r=1}^{r=2}(r^3+r)\,dr\,d\theta = \frac{c}{3\pi}\int_{\theta=0}^{2\pi}\left(\frac{r^4}{4}+\frac{r^2}{2}\right)\Bigg|_{r=1}^{r=2} d\theta =$$

$$\frac{c}{3\pi}\int_{\theta=0}^{2\pi}\left(6-\frac{1}{4}-\frac{1}{2}\right)d\theta = \frac{c}{3\pi}\frac{21}{4}\int_{0}^{2\pi} 1\,d\theta = \frac{7c}{4\pi}(2\pi-0) = \frac{7c}{2}$$

Thus c = 2.

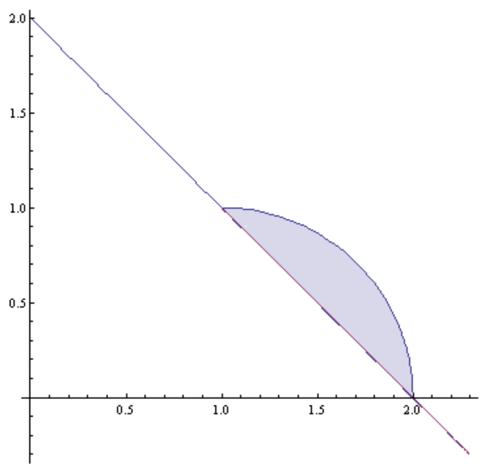
EXTRA CREDIT:

Reverse the order of integration of the following iterated integral. Be certain to *sketch* the region of integration. *Do not evaluate.*

$$\int_1^2 \int_{2-x}^{\sqrt{2x-x^2}} \ln(1+x^2+y^2)\,dy\,dx$$

Solution:

Here is the region of integration. Note that $y = (2x - x^2)^{1/2}$ is the upper half of the circle of unit radius centered at (1, 0).



Hence:

$$\int_1^2 \int_{2-x}^{\sqrt{2x-x^2}} \ln(1+x^2+y^2)\,dy\,dx = \int_0^1 \int_{2-y}^{1+\sqrt{1-y^2}} \ln(1+x^2+y^2)\,dx\,dy$$


WORLD WATER DAY
22ND MARCH