

**Class Discussion: 21 January 2020**

**Rates of change, continued**

8. Table 1.12 gives the populations of two cities (in thousands) over a 17-year period.
- (a) Find the average rate of change of each population on the following intervals:
- (i) 1996 to 2006            (ii) 1996 to 2013  
(iii) 2001 to 2013
- (b) What do you notice about the average rate of change of each population? Explain what the average rate of change tells you about each population.

**Table 1.12**

Year	1996	1998	2001	2006	2013
$P_1$	42	46	52	62	76
$P_2$	82	80	77	72	65

Exercises 10–14 use Figure 1.17.

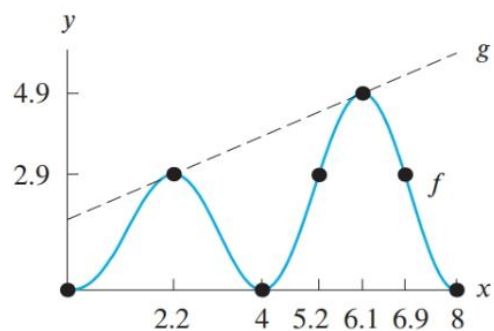


Figure 1.17

10. Find the average rate of change of  $f$  for  $2.2 \leq x \leq 6.1$ .
  11. Give two different intervals on which  $\Delta f(x)/\Delta x = 0$ .
  12. What is the average rate of change of  $g$  between  $x = 2.2$  and  $x = 6.1$ ?
  13. What is the relation between the average rate of change of  $f$  and the average rate of change of  $g$  between  $x = 2.2$  and  $x = 6.1$ ?
  14. Is the rate of change of  $f$  positive or negative on the following intervals?
17. Figure 1.18 shows the percent of the side of the moon toward the earth illuminated by the sun at different times during the year 2008. Use the figure to answer the following questions.
    - (a) Give the coordinates of the points  $A$ ,  $B$ ,  $C$ ,  $D$ ,  $E$ .
    - (b) Plot the point  $F = (15, 60)$  and  $G = (60, 15)$ . Which point is on the graph?
    - (c) During which time intervals is the function increasing?
    - (d) During which time intervals is the function decreasing?

% of face of moon  
toward earth illuminated

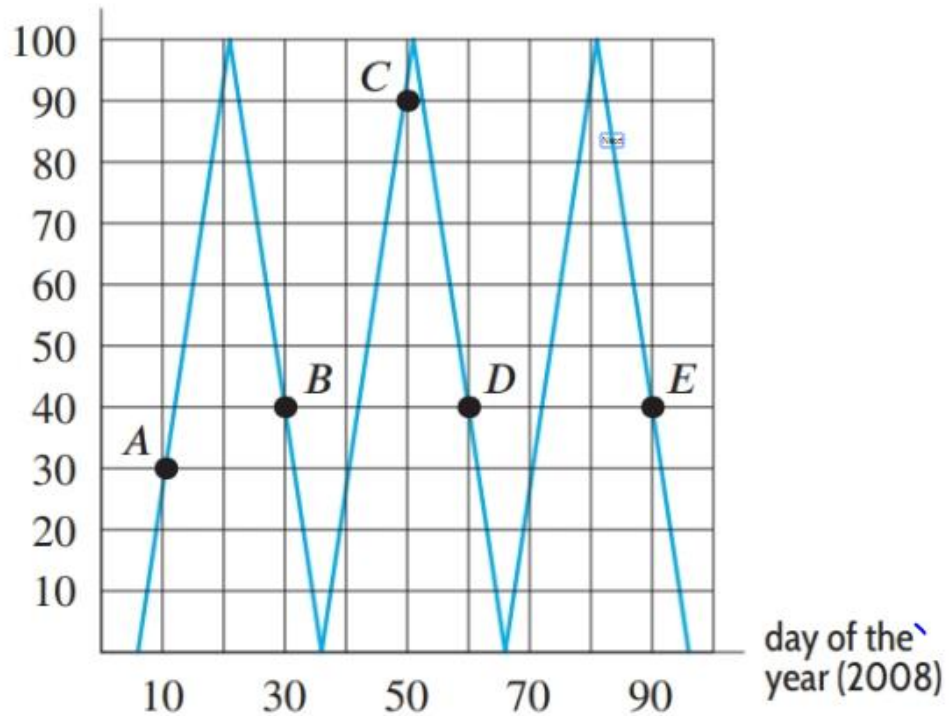


Figure 1.18: Moon phases

Section 1.4 For the function in Exercises 1 – 4 below, state the units for the average rate of change,

$$\frac{f(b) - f(a)}{b - a}$$

1. The population, in people, of a city,  $P = f(t)$ , is a function of the number of years,  $t$ , since 2010.

ANSWER ⊕

WORKED SOLUTION ⊕

2. The number of gallons of gas in a car,  $g = f(m)$ , is a function of the number of miles driven,  $m$ .

3. The number of smartphones,  $N = f(p)$ , purchased is a function of the price  $p$ , in dollars, of the smartphone.

ANSWER ⊕

4. The cost,  $C = f(w)$ , in dollars of buying a chemical is a function of the weight bought,  $w$ , in pounds.

■ In Exercises 5-7, let  $g(t)$  give the market value (in \$1000s) of a house in year  $t$ . What does the statement say about the house?

5.  $g(5) - g(0) = 30$

ANSWER ⊕

WORKED SOLUTION ⊕

6.  $\frac{g(10)-g(4)}{10-4} = 3$

7.  $\frac{g(20)-g(12)}{20-12} = -1$

■ Find the average rate of change of  $g(x) = 2x^3 - 3x^2$  on the interval in Problems 15-18.

15. Between 1 and 3.

ANSWER ⊕

16. Between  $-1$  and 4.

17. Between 0 and 10.

ANSWER ⊕

WORKED SOLUTION ⊕

18. Between  $-0.1$  and  $0.1$ .

19. The value in dollars of an investment  $t$  years after 2010 is given by

$$V = 1000 \cdot 2^{t/6}.$$

Find the average rate of change of the investment's value between 2011 and 2014.

ANSWER ⊕

20. Atmospheric levels of carbon dioxide ( $\text{CO}_2$ ) have risen from 336 parts per million (ppm) in 1979 to 396 parts per million (ppm) in 2014.

21. Sea levels were at a low point about 22,000 years ago.<sup>7</sup> Since then they have risen approximately 120 meters. Find the average rate of ch

■ In Exercises 8-10, let  $s(t)$  give the number of acres of wetlands in a state in year  $t$ . Assuming that the area of wetlands goes down over time

8.  $s(25) - s(0) = -25,000$

9.  $\frac{s(20) - s(10)}{20 - 10} = -520$

ANSWER ⊕

WORKED SOLUTION ⊕

10.  $s(30) - s(20) < s(20) - s(10)$

## PROBLEMS

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■ Find the average rate of change of  $f(x) = x^2 + 3x$  on the interval in Problems 11-14.

11. Between 2 and 4.

ANSWER ⊕

12. Between  $-2$  and 4.

13. Between  $-4$  and  $-2$ .

ANSWER ⊕

WORKED SOLUTION ⊕

14. Between 3 and 1.